

Optimized Comb Filter: Design and Analysis

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ABSTRACT

This paper intends to optimize the decimation filter design of delta-sigma A/D converter. The design focuses on optimization of transfer function of generalized comb filter. A novel design for placing zeroes of this transfer function in the best possible position by optimizing zero's angle is proposed. In this process, genetic algorithm is used. The optimization is done with the aim of increasing Delta Sigma Quantization noise rejection around folding bands. Folded bands refer to the frequency intervals in which cases of quantization noise are folded inside the base band because of the decimation process. The designed filter rejects the most part of the quantization noise, so the following filtering stages could be designed with some simple specifications. To this end a comparison is made between optimized comb filter, classic comb and generalized comb filters. The comparisons are particularly done in the terms of quantization noise power and pass band drop and selectivity.

KEYWORDS: Analog to digital converter, delta-sigma converter, cascade integrator comb filter, decimation filter, sinc filter, genetic algorithm.

1. INTRODUCTION

Delta-sigma analog to digital converters are widely used in electronics industry. These converters are made up of two main sections, sigma-delta modulator and digital decimation filter, which reduces the sampling rate (Fig. 1). The decimation filter decreases oversampling signal sampling rate into the Nyquist rate [1, 2].

One of the issues involved in the decimation process, is realization decimation filters in sigma delta converters, on which several research papers have already been written.

In practical situations, the decimation filter is made use of in the form of simple sinc filter. Although, this filter structure is very simple, but it does not guarantee a satisfactory frequency response [3]. There are two ways to solve such a problem. The first, optimizing the sinc filter by adding compensator filters to it. The second involves designing the optimal filter with a sinc base. The objective of this paper is second method.

Of the methods presented in the previous papers, some have optimized filter stop band attenuation. Among these, one may refer to the method of imposing the zero position of the filter transfer function in specific points. In this method, an increase in the stop band attenuation is done using high-order filters, itself simultaneously increases the circuit complexity and computational load [3]. With this in mind, the recommended approach is to realization decimation filter by releasing zeros position in the transfer function. In this method it is possible to optimize the zeros position so better stop band attenuation obtained [4].

The purpose of this paper it is to increase delta-sigma quantization noise around the folded bands compared to classical comb filter order N. The first attempts in this connection is presented in studies by Hogenauer [5] and Candy [6]. In reference [4] has proposed a third-order modified sinc filter. Reference [7] proposes a new two-stage sharpened comb decimator. Reference [8], proposes a scheme for multi-stage decimation filter, has been used to Cyclotomic polynomials. Reference [3], the author proposes another decimation filter structure based on comb filters. Reference [9] has designed certain types of Kaiser and Hamming filter based on a comb filter. Reference [10] has suggested a simple method to compensate for the decimation filter. A new decimation design in delta-sigma analog to digital converters based on Kaiser and Hamming sharpened filters have been proposed in Reference [11], and reference [12] filters with high order have been generalized. Other relevant literature are given in the references [13 -17].

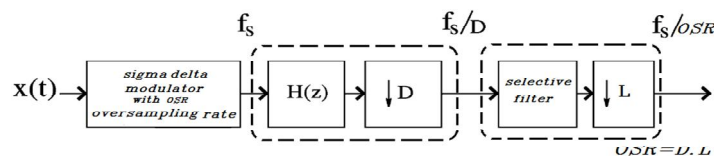


Figure 1. The two-stage decimation filter

The present paper is organized as follows. Section 1 is an introduction about comb filter. In section two, the performance principles filters and genetic algorithms and the reason of its use, are briefly explained. In

section three, new comb filter is proposed. The section four presents how to measure the performance of the filters. The section five deals with the results and compares them. In the final section the conclusion is presented.

2. PERFORMANCE PRINCIPLES OF COMB FILTER

Comb filter as a decimation filter and at the same time anti-aliasing, in the analog to digital converters are used. The oversampled signal can be decimated, with two-stage decimation or more [1], and here the two-stage decimation is used. The first stage is comb filter order N, which decimates by rate reduction parameter D and a finite impulse response filter constitutes the second stage, and can be decimated by rate reduction parameter L[15].

2.1 Folded bands

Anti-aliasing comb filter in the first stage decimation is used because the quantization noise that falls inside the folded bands should be attenuated. Folded bands, refer to the frequency intervals in which cases of quantization noise are folded inside the base band because of the decimation process (Fig. 2)[15].

As mentioned in reference [12] the range of frequency bands is $[(k/D) - f_c; (k/D) + f_c]$, the k value for the even D is $k = 1, \dots, [D/2]$ and for the odd D is $k = 1, \dots, [(D-1)/2]$. Due to reduction in the sampling rate by D in the first stage of decimation, delta sigma quantization noise (that has fallen into frequency bands) folded into the base band and after a few stages from the chain of decimation, extremely affects the accuracy of the signal. This is especially significant as long as quantization noise in the previous stages (which is folded in the base band) had already not been attenuated. As a result, the first stage decimation in the multi-stage structure is usually implemented using a comb decimation filter, placing the zeros in the middle of the folded band, to provide inherent anti-aliasing function [15].

Analog input signal with maximum frequency f_x is sampled by Delta-sigma analog to digital converter order B with rate f_s which much more than $2f_x$. Oversampling ratio (OSR), is defined as $OSR = D.L = f_s/2f_x$.

$f_c = f_x/f_s = 1/2OSR$ as input signal normalized maximum frequency is introduced. Digital signal components in the first input decimation filter belong to $f_d \in [-f_c, f_c]$, in which f_d stands for digital frequency. N, is a comb filter order and B modulator order must be greater than or equal to $N + 1$ which is $B \geq 1$ [1].

Before presenting the main idea of this paper, it seems appropriate comb filter transfer function of order N to refer to [5]:

$$H_{CN}(z) = \left(\frac{1 - z^{-D}}{D(1 - z^{-1})} \right)^N = \frac{1}{D^N} \prod_{i=1}^{D-1} \left(1 - z^{-1} e^{j\frac{2\pi}{D}i} \right)^N \quad (1)$$

Fig. 2 shows a comb filter in which specified intervals around the zeros, represent folded bands. It is noteworthy that in the classic comb filter, zeros are located exactly at the center of folded bands [1 & 2].

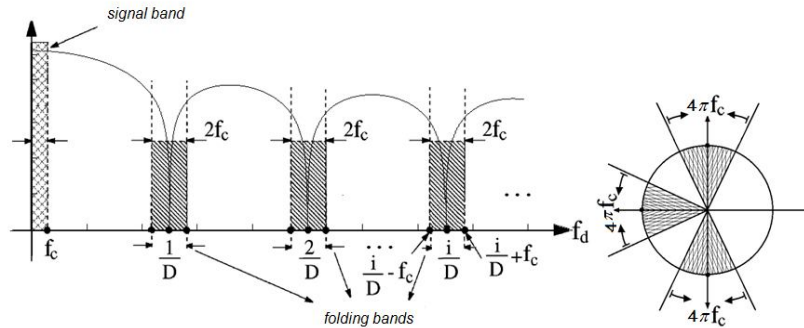


Figure 2. Folded bands In the a classical comb filter

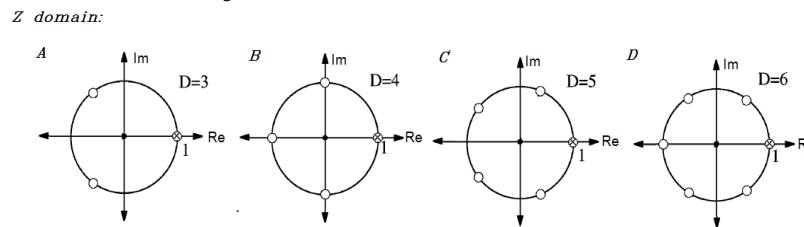


Figure 3. Zeros position for classic comb filter for N = 1.

Fig. 3 shows the position of classic comb filter zeros in the Z domain for $N = 1$ and $D = 1, 2, 3, 4$. Note that zero in the $z = 1$ has been simplified by pole $(1 - z^{-1})$ in the equation (1), [15]. For $N > 1$, the position of zeros does not change. Only with an increase in N , each zero will be of order N . Now the rejection of sigma-delta quantization noise around folded bands compared to classical comb filters must be increased. Because the quantization noise reduction in the folded bands is important, focus will be on reducing the noise power in these bands. The frequency range, except the folded bands and base-band, are called don't care bands. This means that, in these frequency bands, the quantization noise by selective filter (located at the end of the down converter chain) is removed.

2.2 Genetic Algorithm

Let's begin this section with a question. For what reasons genetic algorithm has been used for optimization? The answer: because there are significant differences between the genetic algorithm and most of conventional searches and optimization techniques that are as follows:

- Firstly, genetic algorithm, will search simultaneously with a set of points not with a point.
- Secondly, genetic algorithm follows the probability rules and not the natural ones.
- Thirdly, genetic algorithm acts on a set of properties coded, not on their main values (Except in cases where the real display in strings is used).
- Fourthly, genetic algorithm does not require any kind of derivative or any additional information and only the objective function and way of fitting the raw data, has to determine the direction search.
- Fifthly, genetic algorithm offers a set of potential answers and in cases where no single answer to the question such as multi-objective optimization, genetic algorithm would be useful to simultaneously determine the answers [18].

Some of the definitions used in the genetic algorithm are as follows:

Mutation

Mutation introduces new genetic structures in the population by randomly modifying some of its building blocks. It helps escape from local minima's trap and maintains diversity in the population. Mutation of a bit involves flipping a bit, changing 0 to 1 and vice versa [17].

Crossover

Crossover is the process of taking two parent solutions and producing from them a child. After the selection (reproduction) process, the population is enriched with better individuals. Reproduction makes clones of good strings but does not create new ones.

Elitism

The first best chromosome or the few best chromosomes are copied to the new population. The rest is done in a classical way. Such individuals can be lost if they are not selected to reproduce or if crossover or mutation destroys them. This significantly improves the genetic algorithms performance.

Selection

Selection is the process of choosing two parents from the population for crossing. The purpose of selection is to emphasize fitter individuals in the population in hopes that their off springs have higher fitness. Chromosomes are selected from the initial population to be parents for reproduction.

Fitness

The fitness of an individual in a genetic algorithm is the value of an objective function for its phenotype. For calculating fitness, the chromosome has to be first decoded and the objective function has to be evaluated. The fitness not only indicates how good the solution is, but also corresponds to how close the chromosome is to the optimal one.

Inversion

Inversion operator is a primary natural mechanism to recode a problem. In inversion operator, two points are selected along the length of the chromosome, the chromosome is cut at those points and the end points of the section cut, gets reversed [17].

Genetic algorithms can be called a general search method that follows biological evolution rules. This algorithm is one of the methods of search and optimization which deals with a large collection of answers and due to this characteristic, it provides parallel execution of algorithms and migration between the subsets which result in genetic diversity. Genetic algorithm is the prime example evolutionary methods. This algorithm encodes potential solutions in the form of chromosomes. Implementation of genetic algorithms begins with creating an initial population of chromosomes (answers) that are randomly selected. Genetic algorithm, imposes the best survival law on a group of answers, hoping to get a better answer [17]. In every generation, with the help of the selection process, according to the value of answers and reproduction of the selected answers and with the aid of operations that have followed natural genetics better approximations of the final answer are achieved. This process causes the new generations to be more compatible with the conditions. Each of those populations which are approximations of the final answer, are encrypted as strings (chromosomes) of letters or

numbers. The most common mode is displayed with the numbers 0 and 1. Then a fitting is allocated to each. Fitting is a relative scale which signifies fitness of the individuals for the next generation. Once the fitting of individual members of all the population were identified, each with the probability proportional to their fitness amount can be selected to produce the next generation [19].

The proliferation is used in the genetic algorithm for information exchanged between a pair or more individuals. Following stages of proliferation and mutation, chromosomes are decoded and an objective function value is calculated. Now, if necessary, proliferation and mutation and selection process will be performed again. During this process, the average performance of answers, are expected to increase. The algorithm ends as soon as the target specified, is fulfilled [17 & 18].

3. OPTIMIZED COMB FILTER

In this section, changing the angle of zeros in classical filters is to be dealt with. It is to be noted that for the transfer function with real coefficients, each pair of angles with respect to the horizontal axis of the trigonometric circle must be imaged. Furthermore, zeros must be moved in such a way to be able to be Placed in the range $[(k/D) - f_c; (k/D) + f_c] \forall k \in K_k$.

Generalized comb filter transfer function is presented as follows [15]:

$$H_1(z) = \prod_{i=1}^{D_M} \prod_{p=1}^N \left(1 - 2 \cos\left(\frac{2\pi}{D} i - \alpha_p\right) z^{-1} + z^{-2} \right) \quad (2)$$

In the above equation, $D_M = (D/2) - 1$ represents the even Ds and $D_M = (D - 1)/2$ represents the odd Ds.

$$H_2(z, \alpha_n) = (1 - 2 \cos(\pi - \alpha_n) z^{-1} + z^{-2}) \quad (3)$$

$H_1(z)$ and $H_2(z, \alpha_n)$ are inserted in the following equation. This is to formulate the general equation of a generalized comb filter transfer function [15].

$$\text{D even } \frac{1}{H_{o, ev, 1}} H_1(z) \prod_{n=1}^{\lfloor \frac{N}{2} \rfloor} H_2(z, \alpha_{n+N}) \text{ Neven,}$$

$$\text{Deven } \frac{(1 + z^{-1})}{H_{o, ev, 2}} H_1(z) \prod_{n=1}^{\lfloor \frac{N}{2} \rfloor} H_2(z, \alpha_{n+N}) \text{ Nodd,}$$

$$\text{D odd } \frac{1}{H_{o, od}} H_1(z) \quad (4)$$

And normalization coefficients used in the transfer function, are listed below [15]:

$$H_{o, ev, 1} = \prod_{i=1}^{D_M} \prod_{p=1}^N \left[2 - 2 \cos\left(\frac{2\pi}{D} i - \alpha_p\right) \right] \cdot \prod_{n=1}^{\lfloor \frac{N}{2} \rfloor} [2 + 2 \cos(\alpha_{n+N})]$$

$$H_{o, ev, 2} = 2 \cdot \prod_{i=1}^{D_M} \prod_{p=1}^N \left[2 - 2 \cos\left(\frac{2\pi}{D} i - \alpha_p\right) \right] \cdot \prod_{n=1}^{\lfloor \frac{N}{2} \rfloor} [2 + 2 \cos(\alpha_{n+N})] \quad (5)$$

$$H_{o, od} = \prod_{i=1}^{D_M} \prod_{p=1}^N \left[2 - 2 \cos\left(\frac{2\pi}{D} i - \alpha_p\right) \right]$$

According to the folded band, the right choice for α_p , is $\alpha_p = q_p 2\pi f_c$ and $q_p \in [-1, 1]$. This choice is such that each pair of zeros are placed inside the relevant folded bands [12].

Now, the value of q_p must be chosen. It is desirable to optimize rotation parameters q_p in the transfer function of a generalized comb filter. This may be achieved if the maximum attenuated imposed by the quantization noise of sigma-delta modulator order B around the folded bands is performed, as a function of factors q_p . Delta-sigma quantization noise power inside the folded band can be calculated as follows. The quantization noise is considered as white noise and modulator noise transfer function, is assumed as maximum flat [20&15].

$$P_{qn} = \sum_{k=K_k} \int_{\frac{k}{D} - f_c}^{\frac{k}{D} + f_c} |H_{OCF_N}(f_d)|^2 S_B(f_d) df_d \quad (6)$$

$S_B(f_d)$, is a sigma-delta quantization noise power spectrum density which can be expressed as $S_B(f_d) = S_e(f_d)[2\sin(\pi f_d)]^{2B}$.

$S_e(f_d) = \Delta^2/12f_s$ is sampled noise spectrum density, Δ stands for the number of quantization levels in the delta sigma quantizer and f_s for the delta sigma sampling rate, [1]. Optimization has been done considering all the details of equation (4) by the genetic algorithm, for $q_p \in [-1,1]$ and that q_p 's, which have a minimum P_{q_p} value are registered in Table 1 in which q_p values for different order filters are shown.

Table 1. q_p values for different order filters

q_n values	Comb Filter Order N			
	3	4	5	6
q_1	0.0057	-0.2693	-0.4511	0.7262
q_2	-0.7416	0.8598	0.5734	-0.5385
q_3	0.7582	-0.8283	0.1035	-0.1510
q_4	-1	0.3623	0.9080	0.3283
q_5	-	1.0000	-0.8658	0.9738
q_6	-	0.9990	1	-0.8980
q_7	-	-	1	0.9704
q_8	-	-	-	0.9976
q_9	-	-	-	-1
P_{q_n}	1.0846e-10	1.0548e-12	2.5308e-14	4.7543e-14

4. MEASURING PERFORMANCE OF FILTERS

In this section, the performance of decimation filter will be measured [1&2]. For comparison of the decimation filters, the measurement criteria such as delta-sigma quantization noise power inside the folded bands P_{qn} , pass band drop $d(f_c)$ and selectivity ϕ , have been used. Pass band drop and selectivity are defined by $d(f_c) = |H(f_c)/H(0)|$ and $\phi = |H(f_c)/H((1/D) - f_c)|$ respectively, where $H(f_c)$ is decimation filter frequency response in f_c , $f_c = 1/2DL$ is normalized maximum frequency in the input signal and $((1/D) - f_c)$ constitutes the lower edge of the first folded band. The pass band drop indicates a maximum attenuation of the designed filter at the edge of the useful bandwidth compared to the ideal low pass filter. Selectivity measures the decimation filter ability to stop high-frequency noise components participating in useful signal. Performance comparison between a generalized comb filter with optimized comb filters will be done in the next section in terms of the quantization noise power, selectivity, and pass band drop [6&15].

5. RESULTS AND DISCUSSION

5.1 filter comparison

In this section, optimized comb filter (the filter considered in this article), generalized comb filter (the filter referred to in reference [15]) and classical comb filter are compared (Fig. 4). Clearly that optimized comb filter compared to two other filters has the most quantization noise rejection. Fig. 5 shows this point more accurately, for generalized comb filter and comb filter optimized in a more detailed way. In this figure, zero shift is quite distinctive, in addition increases the quantization noise rejection around folded band that result from this shift is clearly visible.

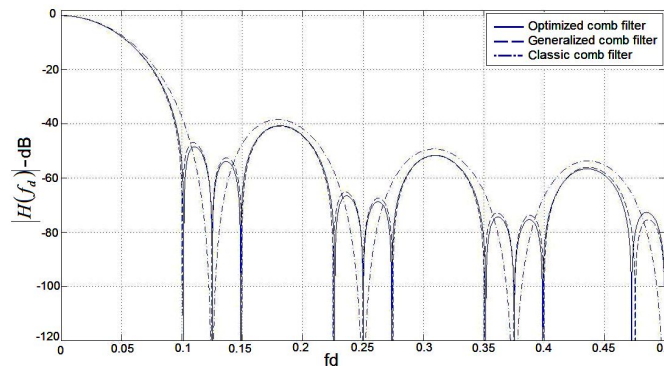


Figure 4. Optimized comb filter, generalized comb filter and classical comb filter for N = 3 and D = 8.

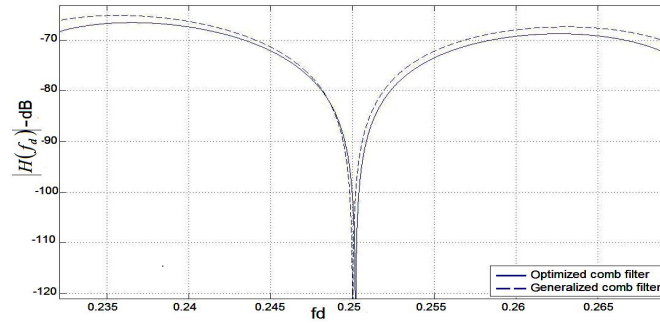


Figure 5. The previous Figure focusing on more details

As shown in Fig. 6 optimized comb filter quantization noise power rejection around the folded bands in all N_s has increased compared to generalized comb filter. In other words, quantization noise power around the folded bands has reduced. A comparison of generalized comb and optimized comb filters shows that for $N=3$, both filters have the same passband drop. On the other hand for $N=4&6$, optimized comb filter has a better performance, but for $N=5$ generalized comb filter performs in a better way (Fig. 7).

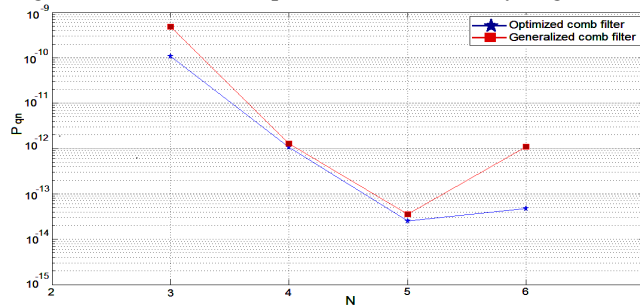


Figure 6. The quantization noise power around the folded bands optimized comb filter and generalized comb filter

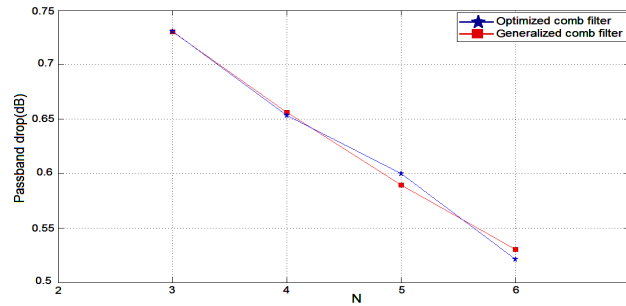


Figure 7. Comparing the passband drop of optimized comb filter and generalized comb filter

The comparison of selectivity parameter of these filters, shows that selectivity is almost equal for $N=3&4$, but for $N=5$ generalized comb filter has a better selectivity value. However, for $N=6$, the selectivity of optimized comb filter is considerably better (Fig. 8).

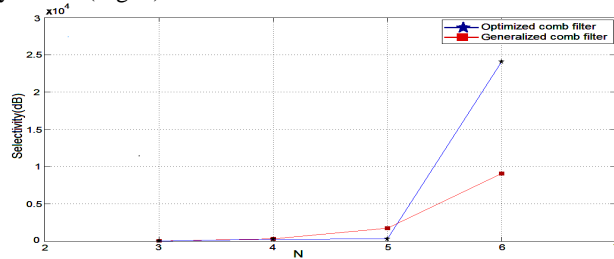


Figure 8. Comparing the selectivity of optimized comb filter and generalized comb filter

6. CONCLUSION

In this article, instead of the method used in reference [15], the genetic algorithm optimization method was used to find more accurate qs in which, the values obtained represent better results. A change in angles has a significant impact on the amount of quantization noise power. As a result, the article attempts to use genetic algorithm in order to get the best angles. It is to be noted that the angles are obtained, not only reduce quantization noise power, but also selectivity and the passband compared to generalized comb filter in some N_s has achieved more optimization.

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