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Presenting a Fuzzy ARIMA Model for Forecasting Stock Market Price Index of Iran

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ABSTRACT

Stock market is as a key element of financial markets and signs of economic growth. Since this market is recognized as a leading variable in economics, it is always the interest of policy makers and economic planners and hence the forecasting of expansions and recessions trends by this indicator has significant importance. At the stock market predicting methods, different approaches are used which they are separable into technical analysis, fundamental analysis, time series and artificial intelligence technique. Nowadays due to the development of science especially in the area of mathematics, combinatorial methods that are often bound by statistics and econometrics are applied. One of these methods is Fuzzy ARIMA or Fuzzy Auto-regressive integrated moving average. This method because of using finite number of data and calculating upper and lower limits that can indicate the best and worst possible status of investigated variables is the interest of policy makers. Therefore in this paper the 50-days data of stock index in Tehran Stock Exchange is calculated and estimated by Fuzzy ARIMA method up through we can introduce an appropriate forecast method for financial variables.

KEYWORDS: ARIMA, Fuzzy ARIMA, Stock Market.
JEL: C22, C53, G12.

INTRODUCTION

Financial sector of every country is the provider of financial resources and real activities which financial assets like money and bank credit, stocks and bonds are transacted and divided into two sectors of monetary and capital markets. Supplying financial resources in long term period and required flexibility in this matter is one of the key functions of capital market. Because of low efficiency of the other kinds of investment, stock market become to a famous investment channel in recent years (Wang &*et al,* 2012).Some of the main functions of stock market are risk transferring and distribution management, information transparency, price discovery, creating competitive markets and also collecting small savings and capitals for financing economic activities and finally to contribute to a more equitable price for Bonds and to accelerate transactions. Because of the effects of hidden factors in the stock price, stock market is volatile. These hidden factors are divided into two sections of qualitative factors including political events, international incidents, economic policy of firms, economic condition, commodity prices index, interest rate, exchange rate and psychological factors and etc. and quantitative factors including open rate, close rate, up and down rates of individual equities (Padhiary& Mishra, 2011). Because of the fluctuations of stock market, forecasting is difficult.

Considering significance of stock market as a financial resources and in result capital accumulation and in following increasing economic growth, many studies on forecasting that have been executed in recent two decades. However exact forecasting is a challenging issue according to non-stationary and noisy nature of stock prices (Wang &*et al*, 2012). Different methods are applied in forecasting output of stock market and are divided into four general categories: Technical analysis, Fundamental analysis, Time series forecasting and artificial intelligence models. In technical methods, market is forecasted by graphical patterns and studying tables describe data in the market. In fundamental methods, the intrinsic value of a share is evaluated and if the current value is estimated below the intrinsic value, the investment will be done. In time series forecasting it is attempting to linear forecasting models done by past data. These models are in two categories of single variable and multi variable regression models and including exponential smoothing, ARIMA, ARCH and GARCH models. ARIMA from statistical methods have very importance and is used widely. This model is an efficient attitude but faces some deficiencies such correlations analysis and is based on the linear correlation hypothesis in time series data. Therefore nonlinear patterns cannot be used by these models. To conquer these limitations, artificial intelligent models forecast by linear and nonlinear patterns (Wang &*et al*, 2012). Recent evidences about artificial intelligence models caused a new era in financial forecasting field. The main advantage of artificial intelligence models are in modeling and forecasting the irregular and nonlinear series. The newest soft calculation tools for financial forecasting are multilayer artificial intelligence, fuzzy

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Logit, fuzzy regression, genetic algorithm, genetic programming and hybrid models (Padhiary& Mishra 2011). According to the importance of stock market forecasting in any economy the objective of this article is to calculate the upper and lower threshold of stock market that are called the maximum and minimum of efficiency by ARIMA and fuzzy regression. So in next section the previous studies are reviewed and then related theoretical basis are stated and in fourth section ARIMA and Fuzzy ARIMA models are estimated and calculated and in the final section the results are presented.

1. LITERATURE REVIEW

Ralevic& et al (2011) in an article have discussed the linear fuzzy regression and its usage in an uncertain environment and forecast stock price using data of stock exchange market of Belgrade by fuzzy linear regression. The results indicate that fuzzy forecasting based on regression model have more forecasting power. Hung (2011) in his article has focused on forecasting volatility in the stock market by fuzzy GARCH and also genetic algorithm. The results show that in-sample estimation and predicting out of sample volatilities when GARCH model has applied clustering and adaptive forecasting has improved a lot. Tseng& et al (2001) have applied the fuzzy ARIMA for forecasting the price of dollar against Taiwan's currency. He showed this method will either present more exact predictions or will give the transactor best and worst condition. In this article firstly points to the limitations of ARIMA model and states when the historical data are less than 50, the exact results and predictions are not achieved. A Persian study is Maleki (2011) that in his own article he compares ARIMA time series model and fuzzy time series model in forecasting global gold price in two different periods.She also used combinational fuzzy ARIMA model for predicting global gold price and the results indicate the short term ARIMA results improved by using fuzzy ARIMA model. Ghavidel, Otadi and Mosleh (2009) in their study based on fuzzy artificial intelligence estimated fuzzy coefficients of labor demand and supply with fuzzy input and output. Based on writers' opinion of this article, the wage level of workers and GDP are as a vague and fuzzy term in labor market and this data estimated by fuzzy regression and coefficients of this regression will be estimated by fuzzy neural network. One of the results of this article is that forecasting by this method is better than Kao and Tanaka method. Khashei and Bijari (2006) forecasted exchange rate by fuzzy ARIMA model. At the end of the article this result compared to fuzzy time series models of Chen (1996) and Watada (1992). The comparison showed the fuzzy ARIMA model is better than the two other models.

2. Theoretical basis of ARIMA and Fuzzy ARIMA forecasting models

3.1. Autoregressive Integrated Moving Average forecasting models

One of the methods for forecasting economic variables that is developed in recent years is ARIMA, which is showed as below in time series literature:

$$Y_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i y_{t-i} + \sum_{i=0}^{q} \beta_i \varepsilon_{t-i} \qquad (1)$$

To forecast *ARIMA (p, d, q),* Box and Jenkins (1976)Forecasting Methodfor non stationary time series which can be stationary by one difference, can be used. Box and Jenkins for choosing a proper ARIMA model for estimation and prediction, proposed this 3-stage method:

1. Parsimony principle

Parsimony principle is one of the Basic Principles of Box Jenkins attitudes and is chose by minimum lag length. There is no doubt that adding new variables to the model will give a better fit, or will increase the amount of R^2 of model; however, this action will reduce the degree of freedom of the model. Box and Jenkins believe selected model by this principle give better forecasting rather models with excessive parameters.

Chosen model has optimum lags when t-statistic of all coefficients are equal or larger than 2. Moreover these coefficients shall not have intensive autocorrelation with each other. The coefficients which have high autocorrelation are unstable and usually in this status, we can omit one or some coefficients of model without any reduction in forecasting power of model.

2. Stationary and invertibility

One of the most important issues in time series data is to be stationary.It is necessary before fitness of any time series pattern; this subject would be investigated by usual stationary tests like Augmented Dickey Fuller test. Another method of recognition of stationary is intuitive study of ACF and PACF that in addition the lags of ARIMA model can be recognized. The basis of using ACF and PACF of sample as an approximation of ACF and PACF of real data production process, is the theory of probability distribution. In this theory it is assumed that the sequence of data is stationary. Thus the autoregressive estimated coefficients should be consistent with above hypothesis.

Model should be invertible in Box Jenkins attitude. $\{y_t\}$ sequence is a invertible if it could be shown by a convergent autoregressive process or with a finite degree. It is important because using ACF and PACF is implicitly is based on this assumption that $\{y_t\}$ sequence can be shown by an autoregressive model.

3. Goodness of fit

Good model has a good fit on data and in OLS, R^2 criteria and mean of sum of squared residuals, are the usual criteria of goodness of fit. The problem of these criteria is that with increasing of existing parameters in model, fitness of model is also betteritself. Parsimony principle demands using Akaike and Schwarz criteria for assessment of overall goodness of model.

The most important thing is that the residuals don't have any autocorrelation. Existence of any autocorrelation in disturbance term is indicating a systematic trend in $\{y_t\}$ sequence which cannot be explained by estimated ARIMA model. If after estimating ARIMA model, the variance of disturbance term is not constant, estimated coefficients of ARIMA model is not reliable anymore. This condition will occur when time series data have large volatilities and subsequently have little changes (Enders, 2010).

3.2. Fuzzy Regression Models

In Fuzzy Theory and Regression Fuzzy, error term is not produced by residuals of estimated values and main or Observation values, but is applied in uncertainty of model's parameters and the distribution possibility in relation to real observations.Totally linear fuzzy regression is as below:

$$y = \tilde{\beta}_0 + \tilde{\beta}_1 x_1 + \ldots + \tilde{\beta}_n x_n = \sum_{i=0}^{n} \tilde{\beta}_i x_i = x'\tilde{\beta} \qquad (2)$$

In above equation X is the vector of independent variables, n is the number of variables and $\tilde{\beta}_i$ of Fuzzy Sets is indicating ith parameter of the model. These fuzzy numbers ($\tilde{\beta}_i$ parameters) are kind-L Dubois and Prade$(a_i, c_i)_L$ with probability distribution as follow:

$$\propto_{\beta_i} (\beta_i) = L\{(\alpha_i - \beta_i / c)\} \qquad (3)$$

Considering L is a function. Fuzzy parameters are also applied in the form of triangular fuzzy numbers.

$$\propto_{\beta_i} (\beta_i) = \begin{cases} 1 - \dfrac{|\alpha_i - \beta_i|}{c_i} & \alpha_i - c_i \le \beta_i \le \alpha_i + c_i \\ 0 & otherwise \end{cases} \qquad (4)$$

And $\propto_{\beta_i} (\beta_i)$ as the membership function of fuzzy set is representing α_i and β_i parameters as the fuzzy center numbers and c_i as expansion around the center. Considering Expansion membership function principle, $y_t = X_t'\beta$ fuzzy number is described as below:

$$\propto_y (y_t) = \begin{cases} 1 - \dfrac{|y_t - x_t\alpha|}{c'|x_t|} & for \quad x_t \ne 0, \\ 1 & for \quad x_t = 0, \quad y_t = 0, \\ 0 & for \quad x_t = 0 \quad y_t \ne 0, \end{cases} \qquad (5)$$

In above equation c and α are related values vector and their expansion around center respectively. This model will minimize whole ambiguities (that are equal to sum of single expansions and are related to each fuzzy model parameters).

$$Minimize \qquad S = \sum_{t=1}^{k} c'|x_t| \qquad (6)$$

Above method considers the condition in which membership value for every y_t is greater than a determined threshold in level $h(h \in [0,1])$. Above criteria has this reality that the output of fuzzy model for all data points of $y_1, y_2, ..., y_t$ shall be greater than Level *h*.Selection of Level *h* value is effective in expansion of fuzzy model parameters (c).

$$\propto_y (y_t) \geq h \qquad for \qquad t = 1,2,..,k \tag{7}$$

The t-Statistic is for non-fuzzy data numbers applied to model production. The problem of finding fuzzy regression parameters is formulated by Tanaka as a linear planning.

$$\begin{aligned} &Minimize \qquad S = \sum_{t=1}^{k} c'|x_t| \\ &\qquad\qquad\qquad x_t'\alpha + (1-h)c'|x_t| \geq y_t \qquad t = 1,2,...,k \\ &subject.to \qquad x_t'\alpha + (1-h)c'|x_t| \leq y_t \qquad t = 1,2,...,k \\ &\qquad\qquad\qquad c \geq 0 \end{aligned} \tag{8}$$

When $\alpha' = (\alpha_1, \alpha_2, ..., \alpha_n)$ and $c' = (c_1, c_2, ..., c_n)$ are unknown variables vector and *S* is the whole Ambiguity.

3.3. Formulating Fuzzy ARIMA Model

The Parameters of ARIMA model are definite $\varphi_1, \varphi_2, ..., \varphi_p$ and $\theta_1, \theta_2, ..., \theta_q$. But in fuzzy regression method, fuzzy parameters $\tilde{\varphi}_1, \tilde{\varphi}_2, ..., \tilde{\varphi}_p$ and $\tilde{\theta}_1, \tilde{\theta}_2, ..., \tilde{\theta}_q$ are used in form of triangular fuzzy numbers. It is necessary to state that utilization of fuzzy parameters will reduce the need to past data (Tseng, 2001).

In this study the presented methodology by Ishibuchi&Tanakais used for the condition that prediction domain would be wide. A Fuzzy ARIMA Model with fuzzy functions and parameters is as below:

$$\tilde{\Phi}_p(B)W_t = \tilde{\theta}_q(b)a_t \tag{9}$$

$$W_t = (1-B)^d (Z_t - \propto) \tag{10}$$

$$\tilde{W}_t = \tilde{\varphi}_1 W_{t-1} + \tilde{\varphi}_2 W_{t-2} + ... + \tilde{\varphi}_p W_{t-p} + a_t - \tilde{\theta}_{p+1} a_{t-1} - \tilde{\theta}_{p+2} a_{t-2} - ... - \tilde{\theta}_{p+q} a_{t-q} \tag{11}$$

$\{z_t\}$ are the observations, $\tilde{\varphi}_1, \tilde{\varphi}_2, ..., \tilde{\varphi}_p$ and $\tilde{\theta}_1, \tilde{\theta}_2, ..., \tilde{\theta}_q$ are fuzzy numbers. Now the equation (11) will convert to this:

$$\tilde{W}_t = \tilde{\beta}_1 W_{t-1} + \tilde{\beta}_2 W_{t-2} + ... + \tilde{\beta}_p W_{t-p} + a_t - \tilde{\beta}_{p+1} a_{t-1} - \tilde{\beta}_{p+2} a_{t-2} - ... - \tilde{\beta}_{p+q} a_{t-q} \tag{12}$$

Fuzzy parameters in this equation are considered as triangular fuzzy numbers like below:

$$\propto_{\beta_i} (\beta_i) = \begin{cases} 1 - \dfrac{|\alpha_i - \beta_i|}{c_i} & \alpha_i - c_i \leq \beta_i \leq \alpha_i + c_i \\ 0 & otherwise \end{cases} \tag{13}$$

Qua $\propto_{\tilde{\beta}} (\beta_i)$ is a membership function of a fuzzy set that would be determined by β_i and α_i parameters. Now by using β_i fuzzy parameters as triangular fuzzy numbers and also expansion principle, *W* membership function is as follows (Tanaka, Ishibuchi, 1992):

$$\propto_{\tilde{W}}(W_t) = \begin{cases} 1 - \dfrac{\left| W_t - \sum_{i=1}^{p} \alpha_i W_{t-i} - a_t + \sum_{i=p+1}^{p+q} \alpha_i a_{t+p-i} \right|}{\sum_{i=1}^{p} c_i \left| W_{t-i} \right| + \sum_{i=p+1}^{p+q} c_i \left| a_{t+p-i} \right|} & for \quad W_t \neq 0, \quad a_t \neq 0 \\ 0 & otherwise \end{cases} \quad (14)$$

Level *h*is the threshold for the value of membership functions of all observations.

$$Z_z(Z_t) \geq h \qquad for\, i = 1,2,\ldots,k \quad (15)$$

In other words S is described like this:

$$S = \sum_{i=1}^{p} \sum_{t=1}^{k} c_i \left| \varphi_{ii} \right| \left| W_{t-i} \right| + \sum_{i=p+1}^{p+q} \sum_{t=1}^{k} c_i \left| \rho_{i-p} \right| \left| a_{t+p-i} \right| \quad (16)$$

ρ_{i-p} is the autoregressive coefficient in the lag of $i-p$ and φ_{ii} is the partialautoregressive coefficient in the ith lag. The Steps of fuzzy ARIMA model is as below:

Phase one: To fit the ARIMA model by existing data in observations (that are non-fuzzy). The result of the optimum answer of coefficients in phase one $\alpha^* = (\alpha_1^*, \alpha_2^*, \ldots, \alpha_{p+q}^*)$ are the Net error that are used as an input data set in phase two.

$$\begin{aligned} &Minimize \quad S = \sum_{i=1}^{p} \sum_{t=1}^{k} c_i \left| \varphi_{ii} \right| \left| W_{t-i} \right| + \sum_{i=p+1}^{p+q} \sum_{t=1}^{k} c_i \left| \rho_{i-p} \right| \left| a_{t+p-i} \right| \\ &subject.to \quad \sum_{i=1}^{p} \alpha_i W_{t-i} + a_t - \sum_{i=p+1}^{p+q} \alpha_i a_{t+p-i} + (1+h) \left(\sum_{i=1}^{p} c_i \left| W_{t-i} \right| + \sum_{i=p+1}^{p+q} c_i \left| a_{t+p-i} \right| \right) \geq W_t \qquad t = 1,2,\ldots k \\ &\sum_{i=1}^{p} \alpha_i W_{t-i} + a_t - \sum_{i=p+1}^{p+q} \alpha_i a_{t+p-i} - (1+h) \left(\sum_{i=1}^{p} c_i \left| W_{t-i} \right| + \sum_{i=p+1}^{p+q} c_i \left| a_{t+p-i} \right| \right) \leq W_t \qquad t = 1,2,\ldots k \\ &c_i \geq 0 \qquad for\, i = 1,2,\ldots, p+q \end{aligned} \quad (17)$$

Phasetwo: Determination of minimum ambiguityby criteria like equation (17) and $\alpha^* = (\alpha_1^*, \alpha_2^*, \ldots, \alpha_{p+q}^*)$ .The number of limitations is equal to the number of observations and fuzzy ARIMA model (Savic&Pedrycz, 1991) is like equation (18):

$$\tilde{W}_t = \langle \alpha_1, c_1 \rangle W_{t-1} + \ldots + \langle \alpha_p, c_p \rangle W_{t-p} + a_t - \langle \alpha_{p+1}, c_{p+1} \rangle a_{t-1} - \ldots - \langle \alpha_{p+q}, c_{p+q} \rangle a_{t-q} \quad (18)$$

Phase three: According to Ishibuchi's ideas, when the domain of fuzzy ARIMA model would be wide, up and down data ofmodel will be omitted. To make a model including all possible condition of fuzzy ARIMA model if data set consist specific differences or outside cases, c_j would be broad. According to Ishibuchi's ideas the data near to upper and lower boundaries will be omitted and then model will be reformulated (Ishibuchi& Tanaka, 1998).

3. Data and estimation

4.1 The Combinational ARIMA and Fuzzy ARIMA

As stated before, combinational method is applied when ARIMA model has high error and it cannot present acceptable results. In these cases by utilization of fuzzy regression concepts, combinational model is attempting to reduce the error of ARIMA model.

As it comes from the literature of combinational model (Tseng, 2001), in cases which available data are less than 100 observations or the aim is short term predictions, ARIMA model does not have required convergence for high accuracy forecasts and the error of this model is high. Hence combinational model by minimization of this error will improve the prediction results. Thus in cases which data numbers are less than 100 or even 50, combinational model would be applied and the possibility of using this model when the number of data is more than mentioned value could not represent the answer. Now the modeling phases for 50-days data of the index of Tehran Stock Exchange are represented as follow:

Phase one: In this phase that has a great significance in modeling of this study and in case of performing wrong; an efficient Fuzzy ARIMA pattern would not be achieved. By Box Jenkins methodology which is described at first, we will start the modeling. Firstly the stationary of the index of Tehran Stock Exchange, that is considered 50-days period, would be tested.

Table1. Surveying the stationary status of overall index of stock market

Test result	Variable
Not rejection of H_0 Hypothesis	Overall index of stock market
Rejection of H_0 Hypothesis	The First difference of overall index of stock market

The Results of test are indicating variables are stationary after taking one difference. In next step by using PACF and ACF, lags of ARIMA model is selected and the best model for estimation is chosen by AIC and SC criteria.

$$d(index_t) = 0.98 d(index_{t-1}) - 0.59\varepsilon_{t-1} - 0.28\varepsilon_{t-2} \qquad (19)$$

Finally for insurance of goodness of fit of model augmented dickey fuller test for study of stationary in residual terms, normality test of error term distribution to ensure the estimated coefficients of model, autocorrelation in terms of determination of optimum lags for model and heteroskedasticityfor lack of conditional heteroskedasticity over time that is very usual in financial data.

Table2.Goodness of fit tests on the model residuals

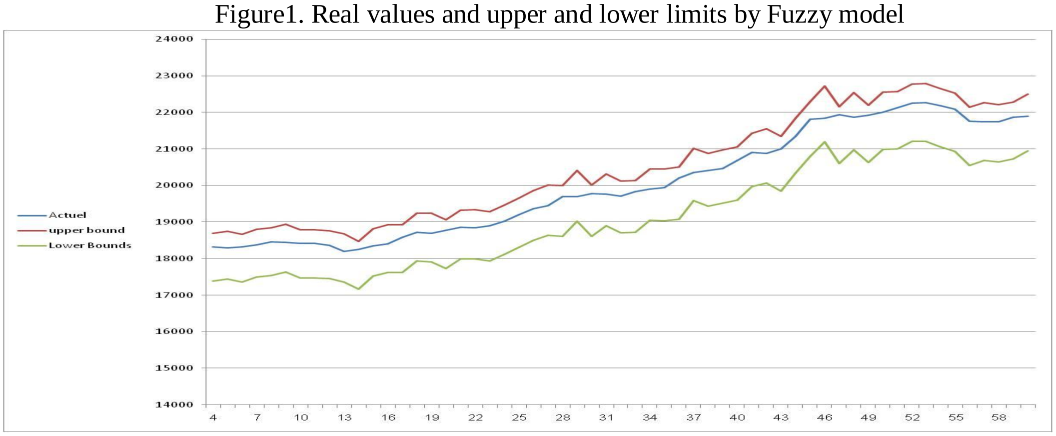
Test	Test Statistic	Probability level	Test result
Augmented Dickey Fuller	τ	0.0	Lack of non-stationary of residual terms
Jarque-Bera test	χ^2	0.6243	Normality of error terms
Breusch-Godfrey test	NR^2	0.9643	No autocorrelation
ARCH test	F	0.2213	No heteroskedasticity

Phase two: Is determination of minimum ambiguity. The result of phase one are the optimum results for parameters without error $\alpha^* = (\alpha_1^*, \alpha_2^*, ..., \alpha_{p+q}^*)$ thatare used as an input for phase two. By putting $\alpha^* = (0.987022, 0.590748, 0.284888)$, fuzzy parameters of model are achieved by equation (17).Fuzzy model is formulated as bellow:

$$\tilde{Z}_t = \langle 0.98, 0.00 \rangle Z_{t-1} - \langle 0.59, 0.03575421 \rangle \varepsilon_{t-1} - \langle 0.28, 0.000 \rangle \varepsilon_{t-2} \qquad (20)$$

The Results of fuzzy model is given in figure1. As it is seen real values are located in fuzzy intervals.

Figure1. Real values and upper and lower limits by Fuzzy model



4.2 Comparison between ARIMA and Fuzzy ARIMA

As it stated ARIMA models are functional for forecasting; however these models have some limitations. Some of the most significant limitations are the need of these models to large number of data for achieving suitable results. The Combinational Fuzzy ARIMA model, because of applying the advantages of both models and also determination of high and low margin in prediction, is appropriate and practical for decision makers of financial markets. Applying this model is useful when the number of data is low and ARIMA model cannot present proper results. Using combinational Fuzzy ARIMA model for 50 historical data could improve the prediction of ARIMA model in short term to a considerable extent. Comparison of the interval of combinational method with 95% confidence interval of usual ARIMA model is indicating improvement in reduction of the interval by proposed model.

Table 3.Real values and upper and lower limits with Fuzzy ARIMA and prediction with ARIMA

Day	Overall index	Forecasted overall index by ARIMA	Lower limit of FUZZYARIMA	Upper limit of FUZZYARIMA
51	22136.5	22083	21003.4	22571.5
52	22260.4	22249	21205.3	22778.6
53	22270.4	22360	21214.□	22797.1
54	22188.3	22330	21061.8	22653.6
55	22090.3	22217	20933.9	22526.5
56	21762.2	21783	20552.4	22139
57	21739.4	21808	20682.8	22262.4
58	21749.5	21764	20651.□	22207.3
59	21867.9	21787	20733.3	22287.8
60	21900.5	21941	20942.9	22498.2

According to the above table, two bands are determined for overall index which show the best and the worst status of bourse index in considered period. Hence policy makers and programmers can plan for the future more exactly by using these tools.

5. Conclusions

Study of the behavior of stock market index because of abrupt changes and volatility and be influenced by economic and political factors is an important and significant matter so that some studies in relation to prediction of stock price is done domestically and abroad and new methods are introduced in order to explaining. According to the importance of forecasting in time series studies, in this study it was attempted to investigate index of Tehranstock exchange by Econometrics and Fuzzy and totally fuzzy regression. In this regard, the data of the total price of the stock is used.

The Results show that performed forecasting by fuzzy regression is more capable of predicting and it can be useful for policy makers and programmers. This indicates new mathematics methods have better performance in modeling of economic variables so that by exact study of theoretical basis of research topic and also using software that have better explaining ability, we can achieve optimal performance with an empirical and scientific basis.

REFERENCES

1. D.A. Savic, W. Pedrycz (1991), Evaluation of fuzzy linear regression models, Fuzzy Sets and Systems 39(1) 51-63.
2. G. Maleki, Gold Price Prediction with using Fuzzy method. MA Thesis, Islamic Azad University Central Tehran Branch. (2010).
3. G.P. Box, G.M. Jenkins, Time Series Analysis: Forecasting and Control, Holden-day Inc., San Francisco, CA, 1976.
4. Ghavidel, Saleh; Otadi, Mahmoud; Mosleh, Maryam (2009); "Estimation of labor market by Fuzzy Neural Network", Economic Modeling Journal, No. 4, Pages 83-101.
5. H. Tanaka, H. Ishibuchi, Possibility regression analysis based on linear programming, in: J. Kacprzyk, M. Fedrizzi (Eds.), Fuzzy Regression Analysis, Omnitech Press, Warsaw and Physica-Verlag, Heidelberg, 1992, pp. 47 -60.
6. Hungj.(2011);" Applying a combined fuzzy systems and GARCH model to adaptively forecast stock market volatility"; Applied Soft Computing, Volume 11, Issue 5, Pages 3938–3945.
7. J. Watada, Fuzzy time series analysis and forecasting of sales volume, in: J. Kacprzyk, M. Fedrizzi (Eds.), Fuzzy Regression Analysis, Omnitech Press, Warsaw and Physica-Verlag, Heidelberg, 1992, pp. 211-227.
8. M. Khashei, M, Bijari,. Using a Fuzzy Autoregressive Integrated Moving Average Model for Exchange rate Forecasting, Quantity methods in engineeringf. PP, 67-75.
   Network Model for Stock Market Prediction"; International Journal of Engineering Science and Technology (IJEST).
9. Padhiary, p.k. & Mishra, A.P.(2011);"Development of Improved Artificial Neural
10. Ralevic, N.M.; Djakovic, V.D.; Andjelic, G.B.; Kovacevic, I.M.; Kiurski, J.S.; Comic, L.L.(2011);"Fuzzy prediction based on regression models: Evidence from the Belgrade - prime market and graphic industry"; Intelligent Systems and Informatics (SISY), 2011 IEEE 9th International Symposium, Access by: http://ieeexplore.ieee.org
11. W. Enders, Applied Econometrics and Time Series, Wiley Inc., Hoboken, US, 2010.
12. Wang,J.J.; Wang, J.ZH.; Zhang, ZH.G.; Guo, SH.P;(2012);" Stock index forecasting based on a hybrid model"; Volume 40, Issue 6, December 2012, Pages 758–766.
13. S.M. Chen, Forecasting enrollments based on fuzzy time series, Fuzzy Sets and Systems 81(3) (1996) 311-319.