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Secondary Goal in DEA Cross Efficiency Evaluation

Saba Sadeghi Gavgani, Majid Zohrehbandian, Esmaile Khorram

Department of mathematics, Karaj Branch, Islamic Azad University, Karaj, Iran

ABSTRACT

Data envelopment analysis (DEA) is a method for estimating efficiency of decision making units (DMU).The method done by means of weighted output to input ratio with the weights being optimal virtual prices of such activities for all units. Cross efficiency evaluation has long been proposed as an alternative method for ranking DMUs. The major problem that reduced the usefulness of cross efficiency evaluation method is that, the cross efficiency score because of alternative optimal cross efficiency may not be unique. In order to resolve this problem, another secondary goal is introduced in this research to reduce the alternative optimal cross efficiency on the base of linear programming and to improve the secondary goal proposed by Doyle et al (1994). A numerical example is then provided to illustrate the capability of this new DEA model.

KEYWORDS: Data envelopment analysis; Decision making units; Cross efficiency; Secondary goal; Rank.

1. INTRODUCTION

Data envelopment analysis (DEA) is a linear programming based technique used to measure the relative efficiencies of homogenous set of decision making units (DMUs) with multiple inputs and outputs. Such an efficiency score is defined by the ratio of weighted outputs over weighted inputs, where a specific weight or so called multiplier is assigned to the corresponding input or output measure, and the weighted output/input is the sum of all outputs/inputs for a given DMU. Many studies have been published dealing with applying DEA in real world [7-8-21]. When inefficient units are recognized, the DEA study also identifies a set of efficient units that can be used as benchmarks for efficiency improvement. A common problem in DEA is that, all efficient DMUs have the same efficiency ratio which makes it difficult to rank efficient DMUs.

The cross-evaluation method was developed as a DEA extension tool that can be utilized to identify best performing DMUs and select them using cross efficiency score that are liked to all units (Sexton et al, (1986) [11]). The main idea of cross evaluation is to use DEA in a peer evaluation instead of self evaluation. It provides a unique ordering of DMUs and eliminates unrealistic weights schemes without requiring the elicitation of weights restrictions from application area expert (Anderson et al, (2002) [1]). Cross efficiency evaluation has been used in various applications, e.g., efficiency evaluations of nursing homes (Sexton et al, (1986) [11]), advanced manufacturing technologies (AMTs) selection (Banker et al (1997) [2], Sun (2002) [12]), Olympic ranking (Wu et al [18]) and so on.

However, the non uniqueness of the DEA optimal weights possibly reduces the usefulness of cross efficiency. Specifically, the cross efficiency scores obtained from the original DEA are generally not unique and depend on the alternative optimal solution used to the DEA linear program. Different approaches have been suggested as a remedy for non uniqueness problem. For instance, Sexton et al (1986) [11] and Doyle et al (1994) [5] propose the use of secondary goals to solve the non uniqueness. They presented aggressive and benevolent model formulations. In using the benevolent model, the idea is to identify the optimal weights that maximize the average efficiency of the other DMUs. In contrast, in using the aggressive model, one seeks weights that minimize the average efficiency of the other units. Liang et al (2008) [9] proposed three alternative secondary goals for cross efficiency evaluation. Wu et al (2008) [17-18] suggested a modified DEA cross efficiency model and applied it for Olympic ranking. Also, Wu et al (2009b) [19] calculated ultimate efficiency scores with weighting cross efficiency value with the weights were being determined using shapely value in cooperative game. Moreover, Wu et al (2009) [20] developed mixed integer programming for cross efficiency evaluation. Wang et al (2010) & (2011) [13] [14] proposed two natural DEA models for the cross efficiency evaluation. More recently, Wang et al (2011) [15] suggested models for cross efficiency based on the ideal and anti ideal DMUs. Finally, Orkcu et al (2011) [10] offered a goal programming model for DEA cross efficiency score.

The aims of this paper are as follows: 1) Improvement of the model suggested by Doyle et al (1994) [5] which sometimes gives same scores for some DMUs and then it is not suitable for ranking in these situations. 2) Most of cross efficiency models have large number of constraints [10,13,14,19, 20] and they occasionally encounter to the infeasibility problem. 3) Most of cross efficiency models are based on solution of mix integer and/or nonlinear problems which are difficult [10,20]. For these reasons suggesting a linear model for computation of cross efficiency scores is attractive. This paper aims at developing a new improved different method with lowest number of constraints for remove pervious problems and choosing more suitable weights with the goal of increasing the efficiency scores of

*** Corresponding Author:** Saba Sadeghi Gavgani, Department of mathematics, Karaj Branch, Islamic Azad University, Karaj, Iran. Email address: sabasadeghi72@gmail.com. Tel & Fax: +984113818828

all DMUs or decreasing the rank order of them, based on multiple criteria decision analysis, in which the ideal DMU and anti ideal DMU are built. As a result scientific contributions of this paper are: 1) complete ranking 2) reduce infeasibility and 3) introduce linear model. The rest of paper organized in following manner section 2 describes briefly the cross efficiency evaluation. The new secondary goal is then developed in section 3. Section 4 demonstrates the model with data set. Finally, conclusion is offered in section 5.

2. Cross efficiency

Suppose there are n DMUs to be evaluated against m inputs and s outputs. Then, the ith input of $DMU_j\ j=1,\ldots,n$ and rth output of it are denoted by x_{ij} and y_{rj}, respectively. Cross efficiency is often calculated as two-phase process, in which the first phase use standard CCR model of Charnes et al (1978) [3]. Consider a DMU, say $DMU_k\ k\in\{1,\ldots,n\}$ whose efficiency related to the following CCR (LP) model:

$$\begin{aligned} E_{kk} = Max \quad & \sum_{r=1}^{s} u_{rk} y_{rk} \\ s.t \quad & \sum_{i=1}^{m} v_{ik} x_{ik} = 1 \\ & \sum_{r=1}^{s} u_{rk} y_{rj} - \sum_{i=1}^{m} v_{ik} x_{ij} \le 0 \qquad j=1,\ldots,n, \qquad (1) \\ & u_{rk} \ge 0 \qquad r=1,\ldots,s, \\ & v_{ik} \ge 0 \qquad i=1,\ldots,m. \end{aligned}$$

If, $E_{kk}=1$ DMU_k is efficient otherwise it is inefficient. We obtain a set of optimal weights $(u_{1k}^*,\ldots,u_{rk}^*,v_{1k}^*,\ldots,v_{mk}^*)$. Then the cross efficiency of DMU_j using the weights of DMU_k chosen in model (1) is:

$$E_{kj} = \frac{\sum_{r=1}^{s} u_{rk}^* y_{rj}}{\sum_{i=1}^{m} v_{ik}^* x_{ij}} \qquad j=1,\ldots,n. \qquad (2)$$

As shown in Table 1 in cross efficiency matrix $E_{kk}=1(k=1,\ldots,n)$ are the optimistic efficiency of n DMUs will have (n-1) cross efficiency and

$$E_j = \frac{\sum_{k=1}^{n} E_{kj}}{n} \qquad j=1,\ldots,n. \qquad (3)$$

Referred to as the cross efficiency score for DMU_j .

Table 1: Cross efficiency matrix

Rated DMU	1	…	n
1	E_{11}	$\cdots$	E_{1n}
$\vdots$	$\vdots$	$\ddots$	$\vdots$
n	E_{n1}	$\cdots$	E_{nn}
Mean	$\bar{E}_1$	$\cdots$	$\bar{E}_n$

Model (1) can also be expressed equivalently in the following deviation variable form:

$$\begin{aligned} Min \quad & \alpha_k \\ s.t \quad & \sum_{i=1}^{m} v_{ik} x_{ik} = 1 \\ & \sum_{r=1}^{s} u_{rk} y_{rj} - \sum_{i=1}^{m} v_{ik} x_{ij} + \alpha_j = 0 \qquad j=1,\ldots,n, \qquad (4) \\ & u_{rk} \ge 0 \qquad r=1,\ldots,s, \\ & v_{ik} \ge 0 \qquad i=1,\ldots,m, \\ & \alpha_j \ge 0 \qquad j=1,\ldots,n. \end{aligned}$$

Where α_k is the deviation variable for DMU_k and α_j the deviation variable for the $DMU_j\,(j=1,\ldots,n)$. For this model DMU_k is efficient if and only if $\alpha_k^*=0$. If DMU_k is not efficient then its efficiency score will be $1-\alpha_k^*$.

It is noticed that model (1) may has multiple optimal solutions. This non-uniqueness of the input and output weights would then damage the use of cross-efficiency and dependency of the optimal solution.. To resolve this problem one remedy suggested by Sexton et al [9] is to introduce a secondary goal which optimizes the input and output weights while keeping unchanged the CCR efficiency by model (1). The most commonly used secondary goals were suggested by Doyle and Green [5] are shown in (5). In this approach an attempt is made to minimize the efficiencies of other DMUs and model is given bellow:

$$
\begin{array}{lll}
Min & \sum_{r=1}^{s}(u_{ro}\sum_{j=1,\,j\neq 0}^{n} y_{rj}) & \\
s.t & \sum_{i=1}^{m}(v_{io}\sum_{j=1,\,j\neq o}^{n} x_{ij})=1 & \\
 & \sum_{r=1}^{s}u_{ro}y_{rj}-\sum_{i=1}^{m}v_{io}x_{ij}\leq 0 \qquad j=1,\ldots,n,\ j\neq o, & \\
 & \sum_{r=1}^{s}u_{ro}y_{ro}-\theta_{oo}^{*}\sum_{i=1}^{m}v_{io}x_{io}=0 & (5)\\
 & u_{ro}\geq 0 \qquad r=1,\ldots,s, & \\
 & v_{io}\geq 0 \qquad i=1,\ldots,m. &
\end{array}
$$

Benevolent formulation for cross efficiency evolution maximizes the cross efficiencies of other DMUs. Aggressive model seem more beneficial when compared to the benevolent model in respect of the discrimination problem. With this information a new secondary goal is examined for better performance in cross evaluation.

3. New secondary goal

Let the inefficiency of DMU_k be α_k^*. The ideal point is then defined

$$
\frac{\sum_{r=1}^{s}u_r y_{rj}}{\sum_{i=1}^{m}v_i x_{ij}}=1 \quad\Rightarrow\quad \sum_{r=1}^{s}u_r y_{rj}-\sum_{i=1}^{m}v_i x_{ij}=1 \quad\Rightarrow\quad \alpha_j=0
$$

In the absence of such an ideal point a reasonable objective is to treat α_j as the goal achievement variables. The lack of discriminating power of basic DEA using a single objective function can overcome by using multiple and more objective functions. Then, the following model is considered in which the secondary goal is to minimize all of inefficiencies and minimize deviation from the ideal point:

$$
\begin{array}{lll}
Min & \alpha_{k1} & \\
 & \vdots & \\
Min & \alpha_{kj} \quad j\neq k & \\
 & \vdots & \\
Min & \alpha_{kn} & \\
s.t & \sum_{i=1}^{m}v_{ik}x_{ik}=1 & \\
 & \sum_{r=1}^{s}u_{rk}y_{rj}-\sum_{i=1}^{m}v_{ik}x_{ij}+\alpha_{kj}=0 \qquad j=1,\ldots,n, & (6)\\
 & \sum_{r=1}^{s}u_{rk}\bar{y}_{rk}-\sum_{i=1}^{m}v_{ik}\bar{x}_{ik}=0 & \\
 & u_{rk}\geq 0 \qquad r=1,\ldots,s, & \\
 & v_{ik}\geq 0 \qquad i=1,\ldots,m, & \\
 & \alpha_{kj}\geq 0 \qquad j=1,\ldots,n. &
\end{array}
$$

If DMU_k is not efficient then we can image this DMU on the efficient frontier $(\bar{x}_k,\bar{y}_k)=(\theta_k^* x_k, y_k)$ and θ^* is the optimal solution of model (1).

In multi objective models we can't find a unique optimal solution that optimizes all objective functions but we can find efficient point or Pareto optimal point.

Definition: A feasible solution $\hat{x} \in X$ in (7) called efficient or Pareto optimal if there is no other $x \in X$ that $f_k(x) \le f_k(\hat{x})\ (k = 1,\ldots,p)$

$$\begin{aligned} &Min \quad (f_1(x),\ldots,f_p(x)) \\ &s.t \quad x \in X \qquad (7) \end{aligned}$$

We state without proof the following theorems [6].

Theorem1: Let $\hat{x} \in X$ be an optimal solution of (8). Then if $\lambda > 0$ then $\hat{x}$ is efficient point in (7).

$$\begin{aligned} &Min \quad \sum_{k=1}^{p} \lambda_k f_k(x) \\ &s.t \quad x \in X \qquad (8) \end{aligned}$$

Theorem2: Let $\lambda_k > 0\ (k = 1,\ldots,p)$ with $\sum_{k=1}^{p} \lambda_k = 1$ be positive weights. If $\hat{x}$ is an optimal solution of (8) then $\hat{x}$ is a properly efficient solution of (7).

Then we can write following model from (6):

$$\begin{aligned} &Min \ \sum_{j=1, j\neq k}^{n} \lambda_j \alpha_{kj} \\ &s.t \quad \sum_{r=1}^{s} u_{rk} y_{rj} - \sum_{i=1}^{m} v_{ik} x_{ij} + \alpha_{kj} = 0 \qquad j = 1,\ldots,n,\ j \neq k \\ &\qquad \sum_{r=1}^{s} u_{rk} \bar{y}_{rk} - \sum_{i=1}^{m} v_{ik} \bar{x}_{ik} = 0 \qquad (9) \\ &\qquad \sum_{i=1}^{m} v_{ik} x_{ik} = 1 \\ &\qquad u_{rk} \ge 0 \qquad r = 1,\ldots,s, \\ &\qquad v_{ik} \ge 0 \qquad i = 1,\ldots m, \\ &\qquad \alpha_{kj} \ge 0 \qquad j = 1,\ldots,n. \end{aligned}$$

However, the weights have to be determined, from theorems $\lambda_j > 0\ (j = 1,\ldots,n,\ j \neq k)$ and $\sum_{j=1}^{n} \lambda_j = 1$. None of them prefers to the others as equal weights are given and image the DMU under evaluation on efficient frontier. Then $\lambda_j = \frac{1}{n-1}, (j = 1,\ldots,n,\ j \neq k)$. The following model is then given:

$$\begin{aligned} &Min \ \frac{\sum_{j=1, j\neq k}^{n} \alpha_{kj}}{n-1} \\ &s.t \quad \sum_{r=1}^{s} u_{rk} y_{rj} - \sum_{i=1}^{m} v_{ik} x_{ij} + \alpha_{kj} = 0 \qquad j = 1,\ldots,n, \\ &\qquad \sum_{r=1}^{s} u_{rk} \bar{y}_{rk} - \sum_{i=1}^{m} v_{ik} \bar{x}_{ik} = 0 \qquad (10) \\ &\qquad \sum_{i=1}^{m} v_{ik} x_{ik} = 1 \\ &\qquad u_{rk} \ge 0 \qquad r = 1,\ldots,s, \\ &\qquad v_{ik} \ge 0 \qquad i = 1,\ldots m, \\ &\qquad \alpha_{kj} \ge 0 \qquad j = 1,\ldots,n. \end{aligned}$$

This new model has some advantages. First it is a LP model and can find accurate optimal solution since we don't have the exact optimal solution in mixed integer programs or NLP programs and need difficult algorithms with high complexity and a large number of secondary goals don't have this condition. Secondly, in this model there are n+2 constrains, the number of constrains are remarkably fewer in comparison with the other secondary goals and adding constraints to model doesn't take a better solution [16], for instance in model of Wu et al [20], there are 3n+2 constraints and in model of Orkcu et al [10] there are 2n+2 constraints. Our model has lowest number of constraints. This means our new model always feasible and stable which are the major problems in cross efficiency evaluation. Moreover, another important advantage of this method is different scores have been chosen in this model and we can use it for complete ranking and selecting most efficient unit.

4. Numerical example

Sexton et al (1986) [11] considered a case of six nursing homes whose input and output data for a given year are reported in Table 1, where the input and output variables are defined as follows:

Input1: staff hour per day,

Input2: supplies per day,

Output1: total Medicare plus Medicaid reimbursed patient days,
Output2: total privately paid patient days

Table2: Inputs & outputs for six nursing homes

DMU	Input1	Input2	Output1	Output2
DMU_1	1.5	0.2	1.4	0.35
DMU_2	4	0.7	1.4	2.1
DMU_3	3.2	1.2	4.2	1.05
DMU_4	5.2	2	2.8	4.2
DMU_5	3.5	1.2	1.9	2.5
DMU_6	3.2	0.7	1.4	1.5

Table3: Results

DMU	CCR	CROSS EFF	OBJECTIVE
DMU_1	1	0.9393	0.193
DMU_2	1	0.9416	0.0705
DMU_3	1	0.9392	0.0644
DMU_4	1	0.9457	0.0353
DMU_5	0.9775	0.94562	0.0518
DMU_6	0.8465	0.935	0.0527

Table4: Ranking with four models

DMU	CCR	Aggressive model	Benevolent model	Model (10)
DMU_1	1	1	1	4
DMU_2	1	2	4	3
DMU_3	1	5	5	5
DMU_4	1	2	1	1
DMU_5	5	4	3	2
DMU_6	6	6	6	6

Table 2 shows the output and input of the six nursing homes. Table 3 shows the result of CCR efficiency in the first column. After solving the model (10) and calculate cross efficiency score, results are shown in Table 3 third column and optimal objective of model are listed in Table 3 forth column. Then, we have different score for cross efficiency and with these results it is easy to rank DMUs. Rank of DMUs with CCR model, Doyle et al (1994) [5] models and our suggested model are in Table 4. The most efficient DMU is DMU_4. DMU_5 Has the third rank but in CCR model it is inefficient since sometimes inefficient DMUs have better performance than efficient ones. Second column of Table 4 shows ranking with CCR model and cannot distinguish them any further. Therefore, the CCR model is always not suitable for selection and ranking DMUs. Third column and forth column of Table 4 represent ranking with aggressive model and benevolent model respectively and both of them have two DMU with same rank and hence Doyle et al (1994) [5] model can't provide complete ranking in cross efficiency. We can then use the objective of model (10) for ranking with respect to the lower objective that has better situation in ranking.

5. CONCLUSION

Cross efficiency evaluation is an important yet practical method for comparing and ranking DMUs. Since the DEA weights are generally not unique, the related cross efficiency may not be unique either. It is this non uniqueness phenomenon that can undermine the usefulness of cross evaluation method, and it is difficult for the DM to make a subjective choice between the two formulations. For the current paper, in order to resolve this problem and extending the model of Doyle et al (1994) [5] by introduction of secondary objective functions, we have proposed a multi objective model and transform it to the linear program model as a secondary goal with considering the performance ranking of DMUs. As a result, we discussed about considerable advantage of this model against other secondary goals. A numerical example has been tested to show the capability of the new DEA model and its effectiveness in discriminating among DMUs. It has been shown that the new DEA model make valid contributions to DEA cross efficiency evaluation and can be seen as improvement and complement to Doyle et al's (1994) [5] model. It is hoped that this new DEA model can provide more insights into the cross efficiency evaluation and enhance the theory and methodology of DEA.

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