J. Basic. Appl. Sci. Res., 3(7)41-55, 2013
© 2013, TextRoad Publication

ISSN 2090-4304
Journal of Basic and Applied Scientific Research
www.textroad.com

On the Observability of a Class of Takagi-Sugeno Fuzzy Systems

Navid Dini, Mohammad B. Menhaj, and Amir A. Suratgar

Electrical Engineering Department, Amirkabir University of Technology, Tehran, Iran

ABSTRACT

Despite all the progress made in the field of fuzzy systems, important concepts such as controllability and observability of fuzzy systems have not been fully discussed in the literature yet. This paper tries to investigate and develop a new unique criterion in order to analyze the observability of a class of Takagi-Sugeno fuzzy systems. In the paper, first an observability criterion for a class of T-S fuzzy systems is introduced, then an observer is designed in order to estimate the states of the T-S fuzzy system, and later a controller is designed based on the estimated states of the system by the observer. The controller is then applied to the original nonlinear system in order to approve the applicability of the proposed observability criterion. The results of the presented example are found to be very promising, showing the viability of the proposed method.
KEYWORDS: T-S fuzzy systems, fuzzy controller, fuzzy observer, observability

1. INTRODUCTION

Since the time that fuzzy systems were introduced for the first time, by Zadeh [1] in 1965, they have gained an important role in control engineering and industry. Fuzzy systems use verbal expressions in order to process qualitative information. These systems can be considered as nonlinear dynamic systems that have the ability to approximate real systems, no matter how complicated, from empirical information, with the use of numerical calculations. The most important benefit of fuzzy systems is in dealing with the uncertainty of the parameters and variables of systems.

In general there are three types of uncertainty, namely *ambiguity*, *imprecision*, and *vagueness*. *Ambiguity* usually refers to a situation that one is working with random variables, for example in rolling a dice, one does not know for sure that which number will come out, but the result will become clear after rolling that dice. This kind of uncertainty can be dealt with probabilistic modeling. The second type of uncertainty, *imprecision*, happens when one cannot measure a parameter with enough precision; it is important to note that in this case the problem is not a lack of information, but it is an imprecision in the information, this problem can be solved by the use of some deterministic intervals for that parameter. Finally, the third kind of uncertainty, *vagueness*, happens when some parameters of a system do not have a clear and explicit definition, for instance when one is asked to name a large number, it is not clear that what number should be considered large. While this last kind of uncertainty cannot be modeled with probabilistic models, or by some intervals of numbers, fuzzy systems can model vagueness, very well. Though, it should be noticed that fuzzy logic is capable of modeling all the three kinds of uncertainties mentioned above.

Fuzzy systems imitate the approximate reasoning procedure performed in the human mind. The control mechanism used in these systems appears in the form of a set of IF-THEN rules. These rules have two parts, namely the antecedent and the consequence parts.

Fuzzy systems are generally classified into two groups, namely Mamdani fuzzy systems, and Takagi-Sugeno(T-S) fuzzy systems. The main difference between these two, lies in the consequence parts of their IF-THEN rules; while the former uses fuzzy sets and linguistic expressions as the consequence part of its rules, the latter uses a linear or affine model in its rules' consequence parts.

Over the past decades fuzzy systems have received a lot of attention, both in literature and application; and a lot of scientists have devoted a great deal of time and effort working on these systems. For example in 1974, E.Mamdani developed the first fuzzy logic controller to control the simple dynamic plant of a steam generator [2]. In 1981 cement kiln was controlled with fuzzy logic by Holmblad and Ostergaard [3], which is known to be the first industrial application of fuzzy systems. Fuzzy systems have also found some important applications in areas other than control engineering, for example the diagnosis of Chicken Pox and Measles have been performed using fuzzy relations[17], or decision support systems in employee recruitment have been designed in the fuzzy context[18], etc.

However, despite all the works done in the area of fuzzy systems, there have been only a few in-depth analyses on control concepts of these systems in the literature, concepts such as controllability, observability, and stability [4-7]. State controllability and observability were first addressed by Kalman [8] in 1960. These concepts have been later discussed by other scientists, for example, controllability and observability of nonlinear systems have been discussed by Hermann and Krener [9].

Kalman [8] defined the concepts of controllability and observability as answers to the following questions, respectively:

-"Is it possible to find an input that can move the state of a system from any initial state to any other final state in a finite time interval?"

-"Is it possible to determine the initial state of a system with regard to the knowledge of the input and output data of that system, in a finite time interval?"

The purpose of this paper is to discuss and determine the observability criteria of a class of T-S fuzzy systems. The T-S fuzzy model is described by a set of fuzzy IF-THEN rules that are a representation of the local input-output relations of a nonlinear system. The main feature of a T-S fuzzy model is to express the local dynamics of each fuzzy rule by a linear system

***Corresponding Author**: Navid Dini, Electrical Engineering Department, Amirkabir University of Technology, Tehran, Iran.
Email: naviddini@aut.

model; and the overall fuzzy model of the system is achieved by fuzzy blending of the linear system models.

The paper is organized in the following order: in section 2 we present some preliminaries and definitions on the linear time-invariant systems. Section 3 introduces the T-S fuzzy model which has been used in this work, both in continuous-time and discrete-time format. In section 4, a T-S fuzzy observer and a T-S fuzzy controller design will be introduced in order to observe the states of the fuzzy model and control the model based on the estimated states of the observer. Section 5 represents the main work of this paper, which will be presenting an observability criterion for the T-S fuzzy systems. Section 6 provides an application example of a cart-poled inverted pendulum. The model of the system will be presented in nonlinear form, and then a T-S fuzzy model will be derived based on this nonlinear mode/l of the system and it will show that the observability criterion introduced in this paper, determines the observability of a T-S fuzzy system properly. And finally, section 7 concludes the paper and provides some suggestions for future works.

2. PRELIMINARY AND DEFENITIONS

In this section, some preliminaries and definitions about the linear time-invariant (LTI) systems, will be reviewed.

2.1 Observability of LTI systems

As explained in [10], observability is a property of coupling between the state and the output of the system and thus involves the matrices $\mathbf{A}$ and $\mathbf{C}$.

Definition1: A linear system is said to be observable at t_0, if $\mathbf{x}(t_0)$ can be determined from the output function $\mathbf{y}_{[t_0,t_1]}$, for $t_0 \in T$ and $t_0 \le t_1$, where t_1 is some finite time belonging to T. If this is true for all t_0 and $\mathbf{x}(t_0)$, the system is said to be completely observable [10].

2.2 Observability Criterion

Consider a linear time invariant system as follows

$$\begin{cases} \dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}u(t) \\ \mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{D}u(t) \end{cases} \tag{1}$$

where $\mathbf{x}(t) \in \Re^n$, $u(t) \in \Re^m$, and $\mathbf{y}(t) \in \Re^p$, represent the state, input and output vectors of the system, respectively. And $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, and $\mathbf{D}$ are constant real valued $n \times n$, $n \times m$, $p \times n$, and $p \times m$ matrices, respectively.

The system (1) is said to be observable, if and only if the $n \times mn$ dimensional matrix (2), which is called the observability matrix, has rank n.

$$\mathbf{\Phi}_O = \begin{bmatrix} \mathbf{C}^T & \mathbf{A}^T\mathbf{C}^T & \mathbf{A}^{2^T}\mathbf{C}^T & \dots & \mathbf{A}^{n-1^T}\mathbf{C}^T \end{bmatrix}^T \tag{2}$$

2.3Similarity transformation

A modal matrix can be formed for the LTI system (1), by the use of the linearly independent eigenvectors $\mathbf{V}_1, \mathbf{V}_2, ..., \mathbf{V}_n$ of matrix $\mathbf{A}$, as shown in (3).

$$M = [\mathbf{V}_1, \mathbf{V}_2, ..., \mathbf{V}_n] \tag{3}$$

Special case: If the state matrix $\mathbf{A}$ of the system (1) is in one of the following forms which are named *the companion form*,

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & & & & \\ 0 & 0 & 0 & \dots & 1 \\ -a_0 & -a_1 & -a_2 & \dots & -a_{n-1} \end{bmatrix} \tag{4}$$

Or

$$\mathbf{A} = \begin{bmatrix} -a_0 & -a_1 & -a_2 & \dots & -a_{n-1} \\ 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & \dots & 0 \\ \vdots & & & \vdots & \\ 0 & 0 & 0 & \dots & 1 \end{bmatrix} \tag{5}$$

Where the characteristic equation of the system will be

$$|\lambda I - A| = \lambda^n + a_{n-1}\lambda^{n-1} + ... + a_0 \tag{6}$$

for both of them; the modal matrix can be formed as the Vandermond matrix, which could be written as (7) and (8) for state matrices (4) and (5), respectively.

$$M=\begin{bmatrix}1 & 1 & 1 & \dots & 1\\ \lambda_1 & \lambda_2 & \lambda_3 & \dots & \lambda_n\\ \lambda_1^2 & \lambda_2^2 & \lambda_3^2 & \dots & \lambda_n^2\\ \vdots & \vdots & & & \vdots\\ \lambda_1^{n-1} & \lambda_2^{n-1} & \lambda_3^{n-1} & \dots & \lambda_n^{n-1}\end{bmatrix} \tag{7}$$

$$M=\begin{bmatrix}\lambda_1^{n-1} & \lambda_2^{n-1} & \lambda_3^{n-1} & \dots & \lambda_n^{n-1}\\ \vdots & \vdots & & & \vdots\\ \lambda_1^2 & \lambda_2^2 & \lambda_3^2 & \dots & \lambda_n^2\\ \lambda_1 & \lambda_2 & \lambda_3 & \dots & \lambda_n\\ 1 & 1 & 1 & \dots & 1\end{bmatrix} \tag{8}$$

With the use of the modal matrix we can transform the original LTI model (1), into a linear model with diagonal state matrices; this transformation is called the *similarity transformation*, which can be performed by defining a new state vector defined as $\mathbf{z}$, such that $\mathbf{x}=\mathbf{Mz}$. The new linear model will be

$$\begin{cases}\dot{\mathbf{z}}=\mathbf{\Lambda z}+\boldsymbol{\beta}\mathbf{u}\\ \mathbf{y}=\boldsymbol{\gamma}\mathbf{z}+\mathbf{Du}\end{cases} \tag{9}$$

where $\Lambda=M^{-1}AM$, $\beta=M^{-1}B$, $\gamma=CM$, and D represent the new realization of system (1). It should be noted that Λ, the state matrix of this new realization, would be a diagonal matrix.

3. TAKAGI-SUGENO FUZZY MODEL

As mentioned in the introduction of this paper, a dynamic system's fuzzy model is described by a set of IF-THEN rules. Also it should be noted that in order to investigate the observability, it is better to introduce a model that has inputs and state variables, so that the observability can be investigated easily.

The fuzzy IF_THEN rules of a T-S fuzzy model, represent local linear input-output relations of a nonlinear system. The main feature of a T-S fuzzy model is to express the local dynamics of each fuzzy rule by a linear system model. Then, the overall fuzzy model of the system is achieved by fuzzy blending of the linear system models.

3.1 The continuous-time T-S fuzzy model

We define the continuous-time T-S fuzzy model of a multi-input multi-output nonlinear system, with the following spaces of states, inputs and outputs, respectively:

$$\mathbf{x}\in\Re^n;\mathbf{u}\in\Re^m;\mathbf{y}\in\Re^q$$

as described in [15], as follows:

R^i:

if $z_1(t)\ is\ A_1^i\ \&.....\&\ z_p(t)\ is\ A_p^i$

then

$$\begin{cases}\dot{\mathbf{x}}^i(t)=\mathbf{A}_i\mathbf{x}^i(t)+\mathbf{B}_i\mathbf{u}^i(t)\\ \mathbf{y}^i(t)=\mathbf{C}_i\mathbf{x}^i(t)\end{cases} \tag{10}$$

Here, R^i refers to the i-th fuzzy rule $(i=1,...,s)$; s is the number of rules; $A_j^i\ (j=1,...,p)$ are fuzzy sets with piecewise continuous polynomial functions; the matrices $\mathbf{A}_i$, $\mathbf{B}_i$, and $\mathbf{C}_i$, are matrices with constant real parameters with dimensions $n\times n$, $n\times m$, and $q\times n$, respectively. And $z_1(t),z_2(t),...,z_p(t)$, are known premise variables that may be functions of state variables, external disturbances and/or time. Here we will use $\mathbf{z}(t)$ to denote the vector containing all the individual elements $z_1(t),z_2(t),...,z_p(t)$.

The final outputs of the fuzzy model can be inferred as the weighted summation of the consequence parts of the fuzzy IF-THEN rules, as follows:

$$\dot{\mathbf{x}}(t)=\sum_{i=1}^{s}\alpha_i\left\{\mathbf{A}_i\mathbf{x}(t)+\mathbf{B}_i\mathbf{u}(t)\right\} \tag{11}$$

and

$$\mathbf{y}(t)=\sum_{i=1}^{s}\alpha_i\left\{\mathbf{C}_i\mathbf{x}(t)\right\} \tag{12}$$

where α_i are calculated from the weights w_i as

$$\alpha_i = \frac{w_i}{\sum_{i=1}^{s} w_i} \tag{13}$$

and the weights w_i are defined as:

$$w_i = \prod_{j=1}^{n} A_j^i \left(x_j(t) \right) \tag{14}$$

3.2 The discrete-time T-S fuzzy model

The continuous-time T-S fuzzy model introduced in equations (10)-(14) can be generalized to give the following discrete-time fuzzy model. This model has also been introduced in [15].

In this paper we discuss our method on the continuous-time T-S fuzzy model, but the results can be easily generalized to the following discrete-time fuzzy model as well.

R^i:

If $z_1(k)$ *is* A_1^i &.....& $z_p(k)$ *is* A_p^i

$$\text{then} \begin{cases} \mathbf{x}^i(k+1) = \mathbf{A}_i \mathbf{x}(k) + \mathbf{B}_i \mathbf{u}(k) \\ \mathbf{y}^i(k) = \mathbf{C}_i \mathbf{x}(k) \end{cases} \tag{15}$$

Here R^i again refers to the i-th fuzzy rule $(i = 1, ..., s)$; s is the number of fuzzy rules; $A_j^i \ (j = 1, ..., p)$ are fuzzy sets. And the matrices $\mathbf{A}_i$, $\mathbf{B}_i$, and $\mathbf{C}_i$ are matrices with constant real parameters and with the dimensions $n \times n$, $n \times m$, and $q \times n$, respectively, and where the state and input vectors $\mathbf{x}(k)$ and $\mathbf{u}(k)$ are defined as

$$\mathbf{x}(k) = \left[x(k) \quad x(k-1) \quad \ldots \quad x(k-n+1) \right]$$
$$\mathbf{u}(k) = \left[u(k) \quad u(k-1) \quad \ldots \quad u(k-m+1) \right]$$

As in the continues-time T-S fuzzy model, here we have $z_1(k), z_2(k), ..., z_p(k)$, as known premise variables that may be functions of state variables, external disturbances and/or time. And we will again use $\mathbf{z}(k)$ to denote the vector containing all the individual elements $z_1(k), z_2(k), ..., z_p(k)$.

The final outputs of this fuzzy model are inferred as follows:

$$\mathbf{x}(k+1) = \sum_{i=1}^{s} \alpha_i \left\{ \mathbf{A}_i \mathbf{x}(k) + \mathbf{B}_i \mathbf{u}(k) \right\} \tag{16}$$

$$\mathbf{y}(k) = \sum_{i=1}^{s} \alpha_i \left\{ \mathbf{C}_i \mathbf{x}(k) \right\} \tag{17}$$

where α^i are calculated from the weights w^i as

$$\alpha_i = \frac{w_i}{\sum_{i=1}^{s} w_i} \tag{18}$$

and the weights w^i are given by:

$$w_i = \prod_{j=1}^{n} A_j^i \left(x(k-j+1) \right) \tag{19}$$

4. THE OBSERVER AND CONTROLLER DESIGN

In this section a fuzzy observer and a fuzzy controller model will be introduced for the T-S fuzzy model (10). The fuzzy observer and controller used in this paper were originally introduced in [13].

For the fuzzy controller design, we should first design the local state feedback controllers based on the pairs $(\mathbf{A}_i, \mathbf{B}_i)$ in each of the fuzzy IF-THEN rules of the system model (10).

Now the fuzzy controller for the T-S fuzzy system (10) would be defined as follows:

Controller rule i:

if $\mathrm{z}_1(t)$ *is* A_1^i &.....& $z_p(t)$ *is* A_p^i

Then $\mathbf{u}(t) = -\mathbf{K}_i \mathbf{x}(t), \ i = 1, 2, ...s$ (20)

The final output of this fuzzy controller will be inferred as follows,

$$\mathbf{u}(t) = -\sum_{i=1}^{s} \alpha_i \mathbf{K}_i \mathbf{x}(t), \; i = 1, 2, ..., s \qquad (21)$$

Where $\mathbf{K}_i$ (i=1,2,…,s), are the controller parameters in every fuzzy IF-THEN rule, and the normalized weights α_i are given by equation (13). Substituting (21) into (11) and (12), we will obtain the following equations, for the final states and outputs of the fuzzy model (10), respectively:

$$\dot{\mathbf{x}}(t) = \sum_{i=1}^{s} \sum_{j=1}^{s} \alpha_i \alpha_j \left(\mathbf{A}_i - \mathbf{B}_i \mathbf{K}_j \right) \mathbf{x}(t) \quad (22)$$

$$\mathbf{y}(t) = \sum_{i=1}^{s} \alpha_i \mathbf{C}_i \mathbf{x}(t) \qquad (23)$$

One of the problems that one may happen to come across, while trying to design a controller is that, not all of the states of a system may be observable; so it is necessary to use a fuzzy observer for the T-S fuzzy system (10), in order to be able to implement the controller (20) on this system.

In order to design a fuzzy observer, we should first design local observers for every linear sub-system of the T-S fuzzy system (10), based on the triplets $(\mathbf{A}_i, \mathbf{B}_i, \mathbf{C}_i)$. The model of the fuzzy observer, introduced in [13], is as follows:

Observer rule i:

if $z_1(t)\, is\ A_1^i \;\&.....\&\; z_p(t)\, is\ A_p^i$

Then

$$\dot{\hat{\mathbf{x}}}(t) = \mathbf{A}_i \hat{\mathbf{x}}(t) + \mathbf{B}_i \mathbf{u}(t) + \mathbf{G}_i \left[\mathbf{y}(t) - \hat{\mathbf{y}}(t) \right] \quad (24)$$

$$\hat{\mathbf{y}}_i(t) = \mathbf{C}_i \hat{\mathbf{x}}(t), i = 1, 2, ..., s$$

Where $\mathbf{G}_i \; (i = 1, 2, ..., s)$ are the observation error matrices. The final estimated state and the final output of this fuzzy observer are

$$\dot{\hat{\mathbf{x}}}(t) = \sum_{i=1}^{s} \alpha_i \left\{ \mathbf{A}_i \hat{\mathbf{x}}(t) + \mathbf{B}_i \mathbf{u}(t) + \mathbf{G}_i \left[\mathbf{y}(t) - \hat{\mathbf{y}}(t) \right] \right\} \quad (25)$$

$$\hat{\mathbf{y}}(t) = \sum_{i=1}^{s} \alpha_i \mathbf{C}_i \hat{\mathbf{x}}(t) \qquad (26)$$

Substituting (26) and (12) into (25), the estimated states of the fuzzy observer will be obtained as follows:

$$\dot{\hat{\mathbf{x}}}(t) = \sum_{i=1}^{s} \alpha_i \left\{ \mathbf{A}_i \hat{\mathbf{x}}(t) + \mathbf{B}_i \mathbf{u}(t) \right\} + \sum_{i=1}^{s} \sum_{j=1}^{s} \alpha_i \alpha_j \mathbf{G}_i \mathbf{C}_j \left[\mathbf{x}(t) - \hat{\mathbf{x}}(t) \right] \qquad (27)$$

With the use of the states estimated by the observer (24), the controller (20) can be rewritten as

Controller rule i:

if $z_1(t)\, is\ A_1^i \;\&.....\&\; z_p(t)\, is\ A_p^i$

then $\mathbf{u}(t) = -\mathbf{K}_i \hat{\mathbf{x}}(t), \; i = 1, 2, ...s \qquad (28)$

The T-S fuzzy observer (24) and controller (20) can be easily generalized for the discrete-time fuzzy model (15)-(19). For the fuzzy controller design, again we should first design the local state feedback controllers based on the pairs $(\mathbf{A}_i, \mathbf{B}_i)$ in each of the fuzzy IF-THEN rules of the system model (16), which will result in the following fuzzy controller:

Controller rule i:

$If \;\; z_1(k) \;\; isA_1^i \;\&.....\&\; z_p(k) \;\; isA_p^i$

Then $\mathbf{u}(k) = -\mathbf{K}_i \mathbf{x}(k), \; i = 1, 2, ...s \qquad (29)$

The final output of this fuzzy controller will be inferred as follows,

$$u(k) = -\sum_{i=1}^{s} \alpha_i \mathbf{K}_i \mathbf{x}(k), \; i = 1, 2, ..., s \qquad (30)$$

Where $\mathbf{K}_i \; (i = 1, 2, ..., s)$, are the controller parameters in every fuzzy IF-THEN rule, and the normalized weights α_i are given by equation (18). Substituting (30) into (16) and (17), we will obtain the following equations, for the final states and outputs of the fuzzy model (15), respectively:

$$\mathbf{x}(k+1) = \sum_{i=1}^{s} \sum_{j=1}^{s} \alpha_i \alpha_j \left(\mathbf{A}_i - \mathbf{B}_i \mathbf{K}_j \right) \mathbf{x}(k) \quad (31)$$

$$\mathbf{y}(k) = \sum_{i=1}^{s} \alpha_i \mathbf{C}_i \mathbf{x}(k) \qquad (32)$$

Now, the fuzzy observer for the discrete-time fuzzy system should be defined based on the triplets (A_i, B_i, C_i), as follows:

Observer rule i:

$If \;\; z_1(k) \;\; isA_1^i \;\&.....\&\; z_p(k) \;\; isA_p^i$

Then

$$\hat{\mathbf{x}}(k+1) = \mathbf{A}_i\hat{\mathbf{x}}(k) + \mathbf{B}_i\mathbf{u}(k) + \mathbf{G}_i\left[\mathbf{y}(k) - \hat{\mathbf{y}}(k)\right] \quad (33)$$
$$\hat{\mathbf{y}}_i(k) = \mathbf{C}_i\hat{\mathbf{x}}(k), i = 1,2,...,s$$

Where $\mathbf{G}_i\ (i = 1,2,...,s)$ are the observation error matrices. The final estimated state and the final output of this fuzzy observer are

$$\hat{\mathbf{x}}(k+1) = \sum_{i=1}^{s}\alpha_i\left\{\mathbf{A}_i\hat{\mathbf{x}}(k) + \mathbf{B}_i\mathbf{u}(k) + \mathbf{G}_i\left[\mathbf{y}(k) - \hat{\mathbf{y}}(k)\right]\right\} \quad (34)$$

$$\hat{\mathbf{y}}(k) = \sum_{i=1}^{s}\alpha_i\mathbf{C}_i\hat{\mathbf{x}}(t) \quad (35)$$

Substituting (35) and (17) into (34), the estimated states of the fuzzy observer will be obtained as follows:

$$\hat{\mathbf{x}}(k+1) = \sum_{i=1}^{s}\alpha_i\left\{\mathbf{A}_i\hat{\mathbf{x}}(k) + \mathbf{B}_i\mathbf{u}(k)\right\} + \sum_{i=1}^{s}\sum_{j=1}^{s}\alpha_i\alpha_j\mathbf{G}_i\mathbf{C}_j\left[\mathbf{x}(k) - \hat{\mathbf{x}}(k)\right] \quad (36)$$

With the use of the states estimated by the observer (33), the controller (29) can be rewritten as

Controller rule i:

If $z_1(k)\ isA_1^i\ \&.....\&\ z_p(k)\ isA_p^i$

then $\mathbf{u}(k) = -\mathbf{K}_i\hat{\mathbf{x}}(k),\ i = 1,2,...s$ (37)

Details on the stability analysis of the county fuzzy observer and controller models of the continuous-time T-S fuzzy model can be found in [13].

5. OBSERVABILITY ANALYSIS

In this section the observability of the Takagi-Sugeno fuzzy model discussed in the previous sections, would be investigated.

In order to determine the observability of the continuous-time T-S fuzzy model (10), we limit our investigation to a special class of this model in which, the linear models in the consequence parts, all have state matrices with distinct eigenvalues, so that we can perform the similarity transformation on them and find a new realization of them with diagonal state matrices.

Consider the T-S fuzzy system to have *n* IF-THEN rules and therefore n linear time-invariant sub-systems as follows

$$s_i = \begin{cases}\dot{\mathbf{x}}(t) = \mathbf{A}_i\mathbf{x}(t) + \mathbf{B}_i\mathbf{u}(t)\\ \mathbf{y}(t) = \mathbf{C}_i\mathbf{x}(t) + \mathbf{D}_i\mathbf{u}(t)\end{cases} \quad (38)$$

Consider all of the sub-systems, s_i .in (38) to have distinct eigenvalues so that we can perform a similarity transformation on them. After this transformation, the sub-systems will transform to the following formations

$$s_i' : \begin{cases}\dot{\mathbf{z}}(t) = \mathbf{\Lambda}_i\mathbf{z}(t) + \boldsymbol{\beta}_i\mathbf{u}(t)\\ \mathbf{y}(t) = \boldsymbol{\gamma}_i\mathbf{z}(t) + \mathbf{D}_i\mathbf{u}(t)\end{cases} \quad (39)$$

where

$\mathbf{\Lambda}_i = diag\left\{\lambda_{i1}, \lambda_{i2}, ..., \lambda_{ij}\right\}; (i = 1,...,s), (j = 1,...,n)$ with s being the number of fuzzy IF-THEN rules and in other words, the number of the sub-systems, and n being the number of states of each of the sub-systems.

Now according to (11) and (12), the final output of the T-S fuzzy system, based on the new sub-systems (39), will be

$$\dot{\mathbf{x}}(t) = \left\{\mathbf{\Lambda}\mathbf{x}(t) + \boldsymbol{\beta}\mathbf{u}(t)\right\} \quad (40)$$

and

$$\mathbf{y}(t) = \boldsymbol{\gamma}\mathbf{x}(t) \quad (41)$$

where

$$\mathbf{\Lambda} = \sum_{i=1}^{s}\mathbf{\Lambda}_i = diag\{\alpha_1\lambda_{11} + \alpha_2\lambda_{21} + ... + \alpha_s\lambda_{s1}, \alpha_1\lambda_{12} + \alpha_2\lambda_{22} + ... + \alpha_s\lambda_{s2}, ...,\ \alpha_1\lambda_{1n} + \alpha_2\lambda_{2n} + ... + \alpha_s\lambda_{sn}\} \quad (42)$$

$$\boldsymbol{\beta} = \sum_{i=1}^{s}\boldsymbol{\beta}_i \quad (43)$$

and

$$\boldsymbol{\gamma} = \sum_{i=1}^{s}\boldsymbol{\gamma}_i \quad (44)$$

Forming the observability matrix (2) for state matrix (42), and vector $\boldsymbol{\gamma}$ in (44), we will have

$$\mathbf{\Phi}_O = \left[\boldsymbol{\gamma}^T \quad \mathbf{\Lambda}^T\boldsymbol{\gamma}^T \quad \mathbf{\Lambda}^{2^T}\boldsymbol{\gamma}^T \quad ... \quad \mathbf{\Lambda}^{n-1^T}\boldsymbol{\gamma}^T\right]^T \quad (45)$$

Now by assuming the vector $\boldsymbol{\gamma}$ to be

$$\boldsymbol{\gamma} = \begin{bmatrix} \Gamma_1 & \Gamma_2 & .... & \Gamma_n \end{bmatrix} \quad (46)$$

And with the state matrix $\boldsymbol{\Lambda}$ in (42), the observability matrix $\boldsymbol{\Phi}_o$, will be written as follows:

$$\boldsymbol{\Phi}_O = \begin{bmatrix} \Gamma_1 & \Gamma_2 & ... & \Gamma_n \\ \Gamma_1\left(\alpha_1\lambda_{11}+\alpha_2\lambda_{21}+...+\alpha_n\lambda_{n1}\right) & \Gamma_2\left(\alpha_1\lambda_{12}+\alpha_2\lambda_{22}+...+\alpha_n\lambda_{n2}\right) & ... & \Gamma_n\left(\alpha_1\lambda_{1m}+\alpha_2\lambda_{2m}+...+\alpha_n\lambda_{nm}\right) \\ \vdots & & & \vdots \\ \Gamma_1\left(\alpha_1\lambda_{11}+\alpha_2\lambda_{21}+...+\alpha_n\lambda_{n1}\right)^{n-1} & \Gamma_2\left(\alpha_1\lambda_{12}+\alpha_2\lambda_{22}+...+\alpha_n\lambda_{n2}\right)^{n-1} & ... & \Gamma_n\left(\alpha_1\lambda_{1m}+\alpha_2\lambda_{2m}+...+\alpha_n\lambda_{nm}\right)^{n-1} \end{bmatrix} \quad (47)$$

According to the observability criterion explained in 2.1.1, in order for the system (10) to be observable, the observability matrix (48) must have rank n , in other words the rows of the matrix should be linearly independent, which obviously happens when all of the terms $\alpha_1\lambda_{11}+\alpha_2\lambda_{21}+...+\alpha_s\lambda_{s1}$, $\alpha_1\lambda_{12}+\alpha_2\lambda_{22}+...+\alpha_s\lambda_{s2}$, … , and $\alpha_1\lambda_{1n}+\alpha_2\lambda_{2n}+...+\alpha_s\lambda_{sn}$ can be proved not to be equal to each other, and none of the terms $\Gamma_1,\ldots,\Gamma_n$, of the vector $\boldsymbol{\gamma}$, be zero.

Theorem1: The continuous-time T-S fuzzy system (10) would be observable, if and only if the, linear sub-systems in its consequence parts, all have distinct eigenvalues and moreover, none of the terms $\Gamma_1,\ldots,\Gamma_n$, in the vector $\boldsymbol{\gamma}$ in (44) be zero.

Proof: In order for matrix (47) to have a full rank, we should have:

$$\begin{aligned} &\alpha_1\lambda_{11}+\alpha_2\lambda_{21}+...+\alpha_s\lambda_{s1} \neq \alpha_1\lambda_{12}+\alpha_2\lambda_{22}+...+\alpha_s\lambda_{s2} \\ &\vdots \\ &\alpha_1\lambda_{11}+\alpha_2\lambda_{21}+...+\alpha_s\lambda_{s1} \neq \alpha_1\lambda_{1n}+\alpha_2\lambda_{2n}+...+\alpha_s\lambda_{sn} \\ &\alpha_1\lambda_{12}+\alpha_2\lambda_{22}+...+\alpha_s\lambda_{s2} \neq \alpha_1\lambda_{13}+\alpha_2\lambda_{23}+...+\alpha_s\lambda_{s3} \\ &\vdots \\ &\alpha_1\lambda_{1(n-1)}+\alpha_2\lambda_{2(n-1)}+...+\alpha_s\lambda_{s(n-1)} \neq \alpha_1\lambda_{1n} \\ &\qquad +\alpha_2\lambda_{2n}+...+\alpha_s\lambda_{sn} \end{aligned} \quad (48)$$

Inequalities (48) can be rewritten in the following form:

$$\begin{aligned} &\alpha_1\left(\lambda_{11}-\lambda_{12}\right)+\alpha_2\left(\lambda_{21}-\lambda_{22}\right)+...+\alpha_s\left(\lambda_{s1}-\lambda_{s2}\right)\neq 0 \\ &\vdots \\ &\alpha_1\left(\lambda_{11}-\lambda_{1n}\right)+\alpha_2\left(\lambda_{21}-\lambda_{2n}\right)+...+\alpha_s\left(\lambda_{s1}-\lambda_{sn}\right)\neq 0 \\ &\alpha_1\left(\lambda_{12}-\lambda_{13}\right)+\alpha_2\left(\lambda_{22}-\lambda_{23}\right)+...+\alpha_s\left(\lambda_{s2}-\lambda_{s3}\right)\neq 0 \\ &\vdots \\ &\alpha_1\left(\lambda_{1(n-1)}-\lambda_{1n}\right)+\alpha_2\left(\lambda_{2(n-1)}-\lambda_{2n}\right)+...+\alpha_s\left(\lambda_{s(n-1)}-\lambda_{sn}\right)\neq 0 \end{aligned} \quad (49)$$

Inequalities (49) will obviously hold, because we have assumed the subsystems $s_1,....,s_s$ to have distinct eigenvalues, and also the normalized weights of the fuzzy rules, $\alpha_1,...,\alpha_s$ cannot be zero all together. So the only criterion for the fuzzy system (10) to be observable, would be

for the terms $\Gamma_1,\ldots,\Gamma_n$, of the vector $\boldsymbol{\gamma}$, not to be zero.

Another condition that should hold in order for the matrix $\boldsymbol{\Phi}_o$, in (47) to be of full rank, is that none of the columns of this matrix be a linear combination of the other rows. This can also be certified to be true because of the distinct eigenvalues assumption. A similar conclusion can be derived for the discrete-time T-S fuzzy system model (15).

6. APPLICATION EXAMPLE

To show the capability of the observability method presented here, an application example of an inverted pendulum, [6], is given in this section. First the T-S fuzzy model of the inverted pendulum will be derived from the nonlinear physical equations of the system in the same format of (10), and then the observability of the T-S fuzzy system will be investigated with the use of a fuzzy observer and a fuzzy controller; first we design a fuzzy observer to estimate the states of the T-S fuzzy model of the inverted pendulum and then a controller will be designed to control this system based on the estimated states of the observer, finally this controller will be applied to the original nonlinear model of the inverted pendulum system to show that the controller designed for the T-S fuzzy mode can control the original nonlinear model ,efficiently. In this way, because the controller designed based on the observed states of the fuzzy model works properly for the nonlinear model too, it would become clear that the T-S fuzzy model was observable.

The diagram of the inverted pendulum can be seen in figure1. We use the dynamical equation of a cart-poled inverted pendulum system given in [6]

$$\ddot{\theta}(t)=\frac{g\sin(\theta(t))-aml\dot{\theta}(t)^2\sin(2\theta(t))/2-a\cos(\theta(t))u(t)}{4l/3-aml\cos^2(\theta(t))} \quad (50)$$

and the output of the system is

$$y(t)=\theta(t) \quad (51)$$

Where θ is the angular displacement of the pendulum; $g=9.8\,m/s^2$ is the acceleration due to gravity; $m=2kg$ and $M=8kg$ are the mass of the pendulum and mass of the cart, respectively. $2l=1m$ is the length of the pendulum. We also have that $a=1/(m+M)$, and finally u would be the control input applied to the cart.

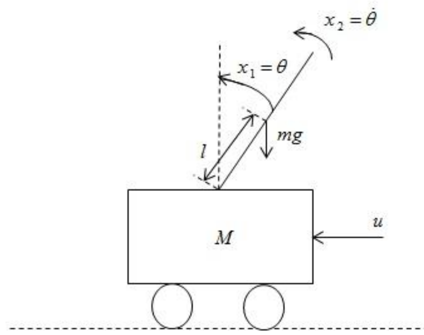

Figure 1. Cart-pole inverted pendulum system

By assuming the states of the system, x_1 and x_2, to be the angular position, θ, and the angular velocity, $\dot{\theta}$, respectively, one can write the state space model of system (32) and (33) as follows:

$$\begin{cases}\dot{x}_1(t)=x_2(t)\\ \dot{x}_2(t)=\dfrac{g\sin(x_1(t))-amlx_2^2(t)\sin(2x_1(t))/2-a\cos(x_1(t))u(t)}{4l/3-aml\cos^2(x_1(t))}\end{cases} \quad (52)$$

$$y(t)=x_1(t)$$

The nonlinear system (52) can be represented by a T-S fuzzy model in the form of the model (10) consisting of eight rules. In order to derive the T-S fuzzy model of this system we use the method, based on *sector nonlinearity*, presented in [15].

Here we assume

$$x_1(t)\in[x_{1\min}\quad x_{1\max}]=\left[-\frac{22\pi}{45}\quad\frac{22\pi}{45}\right] \quad (53)$$

and

$$x_2(t)\in[x_{2\min}\quad x_{2\max}]=[-5\quad 5] \quad (54)$$

Now, in order to derive the T-S fuzzy model of the system, we define the nonlinearities of the state space model, as the *fuzzy variables* of our fuzzy model, as follows

$$z_1=\frac{g\sin x_1/x_1}{4l/3-aml\cos^2(x_1)} \quad (55)$$

$$z_2=\frac{-aml\sin(2x_1)/2}{4l/3-aml\cos^2(x_1)}x_2 \quad (56)$$

and

$$z_3=\frac{-a\cos(x_1)}{4l/3-aml\cos^2(x_1)}. \quad (57)$$

Based on the assumptions made on the intervals of the state variables $x_1(t)$ and $x_2(t)$ in (51) and (52), the minimum and maximum values of the fuzzy variables $z_i\ (i=1,2,3)$,in (53)-(55) are calculated as follows

$$z_{1\min} = 9.567; z_{1\max} = 17.29$$
$$z_{2\min} = -0.0262; z_{2\max} = 0.0262$$
$$z_{3\min} = -0.1765; z_{3\max} = -0.0052$$

The T-S fuzzy rules can be written as follows:

Rule1: If z_1 is A_1^1 and z_2 is A_2^1 and z_3 is B_1^1
Then $\dot{\mathbf{x}} = \mathbf{A}_1\mathbf{x} + \mathbf{B}_1\mathbf{u}$

Rule2: If z_1 is A_1^1 and z_2 is A_2^1 and z_3 is B_1^2
Then $\dot{\mathbf{x}} = \mathbf{A}_2\mathbf{x} + \mathbf{B}_2\mathbf{u}$

Rule3: If z_1 is A_1^1 and z_2 is A_2^2 and z_3 is B_1^1
Then $\dot{\mathbf{x}} = \mathbf{A}_3\mathbf{x} + \mathbf{B}_3\mathbf{u}$

Rule4: If z_1 is A_1^1 and z_2 is A_2^2 and z_3 is B_1^2
Then $\dot{\mathbf{x}} = \mathbf{A}_4\mathbf{x} + \mathbf{B}_4\mathbf{u}$

Rule5: If z_1 is A_1^2 and z_2 is A_2^1 and z_3 is B_1^1
Then $\dot{\mathbf{x}} = \mathbf{A}_5\mathbf{x} + \mathbf{B}_5\mathbf{u}$

Rule6: If z_1 is A_1^2 and z_2 is A_2^1 and z_3 is B_1^2
Then $\dot{\mathbf{x}} = \mathbf{A}_6\mathbf{x} + \mathbf{B}_6\mathbf{u}$

Rule7: If z_1 is A_1^2 and z_2 is A_2^2 and z_3 is B_1^1
Then $\dot{\mathbf{x}} = \mathbf{A}_7\mathbf{x} + \mathbf{B}_7\mathbf{u}$

Rule8: If z_1 is A_1^2 and z_2 is A_2^2 and z_3 is B_1^2
Then $\dot{\mathbf{x}} = \mathbf{A}_8\mathbf{x} + \mathbf{B}_8\mathbf{u}$

The matrices A_i and B_i used in the consequence parts of the T-S fuzzy rules above, can be easily calculated as follows

$$\mathbf{A}_1 = \mathbf{A}_2 = \begin{bmatrix} 0 & 1 \\ z_{1\min} & z_{2\min} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 9.567 & -0.0262 \end{bmatrix};$$

$$\mathbf{A}_3 = \mathbf{A}_4 = \begin{bmatrix} 0 & 1 \\ z_{1\min} & z_{2\max} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 9.567 & 0.0262 \end{bmatrix};$$

$$\mathbf{A}_5 = \mathbf{A}_6 = \begin{bmatrix} 0 & 1 \\ z_{1\max} & z_{2\min} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 17.29 & -0.0262 \end{bmatrix};$$

and

$$\mathbf{A}_7 = \mathbf{A}_8 = \begin{bmatrix} 0 & 1 \\ z_{1\max} & z_{2\max} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 17.29 & 0.0262 \end{bmatrix}.$$

$$\mathbf{B}_1 = \mathbf{B}_3 = \mathbf{B}_5 = \mathbf{B}_7 = \begin{bmatrix} 0 \\ z_{3\min} \end{bmatrix} = \begin{bmatrix} 0 \\ -0.1765 \end{bmatrix};$$

and

$$\mathbf{B}_2 = \mathbf{B}_4 = \mathbf{B}_6 = \mathbf{B}_8 = \begin{bmatrix} 0 \\ z_{3\max} \end{bmatrix} = \begin{bmatrix} 0 \\ -0.0052 \end{bmatrix};$$

From the maximum and minimum values, the membership functions for the fuzzy plant model, which can be seen in figures 2-4, can be represented by

$$z_1(t) = \mu_{A_1^1}(z_1(t)) \cdot 9.567 + \mu_{A_1^2}(z_1(t)) \cdot 17.29$$
$$z_2(t) = \mu_{A_2^1}(z_2(t)) \cdot (-0.0262) + \mu_{A_2^2}(z_2(t)) \cdot 0.0262$$

and

$$z_3(t) = \mu_{B_1^1}(z_3(t)) \cdot (-0.1765) + \mu_{B_1^2}(z_3(t)) \cdot (-0.0052)$$

where

$$\mu_{A_1^1}(z_1(t)) + \mu_{A_1^2}(z_1(t)) = 1$$
$$\mu_{A_2^1}(z_2(t)) + \mu_{A_2^2}(z_1(t)) = 1$$

$$\mu_{B_1^1}\left(z_3(t)\right)+\mu_{B_1^2}\left(z_3(t)\right)=1$$

Therefore, the membership functions can be calculated as:

$$\mu_{A_1^1}\left(z_1(X(t))\right)=\frac{-z_1(X(t))+z_{1\max}}{z_{1\max}-z_{1\min}};\mu_{A_1^2}\left(z_1(X(t))\right)=1-\mu_{A_1^1}\left(z_1(X(t))\right)$$

$$\mu_{A_2^1}\left(z_2(X(t))\right)=\frac{-z_2(X(t))+z_{2\max}}{z_{2\max}-z_{2\min}}$$

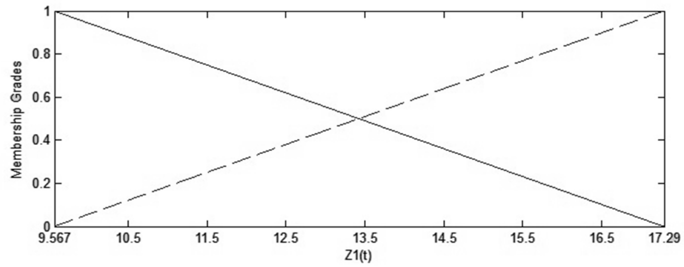

Figure 2- The membership functions for the premise variable $z_1(t)$ of the application example

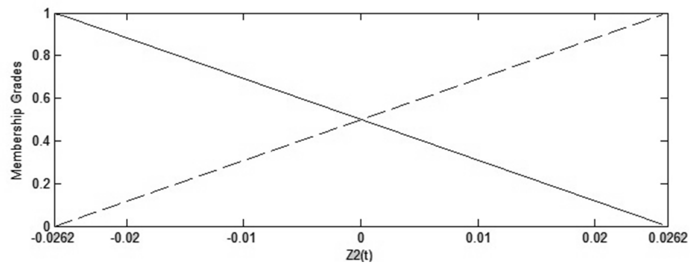

Figure 3- The membership functions for the premise variable $z_2(t)$ of the application example

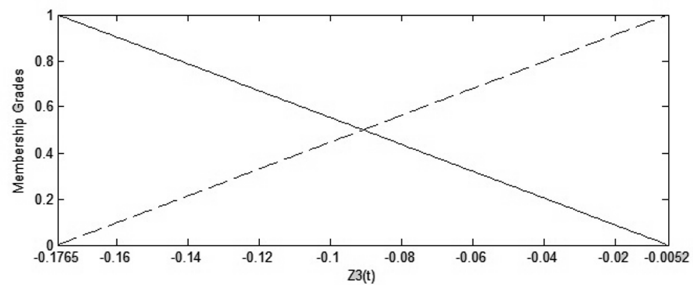

Figure 4- The membership functions for the premise variable $z_3(t)$ of the application example

$$\mu_{A_2^2}\left(z_2(X(t))\right)=1-\mu_{A_2^1}\left(z_2(X(t))\right)$$

$$\mu_{B_1^1}\left(z_3(X(t))\right)=\frac{-z_3(X(t))+z_{3\max}}{z_{3\max}-z_{3\min}}$$

and

$$\mu_{B_1^2}\left(z_3(X(t))\right)=1-\mu_{B_1^1}\left(z_3(X(t))\right)$$

Now we design an observer and a controller for this T-S fuzzy model, based on the observer and controller models (15), and (11). First by choosing the closed loop eigenvalues [-7 -3] for $\mathbf{A}_i-\mathbf{B}_i\mathbf{K}_i\ (i=1,2,...,8)$, we have

$$\mathbf{K}_1=[-71.2011 \quad -22.5144];\mathbf{K}_2=[-2416.7 \quad -764.1923];\mathbf{K}_3=[-71.2011 \quad -22.8113]$$
$$\mathbf{K}_4=[-2416.7 \quad -774.2692];\mathbf{K}_5=[-114.9575 \quad -22.5144];\mathbf{K}_6=[-3901.9 \quad -764.1923]$$
$$\mathbf{K}_7=[-114.9575 \quad -22.8113]\mathbf{K}_8=[-3901.9 \quad -774.2692];$$

Then choosing the closed-loop eigenvalues [-30 -32] for $\mathbf{A}_i-\mathbf{G}_i\mathbf{C}_i\ (i=1,2,...,8)$, the matrices $\mathbf{G}_i\ (i=1,2,...,8)$, are

$$\mathbf{G}_1=\mathbf{G}_2=[61.9738 \quad 967.9433]^T;\mathbf{G}_3=\mathbf{G}_4=[62.0262 \quad 971.1921]^T$$
$$\mathbf{G}_5=\mathbf{G}_6=[61.9738 \quad 975.6663]^T;\mathbf{G}_7=\mathbf{G}_8=[62.0262 \quad 978.9151]^T$$

Figures.5-16 illustrate the simulation results for initial states $\mathbf{x}(0)=\left[\frac{11\pi}{45} \quad 0\right],\left[\frac{11\pi}{45} \quad 3\right]$,and $\left[\frac{22\pi}{45} \quad -2\right]$.

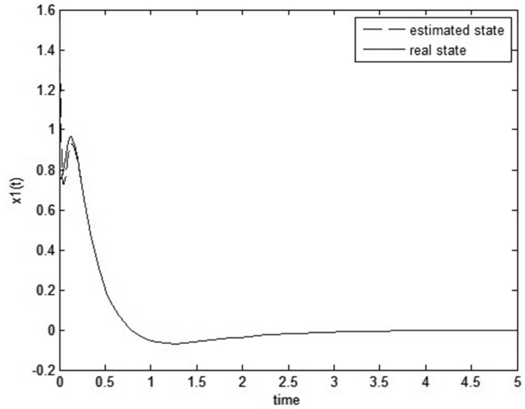

Figure 5- Comparison between the state $x_1(t)$ of the fuzzy T-S model and its estimation, $\hat{x}_1(t)$ by the observer for the initial states $\mathbf{x}(0)=[11\pi/45 \quad 0]$

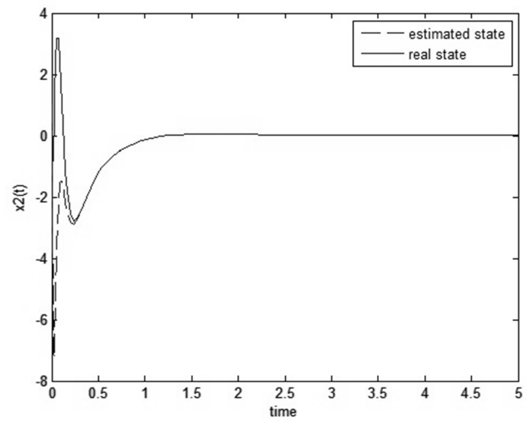

Figure6- Comparison between the state $x_2(t)$ of the fuzzy T-S model and its estimation, $\hat{x}_2(t)$, from the observer for the initial states $x(0)=[11\pi/45 \quad 0]$

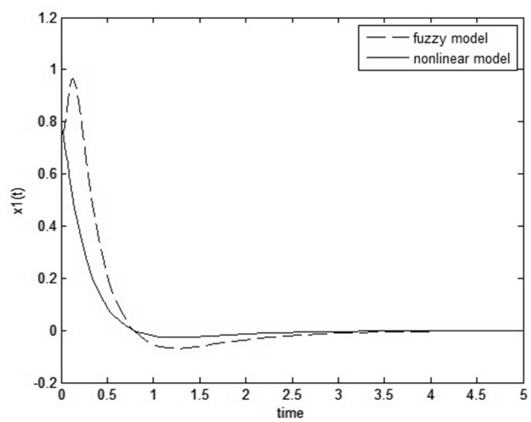

Figure7- Comparison between the state $x_1(t)$, of theT-S fuzzy model and the nonlinear model after applying the controler (28) with the initial states $\mathbf{x}(0)=[11\pi/45 \quad 0]$

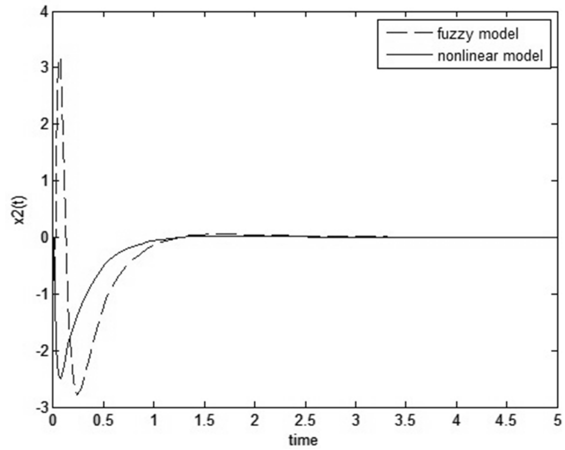

Figure8- Comparison between the state $x_2(t)$, of theT-S fuzzy model and the nonlinear model after applying the controler (28) with the initial states $\mathbf{x}(0)=[11\pi/45 \quad 0]$

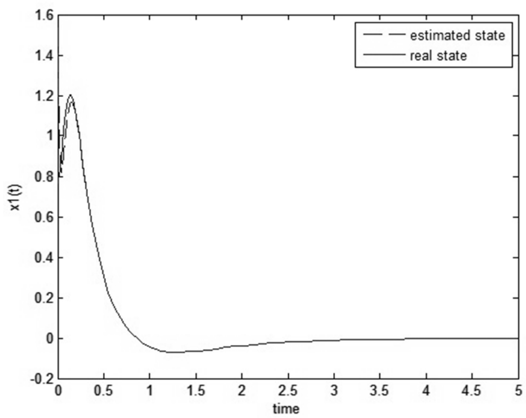

Figure9-Comparison between the state $x_1(t)$ of the fuzzy T-S model and its estimation, $\hat{x}_1(t)$ by the observer for the initial states $\mathbf{x}(0)=[11\pi/45 \quad 3]$

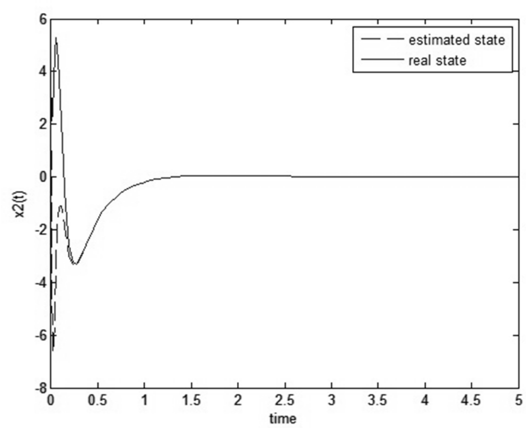

Figure10- Comparison between the state $x_2(t)$ of the fuzzy T-S model and its estimation, $\hat{x}_2(t)$, by the observer for the initial states $\mathbf{x}(0)=[11\pi/45 \quad 3]$

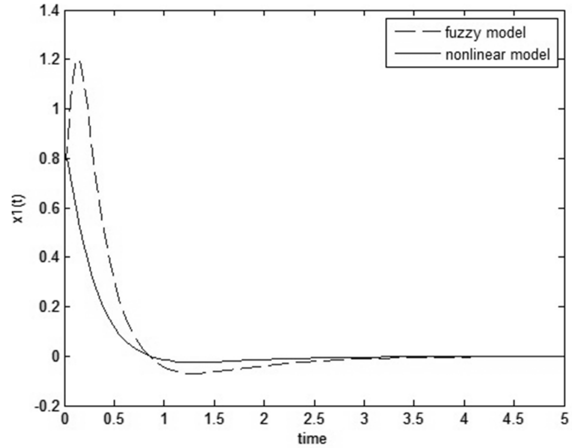

Figure11- Comparison between the state $x_1(t)$, of theT-S fuzzy model and the nonlinear model after applying the controler (28) with the initial states $\mathbf{x}(0)=[11\pi/45 \quad 3]$

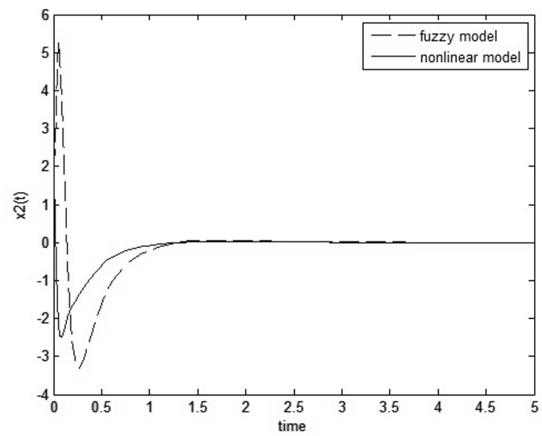

Figure12- Comparison between the state $x_2(t)$, of theT-S fuzzy model and the nonlinear model after applying the controler (28) with the initial states $\mathbf{x}(0)=[11\pi/45 \quad 3]$

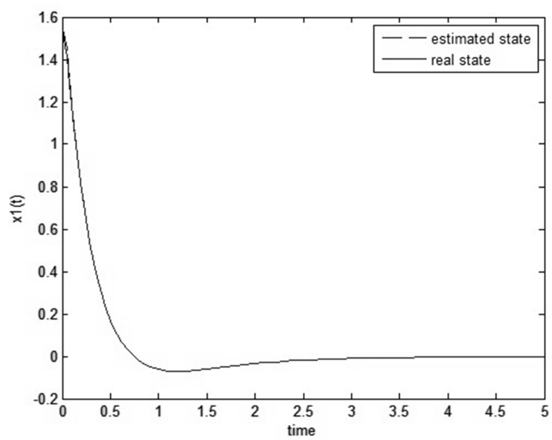

Figure13- Comparison between the state $x_1(t)$ of the fuzzy T-S model and its estimation, $\hat{x}_1(t)$ by the observer for the initial states $\mathbf{x}(0)=[22\pi/45 \quad -2]$

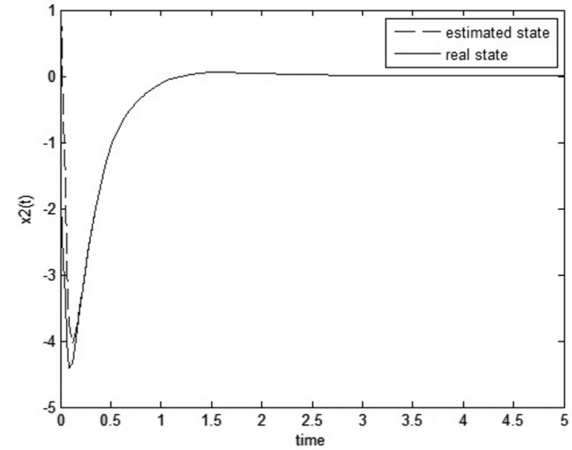

Figure14- Comparison between the state $x_2(t)$ of the fuzzy T-S model and its estimation, $\hat{x}_2(t)$, by the observer for the initial states $\mathbf{x}(0)=[22\pi/45 \quad -2]$

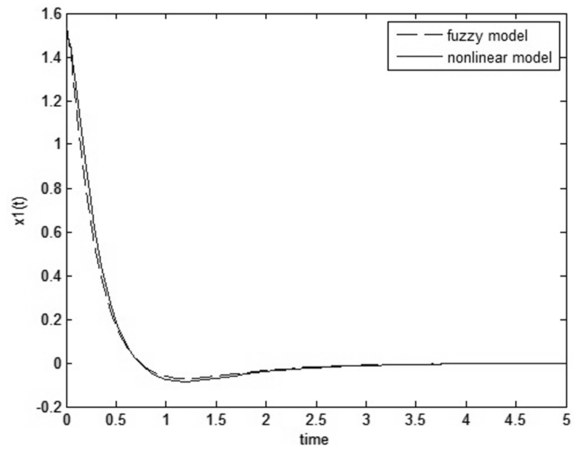

Figure15- Comparison between the state $x_1(t)$, of theT-S fuzzy model and the nonlinear model after applying the controler (28) with the initial states $\mathbf{x}(0)=[22\pi/45 \quad -2]$

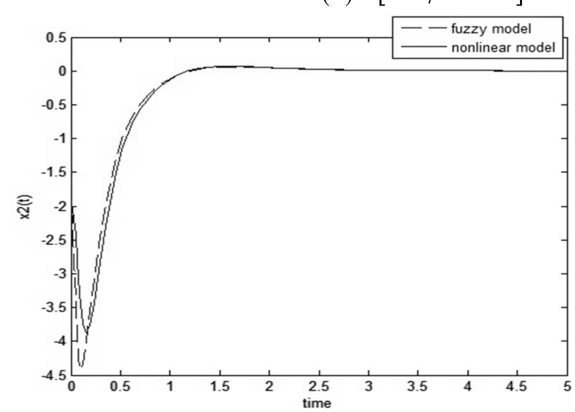

Figure16- Comparison between the state $x_2(t)$, of theT-S fuzzy model and the nonlinear model after applying the controler (28) with the initial states $\mathbf{x}(0)=[22\pi/45 \quad -2]$

The simulation results show that we have modeled the nonlinear dynamic equations of the cart-poled type inverted pendulum system by a T-S fuzzy model, with a very good precision.

In order to investigate the observability of this T-S fuzzy system, we should check the conditions introduced in section5. In order to do that, we should first find out the eigenvalues of the state matrices $\mathbf{A}_1, \mathbf{A}_2, ..., \mathbf{A}_8$.

$$eig(\mathbf{A}_1)=eig(\mathbf{A}_2)=\{-3.1062, 3.08\} \quad ; \quad eig(\mathbf{A}_3)=eig(\mathbf{A}_4)=\{-3.08, 3.1062\}$$

$$eig(\mathbf{A}_5)=eig(\mathbf{A}_6)=\{-4.1712, 4.145\}; eig(\mathbf{A}_7)=eig(\mathbf{A}_8)=\{-4.145, 4.1712\}$$

As it can be seen, all of the eight sub-systems have distinct eigenvalues. So the first condition of observability is satisfied. Now we should make sure that none of the vectors $\gamma_1, \gamma_2, ..., \gamma_8$ have any zero elements. It can be easily seen that the second observability condition is also satisfied.

Having shown both of the conditions to hold for this example, the cart-poled inverted pendulum system in this example would be observable. This is the result that was expected from the original nonlinear model (50), because the output of this system is the angular displacement of the pendulum, θ, and with the states of the system being defined as $x_1 = \theta$ and $x_2 = \dot{\theta}$, it is obvious that both of them are observable from the output data of the system.

As mentioned earlier, in these simulations an observer has been designed for the T-S fuzzy model and a controller has been applied to the system based on this observer, then the same controller has been applied to the original nonlinear model of the system. As the simulation results show, this controller can also control the nonlinear model with a high precision.

7. CONCLUSION

In this paper, observability of a special class of Takagi-Sugeno fuzzy systems was investigated, and a criterion for determining the observability of these systems was presented. Moreover, an application example has been presented in order to show the viability of this criterion. In the example, the T-S fuzzy model has been derived from the nonlinear model of an inverted pendulum; then a controller has been applied to the fuzzy model based on the estimated states of the system from a fuzzy observer, then by applying the controller designed based on the fuzzy model to the original nonlinear model, the efficiency of this method has been shown. With the use of this method, the observability of T-S fuzzy models can be easily and reliably determined. This method not only can help determining the observability of this special class of T-S fuzzy systems, but also would be very useful in determining the observability of nonlinear systems, by determining the observability of their corresponding T-S fuzzy models.

As a recommendation for the future works, working on the observability of Takagi-Sugeno fuzzy systems with *nonlinear* sub-systems in the then parts is suggested. Also, because this class of Takagi-Sugeno fuzzy systems is similar to linear controlled switching systems, it may be possible to try and find a new observability criterion for these systems based on the observability criteria of the switching systems.

REFERENCES

1. Zadeh L.A., 1965.Fuzzy Sets.Information Control, Vol.8, No.3, pp.338-353.
2. Mamdani E.H., 1974.Application of fuzzy algorithms for control of simple dynamic plant. IEEE Proceedings, Vol.121, No.12.
3. Holmblad L.P. and J.J.Ostergarrd, 1981. Control of a cement kiln by fuzzy logic techniques.in Proc., 8th IFAC conf., Kyoto, Japan, pp.809-814.
4. Ding Z., M.Ma andA.Kandel, 2000. On the observability of fuzzy dynamical control systems(I).Fuzzy sets and systems, Vol.111, pp.225-236.
5. Ding Z. andA.Kandel, 2000. On the observability of fuzzy dynamical control systems(II).Fuzzy sets and systems, Vol.115, pp.261-277.
6. Lam H.K., Frank.H.Leung, and Peter K.S.Tam, 2003.Design and stability analysis of fuzzy model-based nonlinear controller for nonlinear systems using genetic algorithms.IEEE Transactions on systems, Vol.33, No.2.
7. Murty M.S.N, and S. Kumar, 2008. On controllability and observability of fuzzy dynamical matrix lyapunov systems. Advances in fuzzy systems, 2008.
8. Kalman R.E., 1960.On the general theory of control systems.Proc.First Internal Congress on Automatic control, Moscow.
9. Hermann R., and A. J. Krener, 1977.Nonlinear controllability and observability.IEEE Transactions on automatic control, Vol. AC-22, No.5, pp.728-740.
10. William L.Brogan, 1991.Modern control theory.Prentice Hall, third edition.
11. Sugeno M., and G.Kang, 1988.Structure identification of fuzzy models.Fuzzy Sets and Systems, Vol.28, No.1, pp.15-33.
12. Kawamoto S., K.Tada, A.Ishigame, and T.Taniguchi, 1992.An approach to stability analysis of second order fuzzy systems.IEEE Transactions on fuzzy systems, pp.1427-1434.
13. Xiao-Jun Ma, Zeng-Qi Sun, and Yan-Yan He, 1998.Analysis and design of fuzzy controller and fuzzy observer. IEEE Transactions on fuzzy systems, Vol.6 , No.1, pp. 41-50.
14. Gilbert E.G., 1963. Controllability and observability in multivariable control systems. J.S.I.A.M Control, Vol.2, No.1, pp. 128-151.
15. Kazuo Tanaka, and Hua O. Wang, 2001.Fuzzy control systems design and analysis; A linear matrix inequality approach. John Wiley and Sons, Inc., pp 5-48, pp 277-289.
16. Kachroo P., 1995.Modeling, analysis and control of fuzzy systems.pp.2046-2051.
17. Mahdi A.A.,A.M.Razali, 2011. The diagnosis of Chicken Pox and Measles using fuzzy relations.Journal of Basic and Applied Scientific Research, 1(7), pp. 679-686.
18. Nobari S.M., 2011. Design of fuzzy decision support systems (FDSS) in employee recruitment. Journal of Basic and Applied Scientific Research, 1(11), pp. 1891-1903.