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Value Efficiency Analysis for Incorporating Preference Information in Data Envelopment Analysis with Interval Data

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ABSTRACT

Korhonen et al. developed a requisite theory for incorporating preference information in the efficiency analysis of Decision Making Units (DMUs). The efficiency of DMU is defined in the spirit of Data Envelopment Analysis (DEA), complemented with Decision Maker's (DM's) preference information concerning the desirable structure of inputs and outputs. Their procedure is being by aid the DM in searching for the most preferred combination of inputs and outputs of DMUs, which are efficient in DEA. So far, a number of DEA models with interval data have been developed. In this paper, we suggest a new model with interval data for estimating value efficiency that is called Interval Value Efficiency Analysis (IVEA).

KEYWORDS:DMU, Most preferred solution, Indifference contour, Value efficiency, Interval data, MOLP

INTRODUCTION

Data Envelopment Analysis (DEA) is a non-parametric method for measuring the relative efficiency of a set of entities, called Decision Making Units (DMU) with common inputs and outputs [1], the same as bank branches, schools and so on.In DEA literature, the performance evaluation along with the efficiency of the included units of an organization is considered as a crucial fact, affecting the whole performance, either directly or indirectly. This is studied by Jafarpour et al. [2]. Dariush Khezrimoghadam provided some of the popular DEA models including CCR, BCC and SBM models by interval data. In addition he discussed some of their properties for benchmarking inefficient DMUs.

One of the purposes of DEA is to empirically estimate the so-called efficient frontier based on the set of available DMUs and to project all DMUs onto this frontier. If a DMU lies on efficient frontier, it is called efficient unit, otherwise it is called inefficient. On the other hand, a DMU is efficient if there is no other unit that can either produce more outputs by consuming the same amount or less of inputs, or produce the same amount or more outputs by consuming less or the same amount of inputs as the DMU under consideration[1].

DEA provides efficiency score and reference units for each inefficient unit. Reference unit is a DMU on the efficient frontier [3], which can be regarded as target unit for inefficient DMUs and it founds by projecting the inefficient unit on efficient frontier.

DEA also provides the user with information about the efficient and inefficient DMUs, as well as the efficiency scores and reference sets for inefficient units.

In DEA literature, each efficient DMU is an equally "good" unit. If Decision Maker (DM) does not prefer an efficient unit, it is necessary somehow incorporate DM's judgments into the analysis. The most important method has been restricting possible values of the multipliers of so-called "dual DEA models". Some ideas can be adopted from research carried out in the Multiple Objective Linear Programming (MOLP) [4].

One of the important issues in MOLP is to provide a DM with a tool, which makes it possible to estimate solution lying on the efficient frontier, which pleases DM. This solution is called DM's *Most Preferred Solution* (MPS). In Joro et al. [5]have been shown that the MOLP and DEA models have a similar structure. Thus theory and approaches develop in MOLP to evaluate solutions on the efficient frontier can also be applied in DEA. We may search solution on the efficient frontier in DEA too.

The most preferred solution plays an important role in this approach, which developed by Halme et al. [6] for incorporating preference into DEA. This approach is called Value Efficiency Analysis (VEA).

Value efficiency analysis is based on the assumption that the DM compares alternatives by using implicitly known value function. This unknown value function is assumed pseudoconcave and strictly decreasing for inputs and strictly increasing for outputs. By this assumption, its maximum occur as the most preferred solution on the efficient frontier. The most important purpose of value efficiency analysis is to estimate a need to increase outputs and/or decrease inputs to reach indifference contour of the value function at the optimum. Because the value function is unknown, the indifference contour cannot be defined precisely.

However, the region consisting of the points less or equally preferred to the MPS can be specified. This region is used in value efficiency analysis [7].

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In this paper, we review the main ideas in value efficiency analysis and the rest of this is organized as follows. In section 2, we review value efficiency concept, and in section 3, we discuss interval value efficiency (IVE) and practical aspects. Concluding remarks are given in section4.

1. Preliminaries

Assume we have $n$decision-making units (DMU) each consuming m inputs and producing s outputs. Let$X \in R_{+}^{m+n}$ and $Y \in R_{+}^{s+n}$ be matrices, consisting of non-negative elements, containing the observed input and output measures for the DMUs. We further assume that there are no duplicated units in the data set. We denote by x_j (the jth column of X) the vector of inputs consumed by DMU_j, and by x_{ij} the quantity of input i consumed by DMU_j. A similar notation is used for outputs.When it is not necessary to emphasize the different roles of inputs and outputs, we denote$U = \binom{y}{-x}$.

The main idea of value efficiency analysis is evaluate efficiency as a distance to an indifference contour of a DM's value function. The distance is measured to the contour that passes through the MPS in the production possibility set (PPS). The evaluation could be done easily, if we explicitly knew the DM's value function. However, in practice this is not realistic. Because the DM's value function is not assumed to be known, we cannot identify indifference contour, but we have approximate it. We do this by finding the region containing the points $\binom{y}{-x} \epsilon R^{m+s}$ less than or equally preferred to the most preferred solution [6].

Halme et al. (1999) assumed that the DM's value function $V(U)$, $U = \binom{y}{-x} \epsilon R^{m+s}$, is pseudoconcave and strictly increasing in U (i.e. increasing in output y and decreasing in input x) and its maximal value $V(U^*)$ occur the most preferred solution $U^* = \binom{y^*}{-x^*} \epsilon R^{m+s}$ on the efficient frontier. When the value function (unknown) $V(U)$ is pseudoconcave, the region containing all points $U = \binom{y}{-x} \epsilon R^{m+s}$ surely less than or equally preferred to the MPS can be characterized by the tangent of hyperplanes of all possible pseudoconcave value functions obtaining maximum value at the MPS [6].

By these hyperplanes we can define a new efficient frontier. We then define efficiency by using standard DEA technique. The resulting score of this technique is called *value efficiency score.*

The value efficiency analysis is illustrated in Figure1. We have five units named A, B, C, D and E, which use same amount of one input and produce two outputs and show output oriented model in output space. Clearly all units except unit B are efficient and the efficiency score of unit B is$\frac{OB}{OB^1}$. But in value efficiency analysis we would like define value efficiency score by ratio$\frac{OB}{OB^4}$. Because of the value function is unknown, this is not possible. If we also knew the tangent of the indifference contour, we could use ratio$\frac{OB}{OB^3}$, but this assumption is rarely realistic. This is why we consider all possible tangents of indifference contour. This idea leads to use ratio $\frac{OB}{OB^2}$ as an approximated measure of actual value efficiency score. A value efficiency score is a minimum need to improve the input (or output) values of a unit to become value efficient.

A value efficiency score of each unit easily can be evaluated by solving linear programming problem. A DMU with vector $U = \binom{y}{-x}$ is value inefficient with respect to any point u^* if the optimum value Z^* of thefollowing problem is strictly positive in following linear programming.

Primal Value Efficiency Model	Dual Value Efficiency Model
$Max\ \ Z = \sigma + \varepsilon(1^t S^+ + 1^t S^-)$ s.t. $Y\lambda - \sigma y_o - S^+ = y_o$ $X\lambda + \sigma x_o + S^- = x_o$ (2.1) $A\lambda + \delta = b$ $S^+, S^- \geq 0$ $\lambda_j \geq 0 \quad if \quad \lambda_j^* = 0 \quad j = 1,2,\ldots,n$ $\mu_j \geq 0 \quad if \quad \mu_j^* = 0 \quad j = 1,2,\ldots,n$ $\varepsilon > 0\ (Non - Archimedean)$	$Min\ \ W = v^t x_o - u^t y_o + \mu^t b$ s.t. $-u^t Y + v^t X + \mu^t A - \gamma = 0$ $u^t y_o + v^t x_o = 1$ (2.2) $u, v \geq 1^t \varepsilon$ $\gamma_j \begin{cases} \geq 0 & if \quad \lambda_j^* = 0 \\ = 0 & if \quad \lambda_j^* > 0 \end{cases} \quad j = 1,2,\ldots,n$ $\mu_j \begin{cases} \geq 0 & if \quad \delta_j^* = 0 \\ = 0 & if \quad \delta_j^* > \end{cases} \quad j = 1,2,\ldots,n$ $\varepsilon > 0\ (Non - Archimedean)$

$\lambda^* \epsilon \Lambda$ and δ^* correspond to the Most Preferred Solution $U^* = \begin{pmatrix} y^* \\ -x^* \end{pmatrix} = \begin{pmatrix} \lambda^* Y \\ -\lambda^* X \end{pmatrix}$.

Some variables allowed having negative value in model 2.2. This modification provides straightforward method to use value judgment in data envelopment analysis. In this model DMU_o is value efficient if and only if $Z^* = W^* = 0$, otherwise it is value inefficient. Parameter σ is called "value inefficient score". It means how much a corresponded unit has to improve input- and/or output- value to become value efficient unit.

When a DMU diagnose value efficient unit, we cannot say that it is equally preferred to the MPS, because we only approximate the true indifference contour of value function of decision maker.

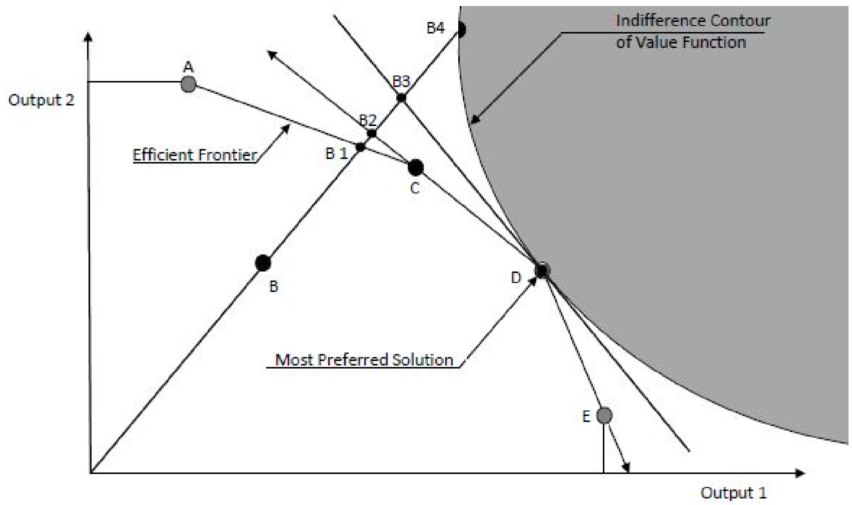

Figure 1. Illustration of value efficiency analysis

2. Interval Value Efficiency Analysis

In this section, we propose an interval value efficiency approach to measure value efficiency of all DMUs under evaluation with interval data.

Let the input and output values of each DMU be located in a certain interval, where x_{ij}^l and x_{ij}^u are the lower and upper bounds of the input i of jth DMU respectively, and y_{rj}^l and y_{rj}^u are the lower and upper bounds of the output r of jth DMU respectively. That is to say $x_{ij}^l \le x_{ij} \le x_{ij}^u$ and $y_{rj}^l \le y_{rj} \le y_{rj}^u$.

Not that always $x_{ij}^l \le x_{ij}^u$ and $y_{rj}^l \le y_{rj}^u$. If $x_{ij}^l = x_{ij}^u$ (or $y_{rj}^l = y_{rj}^u$) then the ith input (or rth output) of DMU_j has a definite value.

If some of input or output of DMU_o is interval then the value efficiency of DMU_o is located in an interval.

The dual value efficiency model for definite data is as follows:

$$
\begin{aligned}
& Min \quad W = v^t x_o - u^t y_o + \mu^t b \\
\text{s.t} \; & -u^t Y + v^t X + \mu^t A - \gamma = 0 \\
& u^t y_o + v^t x_o = 1 \\
& u, v \ge 1^t \varepsilon \\
& \gamma_j \begin{cases} \ge 0 & if \quad \lambda_j^* = 0 \\ = 0 & if \quad \lambda_j^* > 0 \end{cases} \quad j = 1,2,\dots,n \\
& \mu_j \begin{cases} \ge 0 & if \quad \delta_j^* = 0 \\ = 0 & if \quad \delta_j^* > \end{cases} \quad j = 1,2,\dots,n \\
& \varepsilon > 0 \; (Non - Archimedean)
\end{aligned}
\qquad (3\text{-}1)
$$

The upper and lower bounds of the value efficiency of DMU_o are obtained by solving the following problems, respectively.

$$
\begin{aligned}
& Min \quad \underline{W} = v^t x_o^l - u^t y_o^u + \mu^t b \\
\text{s.t} \; & -u^t Y^l + v^t X^u + \mu^t A - \gamma = 0 \\
& u^t y_o^u + v^t x_o^l = 1 \\
& u, v \ge 1^t \varepsilon
\end{aligned}
\qquad (3\text{-}2)
$$

$$\gamma_j \begin{cases} \geq 0 & if \quad \lambda_j^* = 0 \\ = 0 & if \quad \lambda_j^* > 0 \end{cases} \quad j = 1,2,\dots,n$$

$$\mu_j \begin{cases} \geq 0 & if \quad \delta_j^* = 0 \\ = 0 & if \quad \delta_j^* > \end{cases} \quad j = 1,2,\dots,n$$

$\varepsilon > 0\ (Non - Archimedean)$

and

$$Min \quad \overline{W} = v^t x_o^u - u^t y_o^l + \mu^t b$$

s.t $-u^t Y^u + v^t X^l + \mu^t A - \gamma = 0$

$u^t y_o^l + v^t x_o^u = 1$ (3-3)

$$u, v \geq 1^t \varepsilon$$

$$\gamma_j \begin{cases} \geq 0 & if \quad \lambda_j^* = 0 \\ = 0 & if \quad \lambda_j^* > 0 \end{cases} \quad j = 1,2,\dots,n$$

$$\mu_j \begin{cases} \geq 0 & if \quad \delta_j^* = 0 \\ = 0 & if \quad \delta_j^* > \end{cases} \quad j = 1,2,\dots,n$$

$$\varepsilon > 0\ (Non - Archimedean)$$

Considering that the value efficiency of each DMU lies in an interval, all DMUs can be divided into one of the three following classes:

Class1. The set of all DMUs that are value efficient in their best and worst case. In other words:

$$V^{++} = \left\{DMU_j \middle| \overline{W}^* = \underline{W}^* = 0\right\}$$

Class2. The set of all DMUs that are value efficient in their best case, and value inefficient in their worst case. That is

$$V^{+} = \left\{DMU_j \middle| \overline{W}^* > 0 \ , \ \underline{W}^* = 0\right\}$$

Class3. Includes all units which are value inefficient both in their best and worst case. That is to say

$$V^{-} = \left\{DMU_j \middle| \overline{W}^* > 0 \ , \ \underline{W}^* > 0\right\}$$

Definition. DMU_o is interval value efficient (IVE) if and only if $\overline{W}^* = \underline{W}^* = 0$.

Theorem1. If W^*, $\overline{W}^*$ and $\underline{W}^*$ are the optimal value of models 1, 2 and 3 respectively, then $W^* \in \left[\underline{W}^*, \overline{W}^*\right]$.

Proof. Let $u^* = (u_1^*, u_2^*, \dots, u_s^*)$ and $v^* = (v_1^*, v_2^*, \dots, v_m^*)$ are the optimal solution of model 1 for DMU_o , then

$$\begin{cases} x_{io}^l \leq x_{io} & for\ i = 1,2,3,\dots,m \\ -y_{ro}^u \leq -y_{ro} & for\ r = 1,2,3,\dots,s \end{cases}$$

Then we have

$$\begin{cases} \sum_{i=1}^{m} v_i^* x_{io}^l \leq \sum_{i=1}^{m} v_i^* x_{io} \\ -\sum_{r=1}^{s} u_r^* y_{ro}^u \leq -\sum_{r=1}^{s} u_r^* y_{ro} \end{cases}$$

So

$$\sum_{i=1}^{m} v_i^* x_{io}^l - \sum_{r=1}^{s} u_r^* y_{ro}^u + \sum_{k=1}^{n} \mu_k b_k \leq \sum_{i=1}^{m} v_i^* x_{io} - \sum_{r=1}^{s} u_r^* y_{ro} + \sum_{k=1}^{n} \mu_k b_k$$

That is $\underline{W}^* \leq W^*$.

We can easily prove $W^* \leq \overline{W}^*$ likewise.

4. **Illustrative example**

We illustrate our interval value efficiency model with an example. We now apply our approach to some commercial bank branches in Iran that there are 20 branches. Each unit uses three inputs and five outputs. Table 1 shows these data [8]. In Table 2 and 3 interval inputs and outputs of these units are given.

Table1 Inputs and outputs of DMUs.

Inputs	**Outputs**
Payable interest	The total sum of four main deposits
Personnel	Other deposits
Non-performing loans	Loans granted
	Received interest
	Fee

Throughout the example, we us the generalization of traditional CCR model and estimate efficiency score of all DMUs in their best and worst cases. Then, identify the DM's most preferred solution over the set of convex combination of all existing DMUs. We do this by solving multiple criteria problem where we wish to maximize the output and minimize the input.

Reflecting our own bias, we use VIG (Korhonen and Wallenius 1989) software to perform the search for the MPS and present DMU_4 as a DM's Most Preferred Solution.

The information comes from the DMU_4 along with information of models (3.2) and (3.3) enables us to determine the lower bound and upper bound value efficiency of all DMUs at their best and worst case respectively. Based on value efficiency scores, we classify all units to setsV^{++}, V^{+} and V^{-}.

Table2 Input-data for 20 bank branches.

DMU_j	x_{1j}^l	x_{1j}^u	x_{2j}^l	x_{2j}^u	x_{3j}^l	x_{3j}^u
1	5007.37	9613.37	36.29	36.86	87423	87243
2	2926.81	5961.55	18.80	20.16	9945	12120
3	8732.70	17752.5	25.74	27.17	47575	50013
4	945.93	1966.39	20.81	22.54	19292	19753
5	8487.07	7521.66	14.16	14.80	3428	3911
6	13759.35	7359.36	19.46	19.46	13929	15657
7	587.69	1205.47	27.29	27.48	27827	29005
8	4646.39	9559.61	24.52	25.07	9070	9983
9	1554.29	3427.89	20.47	21.59	412036	413902
10	17528.31	6297.54	14.84	15.05	8638	10229
11	2444.34	4955.78	20.42	20.54	500	937
12	7303.27	4178.11	22.87	23.19	16148	21353
13	9852.15	9742.89	18.47	21.83	17163	17290
14	4540.75	9312.24	22.83	23.96	17918	17964
15	3039.58	6304.01	29.32	39.86	51582	55136
16	6585.81	3453.58	25.57	26.52	20975	23992
17	4209.18	8603.79	27.59	27.95	41960	43103
18	1015.52	2037.82	13.63	13.93	18641	19354
19	5800.38	1875.39	27.12	27.26	19500	19569
20	1445.68	2922.15	28.96	28.96	31700	32061

Table3 Output-data for 20 bank branches.

DMU_j	y_{1j}^l	y_{1j}^u	y_{2j}^l	y_{2j}^u	y_{3j}^l	y_{3j}^u	y_{4j}^l	y_{4j}^u	y_{5j}^l	y_{5j}^u
1	2696995	3126798	263643	382545	1675519	1853365	108634.76	125740.28	965.97	6957.33
2	340377	440355	95978	117659	377309	390203	32396.56	37836.56	304.67	749.40
3	1027546	1061260	37911	503089	1233548	1822028	96842.33	108080.01	2285.03	3174
4	1145235	1213541	229646	268460	468520	542101	32362.80	39273.37	207.98	510.93
5	390902	395241	4924	12136	129751	142873	12662.71	14165.44	63.32	92.3
6	988115	1087392	74133	111324	507502	574355	53591.30	72257.28	480.16	869.52
7	144906	165818	180530	180617	288513	323721	40507.97	45847.48	176.58	370.81
8	408163	416416	405396	486431	1044221	1071812	56260.09	73948.09	4654.71	5882.53
9	335070	410427	337971	449336	1584722	1802942	176436.81	189006.12	560.26	2506.67
10	700842	768593	14378	15192	2290745	2573512	662725.21	791463.08	58.89	86.86
11	641680	696338	114183	241081	1579961	2285079	17527.58	20773.91	1070.81	2283.08
12	453170	481943	27196	29553	245726	275717	35757.83	42790.14	375.07	559.85
13	553167	574989	21298	23043	425886	431815	45652.24	50255.75	438.43	836.82
14	309670	342598	20168	26172	124188	126930	8143.79	11948.04	936.62	1468.45
15	286149	317186	149183	270708	787959	810088	106798.63	111962.3	1203.79	4335.24
16	321435	347848	66169	80453	360880	379488	89971.47	165524.22	200.36	399.8
17	618105	835839	244250	404579	9136507	9136507	33036.79	41826.51	2781.24	4555.42
18	248125	320974	3063	6330	26678	29173	9525.60	10877.78	240.04	274.7
19	640890	679916	490508	644372	2946797	3985900	66097.16	95329.87	961.56	1914.25
20	119948	120208	14943	17495	297674	308012	21991.53	27934.19	282.73	471.22

As we can see in the table 4, the unit which $\underline{\theta}_{CCR} < 1$ and $\overline{\theta}_{CCR} < 1$ has its value efficiency positive in both cases ($\underline{W}^* > 0$ and $\overline{W}^* > 0$) and so belongs to set V^- same as DMU_5 and if a decision making unit is efficient at the best and worst case ($\underline{\theta}_{CCR} = 1$ and $\overline{\theta}_{CCR} = 1$) then $\underline{W}^* = 0$ and $\overline{W}^* > 0$ and hence belongs to V^+, same as DMU_1 .

Table4 CCR and Value efficiencies of DMUs.

DMU_j	$\underline{\theta}_{CCR}$	$\overline{\theta}_{CCR}$	$\underline{W}^*$	$\overline{W}^*$	IVE classification
1	1.00	1.00	0.00	4.62	V^+
2	0.21	1.00	0.00	5.97	V^+
3	0.52	1.00	0.00	6.03	V^+
4	1.00	1.00	0.00	0.00	V^{++}
5	0.63	0.77	0.14	1.28	V^-
6	0.91	1.00	0.00	1.31	V^+
7	0.74	1.00	0.00	1.73	V^+
8	1.00	1.00	0.00	8.48	V^+
9	1.00	1.00	0.00	1.06	V^+
10	1.00	1.00	0.00	6.17	V^+
11	1.00	1.00	0.00	1.14	V^+
12	0.33	0.50	0.34	2.86	V^-
13	0.44	0.71	0.18	2.09	V^-
14	0.27	0.73	0.16	3.34	V^-
15	0.42	1.00	0.00	1.10	V^+
16	0.22	1.00	0.00	2.90	V^+
17	1.00	1.00	0.00	6.36	V^+
18	0.27	0.95	0.03	4.33	V^-
19	0.99	1.00	0.00	1.14	V^+
20	0.18	0.98	0.01	0.76	V^-

5. CONCLUSION

In this paper, we provided a new approach for incorporating preference information and value efficiency score in data envelopment analysis based on Halmeet al. approach and interval data. In this method, we estimate value efficiency for all DMUs, and at first efficiency score.

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