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A Method for Finding Most Efficient Decision Making Unit with Interval Data in Data Envelopment Analysis

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ABSTRACT

One of the major issues and application in DEA is obtaining the most efficient decision making unit (DMU). In fact, decision makers are trying to consider the most efficient DMU in several models. Hence they can achieve most efficient unit of the existing units.
In some cases, the data are indefinite and we have disposal their indefinite values so in this paper we are obtaining the most efficient unit when the data are interval.
KEY WORDS: data envelopment analysis, most efficient decision making unit, interval data.

1-INTRODUCTION

Data envelopment analysis (DEA) invented by Charnes, Cooper and Rhodes (1987) [4]. It can determine the relative efficiency of the decision making units (DMU) with multiple inputs and outputs.

DEA performs effectiveness calculations to determine the efficient and inefficient units so efficiency for efficient DMUs is one and for inefficient DMUs is less than one.

In many DEA applications, inputs and outputs is used to specific and the definite phrase but sometimes in some other applications of DEA inputs and outputs are indefinite. As, in cases when data are interval, probabilistic and /or fuzzy, in this paper we will consider interval data and the purpose of this paper is obtaining most efficient decision-making units with using interval data.

Amin and Toloo (2007)[2] expressed integer DEA model for obtaining most efficient unit, as their model obtaining solution in two stages. In many cases, their proposed model could not produce a unique solution as the most efficient DMU. Then Toloo and Nalchegar (2009)[7] extended Amin &Toloo's model(2007)[2] to VRS position and they presented DEA model to find most efficient DMU in BCC state[3], but it also include the same errors.

Moreover, Amin (2009) [1] has proposed an improved DEA model, which could not determine most efficient DMU uniquely. In fact Foroughi (2011)[5] reverse the Amin's model[1] with an example and so Foroughi presented the mixed integer linear programming for finding the most efficient DMU and the difference Foroughi's model with previous models is for to get to the solution by solving one model. But, his model could not get the exact solution as the most efficient unit.

So Ying and Jiang(2012) [8] presented mixed integer linear model for finding most efficient DMU that it is more complete than previous models and strategy for them model is finding the hyper plan that one DMU is one side hyper plan and the other DMUs are the other side of the hyper plan.

Also according to Rostamy et al's paper (2013)[6] discussed in subject most congest DMU, in this paper we present the most efficient DMU. Since in some actual situations the data may be indefinite and we also have interval data, in this paper we presented a method which is finding most efficient DMU when the inputs and outputs are in the interval state.

The construction of this paper is as follows:

In section 2, we express a summary of Ying and Jiang's model for obtaining most efficient DMU. Then in section 3, we present our method for obtaining most efficient DMU in interval data. In section 4 we analyze the data with two examples and in section 5, you will have the conclusion.

2. Ying and Jiang's model for obtaining most efficient DMU:

Suppose there are n DMUs to be evaluated in terms with m inputs and s outputs. For obtaining most efficient DMU, Ying Ming and Peng Jiang model(1) [8] presented as called mixed integer linear programming (MILP) as follows:

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$$
\begin{aligned}
&Min && \sum_{i=1}^{m} v_i \left(\sum_{j=1}^{n} x_{ij}\right) - \sum_{r=1}^{s} u_r \left(\sum_{j=1}^{n} y_{rj}\right) \\
&s.t && \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le I_j \qquad j=1,\dots,n \\
& && \sum_{j=1}^{n} I_j = 1 \\
& && I_j \in \{0,1\} \qquad j=1,\dots,n \\
& && u_r \ge \frac{1}{(m+s)\max\{y_{rj}\}} \qquad r=1,\dots,s \\
& && v_i \ge \frac{1}{(m+s)\max\{x_{ij}\}} \qquad i=1,\dots,m
\end{aligned} \tag{1}
$$

3. To obtain the most efficient DMU on the interval data

Suppose n DMUs are with input variables $X_{ij}\,(x_{1j}, x_{2j}, \dots, x_{mj})$ and output variables $Y_{rj}\,(y_{1j}, y_{2j}, \dots, y_{sj})$ for each j=1,…,n that is $X_{ij} \in \left[X_{ij}^{l}, X_{ij}^{u}\right] \& Y_{ij} \in \left[Y_{ij}^{l}, Y_{ij}^{u}\right]$ and $x_{ij}^{l} > 0$, $x_{ij}^{u} > 0$, $y_{ij}^{l} > 0$, $y_{ij}^{u} > 0$.

Let input and output values of any DMU be located in a certain interval, where x_{ij}^{L} and x_{ij}^{U} the lower and upper bounds of the *i*th input of the*j*th DMU, respectively, and y_{rj}^{L} and y_{rj}^{U} are the lower and upper bounds of the *r*th outputof the *j*th DMU, respectively; that is to say, $x_{ij}^{L} \le x_{ij} \le x_{ij}^{U}$ and $y_{rj}^{L} \le y_{rj} \le y_{rj}^{U}$.

Such data are called interval data, because they are located in intervals. Note that always $x_{ij}^{L} \le x_{ij}^{U}$ *and* $y_{rj}^{L} \le y_{rj}^{U}$.

If $x_{ij}^{L} = x_{ij}^{U}$ then the *i*th input of the *j*th DMU has a definite value. Interval problems are those whose parameter values are located in intervals, their exact values being unable to be identified.

So, we are finding the most efficient DMU in this case with model (1):

$$
\begin{aligned}
&e = Min && \sum_{i=1}^{m} v_i \left(\sum_{j=1}^{n} \left[x_{ij}^{l}, x_{ij}^{u}\right]\right) - \sum_{r=1}^{s} u_r \left(\sum_{j=1}^{n} \left[y_{rj}^{l}, y_{rj}^{u}\right]\right) \\
&s.t && \sum_{r=1}^{s} u_r \left(\sum_{j=1}^{n} \left[y_{rj}^{l}, y_{rj}^{u}\right]\right) - \sum_{i=1}^{m} v_i \left(\sum_{j=1}^{n} \left[x_{ij}^{l}, x_{ij}^{u}\right]\right) \le I_j \qquad j=1,\dots,n \\
& && \sum_{j=1}^{n} I_j = 1 \\
& && I_j \in \{0,1\} \qquad j=1,\dots,n \\
& && u_r \ge \frac{1}{(m+s)\max\left\{\left[y_{rj}^{l}, y_{rj}^{u}\right]\right\}} \qquad r=1,\dots,s \\
& && v_i \ge \frac{1}{(m+s)\max\left\{\left[x_{ij}^{l}, x_{ij}^{u}\right]\right\}} \qquad i=1,\dots,m
\end{aligned} \tag{2}
$$

Because the data are indefinite, so the optimal value of the objective function may be calculated as indefinite. To find interval efficiency, we introduce two models as follows.

$$
\begin{array}{lll}
e^{l} = Min & \sum_{i=1}^{m} v_i (\sum_{j=1}^{n} x_{ij}^{l}) - \sum_{r=1}^{s} u_r (\sum_{j=1}^{n} y_{rj}^{u}) & \\
s.t & \sum_{r=1}^{s} u_r y_{rj}^{l} - \sum_{i=1}^{m} v_i x_{ij}^{u} \le I_j & j = 1,\dots,n \\
 & \sum_{j=1}^{n} I_j = 1 & \\
 & I_j \in \{0,1\} & j = 1,\dots,n \\
 & u_r \ge \dfrac{1}{(m+s)\max\{y_{rj}^{u}\}} & r = 1,\dots,s \\
 & v_i \ge \dfrac{1}{(m+s)\max\{x_{ij}^{u}\}} & i = 1,\dots,m
\end{array} \qquad (3)
$$

$$
\begin{array}{lll}
e^{u} = Min & \sum_{i=1}^{m} v_i (\sum_{j=1}^{n} x_{ij}^{u}) - \sum_{r=1}^{s} u_r (\sum_{j=1}^{n} y_{rj}^{l}) & \\
s.t & \sum_{r=1}^{s} u_r y_{rj}^{u} - \sum_{i=1}^{m} v_i x_{ij}^{l} \le I_j & j = 1,\dots,n \\
 & \sum_{j=1}^{n} I_j = 1 & \\
 & I_j \in \{0,1\} & j = 1,\dots,n \\
 & u_r \ge \dfrac{1}{(m+s)\max\{y_{rj}^{l}\}} & r = 1,\dots,s \\
 & v_i \ge \dfrac{1}{(m+s)\max\{x_{ij}^{l}\}} & i = 1,\dots,m
\end{array} \qquad (4)
$$

Furthermore, we presented theorems that is determined an interval for the optimal value of the objective function of model (2).

Theorem 1: if (u^*,v^*,I^*) is optimal solution of model (2) then (u^*,v^*,I^*) is one feasible solution of model (3) .

Proof: because (u^*,v^*,I^*) optimal solution of model (2) so:

$$\sum_{r=1}^{s} u_r^* y_{rj} - \sum_{i=1}^{m} v_i^* x_{ij} \le I_j^*$$

We want to prove that (u *, v *, I *) satisfy to in the constraints of model (3) and is one feasible solution of model (3).

$$\sum_{r=1}^{s} u_r^* y_{rj}^{l} - \sum_{i=1}^{m} v_i^* x_{ij}^{u} \le \sum_{r=1}^{s} u_r^* y_{rj} - \sum_{i=1}^{m} v_i^* x_{ij} \le I_j^*$$

So

$$\sum_{r=1}^{s} u_r^* y_{rj}^{l} - \sum_{i=1}^{m} v_i^* x_{ij}^{u} \le I_j^*$$

Then we have

$$u_r^* \geq \frac{1}{(m+s)\max\left\{y_{rj}\right\}} \geq \frac{1}{(m+s)\max\left\{y_{rj}^u\right\}}$$

So

$$u_r^* \geq \frac{1}{(m+s)\max\left\{y_{rj}^u\right\}}$$

Also

$$v_i^* \geq \frac{1}{(m+s)\max\left\{x_{ij}\right\}} \geq \frac{1}{(m+s)\max\left\{x_{ij}^u\right\}}$$

So

$$v_i^* \geq \frac{1}{(m+s)\max\left\{x_{ij}^u\right\}}$$

So far we have shown that (u *, v *, I *) is a feasible solution for model (3) thus:

$$e = \sum_{i=1}^{m} v_i^* (\sum_{j=1}^{n} x_{ij}) - \sum_{r=1}^{s} u_r^* (\sum_{j=1}^{n} y_{rj}) \geq \sum_{i=1}^{m} v_i^* (\sum_{j=1}^{n} x_{ij}^l) - \sum_{r=1}^{s} u_r^* (\sum_{j=1}^{n} y_{rj}^u) \geq e^l$$

$$\Rightarrow e \geq e^l$$

Complete the proof.□

Theorem 2: if (u^*,v^*,I^*) is optimal solution of model (4) then (u^*,v^*,I^*) is one feasible solution of model (2) .

Proof: because (u^*,v^*,I^*) optimal solution of model (4) so:

$$\sum_{r=1}^{s} u_r^* y_{rj}^u - \sum_{i=1}^{m} v_i^* x_{ij}^l \leq I_j^*$$

We want to prove that (u *, v *, I *) satisfy to in the constraints of model (2) and is one feasible solution of model (2).

$$\sum_{r=1}^{s} u_r^* y_{rj} - \sum_{i=1}^{m} v_i^* x_{ij} \leq I_j^* \; \sum_{r=1}^{s} u_r^* y_{rj}^u - \sum_{i=1}^{m} v_i^* x_{ij}^l \leq I_j^*$$

So

$$\sum_{r=1}^{s} u_r^* y_{rj} - \sum_{i=1}^{m} v_i^* x_{ij} \leq I_j^*$$

Also

$$u_r^* \geq \frac{1}{(m+s)\max\left\{y_{rj}^l\right\}} \geq \frac{1}{(m+s)\max\left\{y_{rj}\right\}}$$

$$u_r^* \geq \frac{1}{(m+s)\max\left\{y_{rj}\right\}}$$

On the other hand

$$v_i^* \geq \frac{1}{(m+s)\max\left\{x_{ij}^l\right\}} \geq \frac{1}{(m+s)\max\left\{x_{ij}\right\}}$$

So

$$v_i^* \geq \frac{1}{(m+s)\max\left\{x_{ij}\right\}}$$

So far we have shown that (u *, v *, I *) is a feasible solution model (2) thus:

$$e^u = \sum_{i=1}^{m} v_i^* (\sum_{j=1}^{n} x_{ij}^u) - \sum_{r=1}^{s} u_r^* (\sum_{j=1}^{n} y_{rj}^l) \geq \sum_{i=1}^{m} v_i^* (\sum_{j=1}^{n} x_{ij}) - \sum_{r=1}^{s} u_r^* (\sum_{j=1}^{n} y_{rj}) \geq e$$

$$\Rightarrow e^u \geq e$$

Complete the proof.

Thus, according to theorems 1 and 2 we have shown $e^l \leq e \leq e^u$ that the distance most efficient DMU to the choice hyper plane is between values of e^l & e^u .

We had two states to find the most efficient DMU:

State 1: DMU_o In model (3) and model (4) is known as the most efficient unit then we introduce DMU_o as the strongly most efficient DMU.

State 2: two different DMUs in models (3) and (4) are known as the most efficient unit in then according to the manager of strategy introduce related unit as the most efficient DMU.

4-Numerical example:

Example 1: evaluating the performance of 15 bank branches.

In this example, we evaluate 15 units with three inputs and five outputs that below is described and sets of data are shown in Tables 1 and 2.

Input 1: payable interest , Input 2: personnel, Input 3: non-performance loans.

Output 1: the total sum of four main deposits, Output 2: other deposits, Output 3: loans granted, Output 4: received interest, Output 5: fee.

Table 1: Inputs for bank

DMU_j	x_{1j}^l	x_{1j}^u	x_{2j}^l	x_{2j}^u	x_{3j}^l	x_{3j}^u
1	1554.29	3427.89	20.47	21.59	412036	413902
2	2926.81	5961.55	18.8	20.16	9945	12120
3	8732.7	17752.5	25.74	27.17	47575	50013
4	945.93	1966.39	20.81	22.54	19292	19753
5	58.0038	11.87539	27.12	27.26	19.500	19.569
6	13759.35	27359.36	19.46	19.46	13929	15657
7	587.69	1205.47	27.29	27.48	27827	29005
8	4646.39	9559.61	24.52	25.07	9070	9983
9	1445.68	2922.15	28.96	28.96	31700	32061
10	175.2831	362.9754	14.84	15.05	8638	10.229
11	2444.34	4955.78	20.42	20.54	500	937
12	3039.58	63.0401	3.932	3.986	51.582	55.136
13	9852.15	19742.89	18.47	21.83	17163	17290
14	4540.75	9312.24	22.83	23.96	17918	17964
15	7303.27	14178.11	22.87	23.19	16148	21353

Table 2: Output for bank

DMU_j	y_{1j}^l	y_{1j}^u	y_{2j}^l	y_{2j}^u	y_{3j}^l	y_{3j}^u	y_{4j}^l	y_{4j}^u	y_{5j}^l	y_{5j}^u
1	335070	410427	337971	449336	1584722	1802942	176436.81	189006.12	560.26	2506.67
2	340377	440355	95978	117659	377309	390203	32396.65	37836.56	304.67	749.4
3	1027546	1061260	37911	503089	1233548	1822028	96842.33	108080.01	2285.03	3174
4	1145235	1213541	229646	268460	468520	542101	32362.8	39273.37	207.98	510.93
5	640890	679916	490508	684372	2946797	3985900	66097.16	95329.87	961.56	1914.25
6	988115	1087392	74133	111324	507502	574355	53591.3	72257.28	480.16	869.52
7	144906	165818	180530	180617	288513	323721	40507.97	45847.48	176.58	370.81
8	408163	416416	405396	486431	1044221	1071812	56260.09	73948.09	4654.71	5882.53
9	119948	120208	14943	17495	297674	308012	21991.53	27934.19	282.73	471.22
10	7008420	7685930	143780	151920	22907450	25735120	66272521	79146308	58.89	86860
11	641680	696338	114183	241081	1579961	2285079	17527.58	20773.91	1070.81	2283.08
12	2861490	3171860	1491830	2707080	7879590	8100880	10679863	1119623	1203790	4335240
13	553167	574989	21298	23043	425886	431815	45652.24	50255.75	438.43	836.82
14	309670	342598	20168	26172	124188	126930	8143.79	11948.04	936.62	1468.45
15	453170	481943	27196	29553	245726	275717	35757.83	42790.14	375.07	559.85

In above expressed, we mention inputs and outputs of example 1 in the tables 1 and 2. Because data are interval then inputs and outputs values were shown as the lower and upper bounds in these tables.
Symbols exist in the tables are as follow:

x_{ij}^{L} : $i=1,2,3$ $j=1,...,15$ Inputs lower bound

x_{ij}^{U} : $i=1,2,3$ $j=1,...,15$ Inputs upper bound

y_{rj}^{L} : $i=1,2,3$ $j=1,...,15$ Outputs lower bound

y_{rj}^{U} : $i=1,2,3$ $j=1,...,15$ Outputs upper bound

However, to achieve most efficientDMU of 15 branch of the bank to resolving models (3) and (4) so we can find the most efficient unit in each model and then presented the most efficient DMU according to conclusions and manager strategy.

The results are as follows:

$model(3):$

$e^{l} = -1.4152167286 \qquad (I_{12}=1)$

$Lower: V(1)=0.00001209\ ,\ V(2)=0.00431630\ ,\ V(3)=0.0000003$

$U(1)=0.000000\ ,\ U(2)=0.00000080\ ,\ U(3)=-0.0000001\ , U(4)=0.00000003, U(5)=0.00000020$

$model(4):$

$e^{u} = 1.35880326180 \qquad (I_{12}=1)$

$Upper: V(1)=0.00000908\ ,\ V(2)=0.00431630\ ,\ V(3)=0.0000003$

$U(1)=0.00000007\ ,\ U(2)=0.00000008\ ,\ U(3)=0.000000\ , U(4)=0.00000000, U(5)=0.00000013$

As be seen in the 15-branch in the example above, branch (12) is known as the most efficient unit in the model (3) and model (4).After considering the material presented in chapter (3), occur state 1 then we're introducing DMU_{12}a strongly most efficient unit because in both models is known as the most efficient DMU.

Example 2: we have considered 12 decisions units of two-inputs and three outputs that inputs and outputs are shown in Tables 3 and 4.

Table 3: inputs for Example 2

DMU	x_1^l	x_2^l	x_1^u	x_2^u
1	4	7	8	10
2	2	5	12	15
3	1	4	11	14
4	2	3	12	13
5	1	4	6	8
6	6	2	16	12
7	4	6	14	16
8	1	4	11	14
9	5	6	15	16
10	3	6	13	16
11	8	9	18	19
12	5	6	15	16

Table 4: Output for Example 2

DMU	y_1^l	y_2^l	y_3^l	y_1^u	y_2^u	y_3^u
1	6	7	8	16	17	18
2	5	6	7	15	16	17
3	2	4	7	12	14	17
4	2	5	7	12	15	17
5	9	10	11	19	20	21
6	7	8	9	17	18	19
7	4	6	15	14	16	25
8	10	11	13	20	21	23
9	6	7	10	16	17	20
10	7	10	16	17	20	26
11	2	14	15	12	24	25
12	3	5	7	6	8	9

The results are as follows:

$$\text{mod}el\,(3):$$

$$e^l = 37.352887 \qquad\qquad (I_{12} = 1)$$

$$Lower : V(1) = 1.00000 \;,\; V(2) = 0.010526$$

$$U(1) = 0.010000 \;,\; U(2) = 0.008333 \;,\; \mathrm{U}(3) = 0.007692$$

$$\text{mod}el\,(4):$$

$$e^u = 40.289553 \qquad\qquad (I_6 = 1)$$

$$Upper : V(1) = 0.025 \;,\; V(2) = 0.241$$

$$U(1) = 0.020 \;,\; U(2) = 0.014 \;,\; \mathrm{U}(3) = 0.013$$

In this example, most efficient DMU in the model (3) and (4) are different and one DMU is not known as the most efficient unit of both models and as you see, DMU_{12} is known as the most efficient unit with solving model (3) and DMU_6 is known as the most efficient unit with solving model (4).In this case, the manager can presented according to the conditions and related strategy one of them as most efficient DMU.

5-CONCLUSIONS

An applications of DEA is obtain most efficient decision-making unit that in this paper, we obtain the most efficient unit when the data are indefinite. We could to achieve this goal and obtained most efficient unit when the data are interval. In this case, we must to solve two models and most efficient unit obtained in both models and the most efficient unit is selected according to the conditions. As we have seen, the probability is that a DMU in two model is known as most efficient unit such as Example 1, the branch 12 (DMU12) is known as the most efficient unit in of both models and it can be introduced as strongly most efficient DMU. Also the probability is that a DMU in a model and another DMU on another model are known as the most efficient DMU such as example 2 in which case the unit is known as the most efficient unit according to manager strategy.

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