J. Basic. Appl. Sci. Res., 3(8)1-7, 2013
© 2013, TextRoad Publication

ISSN 2090-4304
Journal of Basic and Applied Scientific Research
www.textroad.com

A Redshift Phenomenon in Relativistic Binary System

A.B.Morcos[1,2,3,4]

[1]Department of Astronomy, National Research Institute of Astronomy and Geophysics, Helwan, Cairo, Egypt.
[2]Center of theoretical physics (CTP), British University in Egypt (BUE).
[3]Egyptian Relativity Group.
[4]Leader of the group of extra-solar planets group, in Kottamia Center of Scientific Excellence for Astrophysics and Space Science

ABSTRACT

In previous trials, similar techniques have been used to study the solar limb effect phenomenon, by using Schwarzschild solution and Lense-Thirring Field. A more general formula for the limb effect of rapidly rotating stars was done by the author. In this work a modified Curzon exact solution for Einstein's field equations has been used to study the red-shift of a binary system. In this treatment, it is assumed that the primary star is massive with respect to the secondary one and its center of mass is coincident to the center of rotating polar coordinate system. The secondary star is assumed to rotate around the primary star and Earth's observer rotates with the Earth. A general theoretical formula for the redshift of binary system is obtained. This formula may be useful in studying different cases of binary systems and may through the light on third body of some binary systems.

KEYWORDS: Binary Pulsars, Redshift, Relativistic Binary Stars, Curzon Solution, Axial Symmetry.

INTRODUCTION

The discovery of the first double-neutron-star binary PSR 1916+13 by Hulse-Taylor**,** and many other binary systems as PSR B1913+16 and PSR B1534+12, introduced the possibility of strong observational tests of gravity in the strong fields. As it is well known that the received signals from celestial objects are our sources of knowledge about them. The received signals from these objects come to us via different carriers as photons, neutrinos, gamma ray and x-ray. For this reason we consider here one of the important phenomenon**,** i.e. the red-shift phenomenon for photons and a similar shift for other carriers. The problem of the limb effect phenomenon for the solar disk has been studied theoretically by using the generalized redshift formula given by Mikhail et al. (2002). Many authors have attempted to find satisfactory interpretation for this effect theoretically in the frame work of GR (cf. Mikhail et al. (2002), Wanas et al. (2008)). The generalized formula of the redshift not only used to study the limb effect in the solar disk but also to study the redshift of rabidly rotating stars (Morcos (2013)) and the redshift of static binary system (Wanas et. al (2012)). Waleed and Wanas (2010) considered a modified Curzon solution for Einstein field equations to describe the field of binaries. A more general formula for red-shift for binary system was written in the case of a stationary observer on the Earth surface.

The aim of this paper is deriving a more general formula for the redshift of (relativistic) binary systems. We are going to use an exact axial symmetric solution of Einstein's field equations and the Kermak McCrea and Whittaker (KMW) theorems on null geodesics to derive a more general formula for redshift of relativistic binary system. This has been delivered by using the adjusted Curzon given by Wanas et al. (2012), but in our treatment we considered the center of rotating polar coordinate system and the center of massive primary star are coincident. Also, the rotational motion of the secondary component of the binary system around its center of mass is considered in our calculations, as well as the rotation of the observer with the Earth around its axis is considered. Generally our treatment is more general than other treatments.

In the first section, a review about developed Kermack, Mecrea and Whittakar Formula is given. In the second section the modified Curzon solution is displayed. The calculated values for vectors and null-vectors along the world lines of stars and Earth are shown in the third section. The general formula for the redshift of binary system and conclusion are given in the last two sections.

[*]**Corresponding Author:** A.B. Morcos, Department of Astronomy, National Research Institute of Astronomy and Geophysics, Helwan, Cairo, Egypt. e.mail:fadymorcos@hotmail.com

DEVELOPED KERMACK, MECREA AND WHITTAKER FORMULA

Mikhail et al. (2002) developed the formula given by Kermack, Mecrea, and Wittaker (KMW) (1933) to be suitable for measuring the total redshift of a celestial object. This was instead of using the formula for studying cosmological redshift only. In what follows a short review about the developed formulae will be given. Let us consider two points S_1 and S_2 on the world line (W_s) of a celestial object in the space time. The two points S_1 and S_2 are considered as the emitters of signals on celestial object at wavelengths λ_1 and λ_2 respectively. An observer O is moving in the field of the celestial object and its world line is (W_o), receiving the signals emitted by S_1 and S_2. As it is well known the received signals depend on the line of sights. If it is assumed now that, there are two null-trajectories T_r and T_o passing radially and oblique between the observer point O on its world line and S_1 and S_2 on celestial object respectively. Let the received wavelengths from S_1 and S_2 at O are $\underset{0}{\lambda_1}$ and $\underset{0}{\lambda_2}$ respectively. By using KMW theorems, $\underset{0}{\lambda_1}$ and $\underset{0}{\lambda_2}$ can be expressed as

$$\underset{0}{\lambda_1} = \frac{\left[\rho_\mu \eta^\mu\right]_{S_1}}{\left[\varpi_\mu \eta^\mu\right]_O} \lambda_1 \tag{1}$$

$$\underset{0}{\lambda_2} = \frac{\left[\rho_\mu \zeta^\mu\right]_{S_2}}{\left[\varpi_\mu \zeta^\mu\right]_O} \lambda_2 \tag{2}$$

where ρ^μ is the unit vector along the world line of the emitter W_s and ϖ^μ is the directional unit-vector along the world line of the observer W_o at a certain instant. And ζ^μ is the transport null-vector along the oblique null trajectory T_o and η^μ is the transport null-vector along the radial null trajectory T_r. The suffixes O, S_1 and S_2 denote that the expressions between the brackets are evaluated at O, S_1 and S_2 respectively.

By considering that the two points S_1 and S_2 are identical, so

$$\lambda_1 = \lambda_2 = \lambda \tag{3}$$

The redshift of the signals is given by:

$$\Delta Z = \frac{\underset{0}{\lambda_1} - \underset{0}{\lambda_2}}{\lambda}. \tag{4}$$

By using (1) and (2), the redshift can be expressed as

$$\Delta Z = \frac{\left[\rho_\mu \eta^\mu\right]_{S_1}}{\left[\varpi_\mu \eta^\mu\right]_O} - \frac{\left[\rho_\mu \zeta^\mu\right]_{S_2}}{\left[\varpi_\mu \zeta^\mu\right]_O} \tag{5}$$

A MODIFIED CURZON SOLUTION

Curzon's exact solution of Einstein's equations is an exact axial symmetric stationary solution. Wanas et al. (2012) modified Curzon solution to describe the gravitational field of the two-body system and calculate the redshift of spectral lines from binary system. They have adjusted Curzon metric in such way to describe the gravitational field of a binary system. They wrote Curzon's modified metric in the polar coordinate system (r, θ, ϕ, t) in the form

$$ds^2 = g_{rr}dr^2 + g_{\theta\theta}d\theta^2 + g_{\phi\phi}d\phi^2 + g_{tt}dt^2 + 2g_{\theta\phi}d\theta d\phi + 2g_{\theta t}d\theta dt + 2g_{\phi t}d\phi dt, \tag{6}$$

where the metric functions are given by

$$g_{rr} = -e^{\mu},$$

$$g_{\theta\theta} = -r^2 \frac{\cos^2\theta \sin^2(\phi+\omega t)e^{\mu} + \cos^2(\phi+\omega t)e^{-\nu}}{\left[\cos^2\theta + \sin^2\theta\cos^2(\phi+\omega t)\right]},$$

$$g_{\theta\phi} = -\frac{r^2 \sin\theta\sin(\phi+\omega t)\cos\theta\cos(\phi+\omega t)}{\left[\cos^2\theta + \sin^2\theta\cos^2(\phi+\omega t)\right]}\left(e^{\mu} - e^{-\nu}\right),$$

$$g_{\theta t} = -\omega\frac{r^2 \sin\theta\sin(\phi+\omega t)\cos\theta\cos(\phi+\omega t)}{\left[\cos^2\theta + \sin^2\theta\cos^2(\phi+\omega t)\right]}\left(e^{\mu} - e^{-\nu}\right),$$

$$g_{\phi\phi} = -r^2\sin^2\theta \frac{\cos^2\theta \sin^2(\phi+\omega t)e^{-\nu} + \cos^2(\phi+\omega t)e^{\mu}}{\left[\cos^2\theta + \sin^2\theta\cos^2(\phi+\omega t)\right]},$$

$$g_{\phi t} = -\omega r^2\sin^2\theta \frac{\cos^2\theta \sin^2(\phi+\omega t)e^{-\nu} + \cos^2(\phi+\omega t)e^{\mu}}{\left[\cos^2\theta + \sin^2\theta\cos^2(\phi+\omega t)\right]},$$

$$g_{tt} = -\Big[\omega^2 r^2\sin^2\theta\left(\cos^2\theta\sin^2(\phi+\omega t)e^{-\nu} + \cos^2(\phi+\omega t)e^{\mu}\right)$$
$$-\left(\cos^2\theta + \sin^2\theta\cos^2(\phi+\omega t)\right)e^{\nu}\Big] \Big/ \left[\cos^2\theta + \sin^2\theta\cos^2(\phi+\omega t)\right],$$

and the potential functions are

$$\nu\left(r,\theta,\phi,t\right) = -2\left[\frac{m_1}{r_1} + \frac{m_2}{r_2}\right]$$

$$\mu\left(r,\theta,\phi,t\right) = 2\left[\frac{m_1}{r_1} + \frac{m_2}{r_2}\right] - r^2\left(\cos^2\theta + \sin^2\theta\cos^2(\phi+\omega t)\right)\left[\frac{m_1^2}{r_1^4} + \frac{m_2^2}{r_2^4}\right] + \frac{m_1 m_2}{a^2}\left[\left(\frac{r^2 - a^2}{r_1 r_2}\right) - 1\right],$$

where

$$\left.\begin{aligned} r_1^2 &= (r+a)^2 - 2a\, r(1+\sin\theta\sin(\phi+\omega t)), \\ r_2^2 &= (r+a)^2 - 2a\, r(1-\sin\theta\sin(\phi+\omega t)) \end{aligned}\right\}$$

are the distances between a potential point and the point masses m_1 and m_2 respectively, and a is the distance between m_1 and m_2 .

GENERAL ASSUMPTIONS FOR CALCULATING BINARY STARS REDSHIFT

We are going to use the modified vacuum solution of Curzon to describe the field of compact binaries especially in the case of double binaries, where there is no matter or non-gravitational fields are present in both regions between the two pulsars, or outside them. In order to calculate the redshift of the pulses coming out from a binary system, and using equation (5), we assume that:

1- The gravitational field of the primary star is axially symmetric and will be given by (modified Curzon), while the field of the secondary is a spherically symmetric (Schwarzschild field).
2- The mass of the secondary star is very small with respect to the primary one. The secondary rotates around the primary in a circular orbit of radius (a) at an angular velocity V_1.
3- The observer at the Earth's surface is rotating with the Earth around its axis at a rotational velocity V_2. The Earth's observer observes the primary star radially while he observes the secondary in an oblique line of sight.

4- The binary system is at a distance (b) from the Earth's observer.
5- The Earth's field is assumed to be Schwarzschild spherical symmetric field.

From the previous assumptions it is clear that we are going to calculate the values of the two null-transport vectors η^{μ} and ζ^{μ} along the null-geodesic trajectories T_r (radial and passing through primary star) and T_o (oblique and passing through secondary star). The two unit vectors ρ^{μ} and ϖ^{μ} will be calculated along the world lines of the primary, secondary stars and at the Earth's world line respectively.

In the next section, we are going to calculate the values of vector and null-vectors required to change in redshift.

GENERAL FORM FOR THE SOLUTION OF EQUATION OF MOTION

As it is assumed in the previous section that the field of both the secondary star and the Earth are spherically symmetric , then the transport vectors along the world line of the secondary star $\underset{S2}{\rho^{\mu}}$ and the null vectors, $\underset{o}{\eta^{\mu}}$ & $\underset{o}{\zeta^{\mu}}$ at the Earth and the unit vector $\underset{o}{\varpi^{\mu}}$ along Earth's world line can be evaluated in Schwarzschild field.

As it is well known the ordinary Schwarzschild's space-time is given by,

$$ds^2 = -\gamma^{-1} dr^2 - r^2 (d\theta^2 + \sin\theta d\phi^2) + \gamma dt^2 \tag{7}$$

where $\gamma = 1 - \frac{m_2}{r}$, $m_2 = \frac{GM_2}{c^2}$ and M_2 is the mass of the secondary in CGS units. The equation of motion of a free test particle can be put (cf. Adler et al., 1975, p. 54) into the form

$$\frac{d^2 x^{\alpha}}{dP^2} + \begin{Bmatrix} \alpha \\ \beta\sigma \end{Bmatrix} \frac{dx^{\beta}}{dP} \frac{dx^{\sigma}}{dP} = 0 \tag{8}$$

where P is the affine parameter characterizing the trajectory of the particle which is existing in the Schwarzschild field, then the solution of equation (8) by using (7) is, in general form , a set of differential equations given by **Error! Bookmark not defined.**:

$$\frac{dr}{dP} = \sqrt{\alpha^2 - \gamma\left(E^2 + \frac{\gamma^2}{r^2}\right)} \tag{9}$$

$$\frac{d\theta}{dP} = 0, \quad \theta = \pi/2 \tag{10}$$

$$\frac{d\phi}{dP} = \frac{l}{r^2} \tag{11}$$

$$\frac{dt}{dP} = \frac{\alpha}{\gamma} \tag{12}$$

where α, l are two constants of integration, and E is **a** parameter takes the following values:

$$E = \begin{cases} 0 \text{ , for a photon ;} \\ 1 \text{ , for a material particles} \end{cases}$$

As it is assumed, that the rotational velocity of the secondary star around the primary is V1, then we can write

$$\frac{d\phi}{dt} = V_1$$

$$\frac{d\phi}{dP} = V_1 \frac{dt}{dP} = V_1 \frac{\alpha}{\gamma(a)}$$

where $\gamma(a) = 1 - \frac{m_2}{a}$, m_2 is the mass of the secondary stars.

The components of the unit-vector ρ_μ along the secondary star's world line are

$$\begin{aligned}\rho^\mu &\equiv \left(\frac{dr}{dP}, \frac{d\theta}{dP}, \frac{d\phi}{dP}, \frac{dt}{dP}\right) \\ &= \left(0,\ 0\ ,\ V_1 \frac{\alpha}{\gamma(a)}\ ,\ \frac{\alpha}{\gamma(a)}\right)\end{aligned} \tag{13}$$

Since, $\rho^\mu \rho_\mu = 1$.

Hence, using (7) and (13) we get,

$$\rho_{\mu o} = \left(0, 0, -4\frac{a^2 V_1}{\sqrt{-4V_1^2 a^2 + \gamma(a)}}, \frac{\gamma(a)}{\sqrt{-4V_1^2 a^2 + \gamma(a)}}\right). \tag{14}$$

Since the observer is rotating around the Earth's axis of symmetry at a velocity V_2, following the same set of equations from (9) to (12), taking into our consideration that $\varpi^\mu \varpi_\mu = 1$, then the unit vector along its world line is

$$\varpi_{\mu o} = \left(0,\ 0\ ,\ \frac{-4V_2 b^2}{\sqrt{\gamma_b - 4b^2 V_2^2}}\ ,\ \frac{\gamma_b}{\sqrt{\gamma_b - 4b^2 V_2^2}}\right) \tag{15}$$

where $\gamma_b = 1 - \frac{2m_2}{b}$.

Now we are going to evaluate the null vectors η^μ & ζ^μ and the transport vector ρ^μ of the primary star, assuming that the gravitational field of this star is axially symmetric and is given by the modified Curzon metric (6), we have

$$\begin{aligned}\rho_{\mu s1} = \Bigg\{ &\frac{e^{\mu(a)}\sqrt{-e^{\mu(a)} - V_1^2\omega^2 a^2 e^{\mu(a)} + V_1^2 e^{\nu(a)}}}{e^{\mu(a)} + V_1^2\omega^2 a^2 e^{\mu(a)} - V_1^2 e^{\nu(a)}}, 0\ , \\ &\frac{\omega a^2 e^{\mu(a)} V_1 \sqrt{-e^{\mu(a)} - V_1^2\omega^2 a^2 e^{\mu(a)} + V_1^2 e^{\nu(a)}}}{e^{\mu(a)} + V_1^2\omega^2 a^2 e^{\mu(a)} - V_1^2 e^{\nu(a)}}\ , \\ &-\frac{\left(-\omega^2 a^2 e^{\mu(a)} + e^{\nu(a)}\right) V_1 \sqrt{-e^{\mu(a)} - V_1^2\omega^2 a^2 e^{\mu(a)} + V_1^2 e^{\nu(a)}}}{e^{\mu(a)} + V_1^2\omega^2 a^2 e^{\mu(a)} - V_1^2 e^{\nu(a)}}\Bigg)\end{aligned} \tag{16}$$

$$\begin{aligned}w_{\mu s1} = \Bigg(&\frac{e^{\mu(b)}\sqrt{-e^{\mu(b)} - V_2^2\omega^2 b^2 e^{\mu(b)} + V_2^2 e^{\nu(b)}}}{e^{\mu(b)} + V_2^2\omega^2 b^2 e^{\mu(b)} - V_2^2 e^{\nu(b)}}\ , 0\ , \\ &-\frac{\omega b^2 e^{\mu(b)} V_2 \sqrt{-e^{\mu(b)} - V_2^2\omega^2 b^2 e^{\mu(b)} + V_2^2 e^{\nu(b)}}}{e^{\mu(b)} + V_2^2\omega^2 b^2 e^{\mu(b)} - V_2^2 e^{\nu(b)}}, \\ &-\frac{\left(-\omega^2 b^2 e^{\mu(b)} + e^{\nu(b)}\right) V_2 \sqrt{-e^{\mu(b)} - V_2^2\omega^2 b^2 e^{\mu(b)} + V_2^2 e^{\nu(b)}}}{e^{\mu(b)} + V_2^2\omega^2 b^2 e^{\mu(b)} - V_2^2 e^{\nu(b)}}\Bigg)\end{aligned} \tag{17}$$

Now we can calculate the values of the transport null-vector η^{μ} & ζ^{μ} along the null-trajectories Tr and T0 at the primary star S1 and Earth. Using η^{μ}_{o} & ζ^{μ}_{o} to express their values at Earth, and η^{μ}_{s1} & ζ^{μ}_{s2} at star. The values of these null- vectors are given by:

$$\eta_{o}^{\mu} = \left[-\frac{e^{\mu(b)}C}{\gamma(b)}, -b^{2} e^{(-\nu(b))} \sqrt{C^{2}}, 0, 0 \right] \tag{18}$$

$$\eta_{s1}^{\mu} = \left[-\frac{e^{\mu(a}C}{\gamma(a)}, -r^{2} e^{(-\nu(a))} \sqrt{C^{2}}, 0, 0 \right] \tag{19}$$

$$\zeta_{o}^{\mu} = \left(-\frac{e^{\mu(b)}\mathrm{D}}{\gamma(b)}, -b^{2} e^{-\nu(b)} \sqrt{-\frac{-D^{2}b^{2}+\gamma(b)L^{2}}{b^{2}}}, -\omega e^{\mu(b)}L, -\frac{\left(\omega^{2} b^{2} e^{\mu(b)} - e^{\nu(b)}\right)L}{b^{2}} \right) \tag{20}$$

$$\zeta_{s2}^{\mu} = \left(-\frac{e^{\mu(a)}\mathrm{D}}{\gamma(a)}, -b^{2} e^{-\nu(a)} \sqrt{-\frac{-D^{2}a^{2}+\gamma(a)L^{2}}{a^{2}}}, -\omega e^{\mu(a)}L, -\frac{\left(\omega^{2} a^{2} e^{\mu(a)} - e^{\nu(a)}\right)L}{a^{2}} \right) \tag{21}$$

THEORETICAL RELATION OF REDSHIFT

Here, we found the value of redshift of a binary system one of them is massive and the coordinate system is fixed at its center and rotate with coordinate system at an angular velocity ω, while the secondary star is of small mass with respect to primary and rotates around the primary at a rotational velocity V1. If we use the equations from (14) to (21) in equation (5), we have

$$\begin{aligned}
\Delta Z = {} & \gamma(b)\mathrm{L}\Big(-b^{2}\left(\mathrm{e}^{\mu(b)}\right)^{2} V_{1}\gamma(a)\omega^{2}a^{2}\mathrm{e}^{\mu(a)} - b^{4}\left(\mathrm{e}^{\mu(b)}\right)^{2} V_{1}\gamma(a)\omega^{4}a^{2}\mathrm{e}^{\mu(a)}V_{2}^{2} \\
& + b^{2}\mathrm{e}^{\mu(b)}V_{1}\gamma(a)\omega^{2}a^{2}\mathrm{e}^{\mu(a)}V_{2}^{2}\mathrm{e}^{\nu(b)} + b^{2}\left(\mathrm{e}^{\mu(b)}\right)^{2} V_{1}\gamma(a)\mathrm{e}^{\nu(a)} + b^{4}\left(\mathrm{e}^{\mu(b)}\right)^{2} V_{1}\gamma(a)\mathrm{e}^{\nu(a)}v_{2}^{2}\omega^{2} \\
& - b^{2}\mathrm{e}^{\mu(b)}V_{1}\gamma(a)\mathrm{e}^{\nu(a)}v_{2}^{2}\mathrm{e}^{\nu(b)} + \mathrm{e}^{\mu(a)}a^{2}V_{2}\gamma(b)\omega^{2}b^{2}\left(\mathrm{e}^{\mu(b)}\right)^{2} + \mathrm{e}^{\mu(a)}a^{2}V_{2}^{3}\gamma(b)\omega^{4}b^{4}\left(\mathrm{e}^{\mu(b)}\right)^{2} \\
& - 2\mathrm{e}^{\mu(a)}a^{2}V_{2}^{3}\gamma(b)\omega^{2}b^{2}\mathrm{e}^{\mu(b)}\mathrm{e}^{\nu(b)} - \mathrm{e}^{\mu(a)}a^{2}V_{2}\gamma(b)\mathrm{e}^{\nu(b)}\mathrm{e}^{\mu(b)} + \mathrm{e}^{\mu(a)}a^{2}v_{2}^{3}\gamma(b)\left(\mathrm{e}^{\nu(b)}\right)^{2} \Big) \\
& \div \Big(\mathrm{e}^{\mu(b)}\left(\sqrt{-\mathrm{e}^{\mu(a)} - V_{1}^{2}\omega^{2}a^{2}\mathrm{e}^{\mu(a)} + V_{1}^{2}\mathrm{e}^{\nu(a)}}\right)\left(\gamma(a)a^{2}\sqrt{-\mathrm{e}^{\mu(b)} - V_{2}^{2}\omega^{2}b^{2}\mathrm{e}^{\mu(b)} + V_{2}^{2}\mathrm{e}^{\nu(b)}}\right) \\
& \left(-\mathrm{e}^{\mu(b)}(D)b^{2} - V_{2}L\gamma(b)\omega^{2}b^{2}\mathrm{e}^{\mu(b)} + V_{2}L\gamma(b)\mathrm{e}^{\nu(b)}\right)\Big),
\end{aligned} \tag{22}$$

where L and D are constant of integrations.
This relation seems to be complicated, but all its terms can be determined easily from observations.

CONCLUSION

The equation (22) represents a theoretical general relation for calculating the redshift of a relativistic binary system, in which the primary star is more massive with respect to the secondary one. Also, it is assumed that the center of the polar coordinate is coincident to the center of mass of primary and the primary rotate with the coordinate system. The secondary star of the system is rotating around the primary and also the observer is assumed to rotate with Earth around its axis. This situation is to near to the true situation of a binary system in our Galaxy.

If it is assumed that the coordinate system is at rest this relation will tend to a very easy form, depends only on the masses of the two stars, beside the rotational speeds of secondary star and the observer. Although the modified Curzon solution, is a suitable for studying a binary pulsars (Wanas et al. (2012)), but from the relation (22) and the assumed conditions, one can use it in case of binary systems. This relation can be adapted also to study a system of binary stars system approximately equal in mass and both of them rotate around the center of mass of that system.

Moreover the obtained relation may help to study the gravitational radiation by compact binaries. Furthermore, this may be used to study the restricted three body problem. Considering the motion of a third point-like particle in the field of a binary system, without applying the perturbation technique, that may help in solving many problems in celestial mechanics.

REFERENCES

1. Adler, R. , Bazin, M., and Schiffer, M., 1975 "INTRODUCTION TO GENERAL RELATIVITY , 2ed ed., Mc Graw Hill, pp. 54
2. Kermack, W.O., Mc Crea, W.H. and Wittaker, E.T., 1933, Proc. Roy. Soc. Edin.53, 31.
3. Mikhail, F.I., Wanas, M.I. and Morcos, A.B., 2002, Astrophys. Space Sci., 280, (3): pp 223- 233,
4. Morcos, A.B., June 2013, accepted to be published in NRIAG JAG. It is a peer-reviewed scientific journal published by Elsevier.
5. Waleed El Hanafy and Mamdouh Wanas, 2010: RELATIVISTIC GRAVITY AND COMPACT BINARY

   OBSERVATIONS: A Complete Guide to Binary Pulsar Analysis, a Step towards an exact solution of Two Body Problem in General Relativity, LAP LAMBERT Academic Publishing, pp 175-200
6. Wanas, M.I, Awadalla, N.S. and El Hanafy, W.S., 2012, arXiv: 1002.3399v5 [gr-qc].
7. Wanas, M.I, Morcos, A.B. and El Gamal, S.I., 2008. Lense-Thirring Field and the Solar Limb Effect, Proc. MERIEM-IAU regional Meeting, (1): pp 173-177 (http://arxiv.org/abs/1008.1028)