

Secondary Flow Modeling in a Compound Meandering River

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ABSTRACT

The present paper was formulated in order to present a method for simulation of transverse distribution of velocity in bend; this is of high importance in bends and of high application in conservation of river banks, transfer pattern, sediments deposition, etc. The presented method is based upon simulation of secondary flow's cells in Meander Rivers. In order to achieve an improved prediction of mean velocity in depth, secondary flow was defined as a function of bed shear stress and flow velocity. In meander channels, the proposed method's relative error is 4 and has better determination coefficient (R^2 =94%) than the method proposed by Ervine et al. (R^2 =70%).

KEYWORDS: velocity transverse distribution, secondary flow, river, meander.

1- INTRODUCTION

Studying flow hydraulics in bends has always been in the focus of researchers. In bend, variations of flow depth, velocity, and shear stress in width of rivers are always high causing secondary flow and friction of outer bend as well as sedimentation in inner bend. Meander rivers have always gained attention by engineers and authors due to friction of outer bend and necessity of water supply equipment in addition to flood channels in outer bend, to name a few. Among them, simulation of flow velocity transverse distribution in river bends is very important and it is applied in conservation of river banks, transfer pattern, sediments deposition, pollution spread, flood control, and flood channel design. Flow structure in river bends in non-flood flows due to development of secondary flow is complicated and three-dimensional. 2-D and 3-D models are usually adopted in order to determine velocity transverse distribution in Meander Rivers. These models have complicated structures and long execution time. The present study attempted to use a simpler method. For this aim, velocity transverse distribution is estimated and simulated via Navier-Stokes equation (semi 2-D). Shiono and Knight (1988) presented a simple semi 2-D mathematical model by use of Navier-Stokes coherence and momentum equations for estimation of mean velocity transverse distribution in depth in simple and compound channels with direct route (hypothesizing uniform and persistent flow) [6]. The effects of secondary flow were neglected in this mathematical model. Shiono and Knight (1991) proposed a more thorough semi 2-D model for channels and the rivers with compound channel through addition of secondary flow term into the previous mathematical model [7]. Numerical solution of semi 2-D mathematical method of Knight and Shiono(1996) was performed through limited components method proposed by Knight and Abril (1996) [3,4]. The results obtained from this numerical solution in Wallingford experimental channel were in agreement with those obtained for velocity transverse distribution. Ervine et al. (2000) proposed an analytic semi 2-D model for analyzing flow hydraulics in compound channels with direct and meander route [2]. Being similar to Shiono and Knight mathematical method (1991), this model considered a simple term for interference of secondary flows [7]. The results obtained from solving this model in experimental channels with compound channels as well as flooded rivers showed that secondary flow is very effective on simulation of velocity transverse distribution in meander route. It is noteworthy that the present study only considered flood flows without taking basic flows into account. Spooner and Shiono (2003) presented a 2-D mathematical model with curvilinear coordinates without taking effect of energy loss caused by centrifugal force and secondary flow into account in order to predict velocity transverse distribution and bed shear stress in meander compound channels [9]. In the present paper, velocity transverse simulation was performed in river channel in basic flow.

2- METHODS AND MATERIALS

2-1- Principles of mean semi 2-D mathematical model in depth in curvilinear coordinates

Shiono and Knight (1991) presented the following equation for solving transverse distribution of flow velocity in direct compound channels [7]:

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$$\rho g H S_0 - \rho \frac{f}{8} u_d^2 \sqrt{1 + \frac{1}{s^2}} + \frac{\partial}{\partial y} \left\{ \rho \lambda H^2 \left(\frac{f}{8} \right)^{1/2} u_d \frac{\partial u_d}{\partial y} \right\} = \frac{\partial H (\rho \overline{UV})_d}{\partial y}$$
(1)

where u_d , ρ , g, λ , f, s, H, S0, and y stand for mean velocity in depth, density of water, gravitational acceleration, dimensionless coefficient of viscosity for agitated flow, Darcy-Weisbach friction coefficient, lateral gradient of channel, flow depth in each point, channel bed gradient, and transverse direction,

respectively. The term on the right is related to the effect of secondary flow where U and V stand for time mean velocities along longitudinal and latitudinal directions.

Shiono and Knight (1991) defined the effect of secondary flow as a function of bed shear stress [7]:

$$\frac{\partial H(\rho \overline{U}\overline{V})_d}{\partial y} = \beta \left(\rho g S_0 H\right)$$
(2)

Where β is secondary flow coefficient which is 0.15 and -0.25 for main channel and floodplains, respectively. Ervine et al. (2000) hypothesized that \overline{U} and \overline{V} are functions of mean velocity in depth u_d [2]:

$$U = k_1 u_d , V = k_2 u_d \rightarrow \frac{\partial H(\rho \overline{U} \overline{V})_d}{\partial y} = \frac{\partial k}{\partial y} \frac{\partial u_d^2}{\partial y}$$

Where k_1 , k_2 , and k_3 are proportion coefficients. Coefficient *k* is dependent on flow depth, floodplain roughness, and meander degree of the river. By calibrating the mathematical model, proportion coefficient range in main channel and floodplains for Meander Riverswere found to be 1-5.5 and 0-2, respectively. The value of this coefficient in wide rivers with direct route is negligible [2].

(3)

Recent studies have shown that application of both hypotheses in meander compound channels is limited due to centrifugal acceleration and agitated flow stresses.

In meander channels, in addition to bed shear stress, centrifugal acceleration, agitated flow stresses, and other mechanisms are effective on simulation of transverse distribution of velocity and flow rate. The equation for flow movement in meander route in a compound channels is as follows [9]:

$$HS_{0} - \left(\frac{r_{in} + y_{in}}{r_{in}}\right)\rho\frac{f}{8}\left(\overline{U}_{d}^{2}\right)\sqrt{1 + S_{oy}^{2}} + \left(\frac{r_{in} + y_{in}}{r_{in}}\right)\frac{\partial}{\partial y}\left[\rho\lambda H^{2}\left(\frac{f}{8}\right)^{1/2}\frac{1}{2}\frac{\partial\overline{U}_{d}^{2}}{\partial y}\right] + \frac{2}{r_{in}}\left[\rho\lambda H^{2}\left(\frac{f}{8}\right)^{1/2}\frac{1}{2}\frac{\partial\overline{U}_{d}^{2}}{\partial y}\right] = \rho\left[\left(\frac{r_{in} + y_{in}}{r_{in}}\right)\frac{\partial H(U\overline{V})_{d}}{\partial y} + \frac{2H(\overline{U}\overline{V})_{d}}{r_{in}}\right]$$
(4)

Where R_{in} and Y_{in} stand for inner radius of channel bend and distance of curve's inner channel (Fig. 1). The term on the right indicates to the effect of secondary flow which is of high importance in meander experimental compound channels unlike direct natural rivers. It is noteworthy that in direct rivers, $Y_{in}=0$; therefore, Equation (4) is simplified into Equation (1).



Figure 1: the parameters in the differential equation (4) in a meander channel [5].

3- THE RECOMMENDED METHOD FOR INTERFERENCE OF SECONDARY FLOW

In the present study, the mean 2-D mathematical method in depth which is based upon coherence and momentum relations with curvilinear coordinates was adopted. For simulation of secondary flow, both hypotheses presented by Shiono and Knight (1991) and Ervine et al. (2000) were considered [7, 2]:

$$\Gamma = \left[\frac{\partial H(\bar{U}\bar{V})_d}{\partial y} + \frac{2}{r_{in} + y_{in}} H(\bar{U}\bar{V})_d\right] = \{\Gamma_1 + \Gamma_2\} = \left\{\left[\beta_s \rho g S_o H\right] + \left[\frac{\partial HK\bar{U}^2_d}{\partial y} + \frac{2}{r_{in} + y_{in}} HK\bar{U}^2_d\right]\right\}$$
(5)

According to the model proposed by Kordi, (2011) for meander compound channels, the model was divided into 5 sections as follows for presenting the mathematical model for river channels when the flow is only in the main channel [5]:



Figure 2: division of transverse channel according to secondary flows structure[5].

In proposed model of Kordi (2011) [7], a k,β is defined for each section where β and k stand for secondary flow coefficient in Shiono and Knight (1991) relation (2) and Ervine et al (2000) [7, 2].; transverse velocity in meander channels is obtained by using the following dimensionless equations and numerical solution of the Equation (5).

The area 1: In this area which is placed in outer wall of channel, K=0%, β is a function of sinusoidal function, relative radius, and distance and it is estimated via the following dimensionless equation according to FCF data:

$$\beta = 0.16 \left[Si^{3.88} \left(\frac{y_a}{y_a + L_2} \right)^{10.31} R_r^{1.045} \right]$$
(6)

where Si, y_a , L_2 , R_r stand for sinusoidal, the distance from the origin coordinate to the point a, width of the area 2, and relative radius, respectively with R_r estimated as follows:

$$R_r = \frac{R}{R + y_{in}} \tag{7}$$

The area 2:In this area which is placed in outer bend of channel, K=0.1%. First, β in the point *a* is estimated from the Equation (6) and then it is estimated in the point *b* from the Equation (8). B from the points *a*tob changes linearly.

$$\beta = 0.796 \left[Si^{1.508} \left(\frac{y_b}{y_b + L_2} \right)^{6.243} R_r^{0.57} \right]$$
(8)

Where y_h is the distance from the point *b* to the origin of coordinate.

The area 3: in this area, K varies linearly from 0.1% in the point *b* to 0.25% in the point *c*. β in this area varies linearly from the point *b* (obtained from the Equation 8) to the point *c* (obtained from the Equation 9):

$$\beta = 0.324 \left[Si^{53.19} \left(\frac{y_c}{y_c + L_4} \right)^{309.84} R_r^{6.617} + D_r^{-0.468} \right]$$
(9)

Where y_c is the distance from the point c to the origin of coordinate.

The area 4: in this area, K=0.25% and β varies linearly from the point *c* (obtained from the equation 9) to the point *d* (obtained from Equation 10).

$$\beta = 0.304 \left[Si^{3.17} \left(\frac{y_d}{y_d + L_4} \right)^{1.832} R_r^{1.65} + D_r^{3.974} \right]$$
(10)

Where y_d is the distance of the point d from the origin coordinate.

The area 5: in this area, K=0 and β is estimated from the Equation (10).

The proposed model in the present paper was based upon the model recommended by Kordi, (2011) [5]. Since Kordi's model is for meander compound channel, the areas 1 and 5 are eliminated as they are for floodplains. Furthermore, in the areas 2, 3, and 4, the relative depth D_r is related to compound channel and therefore, it is eliminated. The proposed model was changed as follows through calibration of Kordi's Equations.

For presenting a mathematical model for meander channels, each channel was divided into three sections. Figure 3 depicts this division. The areas 1 and 3 are in slope walls of channel with the areas 1 and 3 in outer and inner bends of channel, respectively.



Figure 3: different areas in river channel

The area 1: in this area which is placed in outer wall of channel, it begins from river bank and continues until the beginning of river bed. β in this area is dependent on sinusoidal function, relative radius, and the distance of coordinate origin from the point *a*. for estimation of velocity, β is first obtained in the point *a* from the Equation (11) and then in the point *b* from the Equation (12). β varies linearly from the points *a* to *b*. the dimensionless equations are as follows:

$$\beta = 0.0263 \left[Si^{1.377} \left(\frac{y_a}{y_a + L_1} \right)^{2.174} R_r^{6.312} \right] (11)$$

$$\beta = 0.1234 \left[Si^{1.283} \left(\frac{y_b}{y_b + L_1} \right)^{6.595} R_r^{0.586} \right]$$
(12)

where Si, y_a , y_b , L_1 , and R_r stand for sinusoidal function, the distance of coordinate origin from the point *a*, the distance of the point *b* from coordinate origin, the width of area 1, and relative radius respectively with R_r estimated as follows:

$$R_r = \frac{R}{R + y_{in}}(13)$$

And K in this area is dependent on relative radius, water depth, and area length and it is obtained as follows: $K_1 = 0.0012(Rr\frac{H}{L_1})$ (14)

The area 2: β in this area varies linearly from its value in the point *b* which is obtained from the Equation (12) to that in the point *c* which is obtained from the Equation (13):

$$\beta = 0.0174 \left[Si^{1.311} \left(\frac{y_c}{y_c + L_3} \right)^{6.409} R_r^{1.643} \right] (15)$$

Where y_c is the distance of the point *c* from coordinate origin. Moreover, K in this area is obtained from the following dimensionless equation:

$$K_1 = 0.006 (Rr \frac{H}{L_2})$$
 (16)

The area 3: this area is placed in inner wall of river so that its beginning is the end of river bed and its end is inner bank of river. β in this area varies linearly from its value in the point *c* from Equation (15) to that in the point *d* from Equation (17).

$$\beta = 0.0266 \left[Si^{1.397} \left(\frac{y_d}{y_d + L_3} \right)^{21.19} R_r^{.004} \right] (17)$$

Where y_d is the distance of the point *d* from coordinate origin. Also, K in this area is obtained from the following dimensionless equation:

$$K_1 = 0.06 \left(Rr \frac{H}{L_3} \right) (18)$$

4- THE ADOPTED DATA

The adopted data were from 2 big and meander branches of Kaskaskia River, one of the most important river of the US [1] and 523 km long. An equation for simulation of velocity transverse distribution in river bend is derived via numerical analysis of transverse velocity data in 30 channels in different bends along the river.

5- RESULTS AND CONCLUSION

The results obtained from application of the proposed numerical model in solving velocity transverse distribution in a few channels of Kaskaskia River were illustrated in Figure 4. In order to show the precision of the proposed model which present favorable results in different angels of river bend, charts of the channels were chosen from different angles of bend in the first branch of the river (Figure 4) and the situation of channels were shown in Figure 5.



Figure 4: comparing real velocity with estimated velocity of the proposed model



Figure 5: position of channels in the river[1].

As shown in the above charts, the recommended model is in a suitable agreement with reality and it can be used for estimation of velocity transverse distribution in river bend.

As it can be seen from the position of the channel 7 which is on the top of river bend, according to predictions, the lowest and highest velocity values are in inner and outer bends, respectively.

Comparison of the results obtained for flow velocity in the main channel by the proposed method and the method proposed by Ervine and et al. (2000) showed that the proposed method in the present study has a higher accuracy [2]. Determination coefficient in this equation was derived to be 94% for all channels which is better than that in the method proposed by Ervine et al. (70%). Considering the importance of flow velocity in main channel in estimations of sediment shift and pollutants spread in flood situation, the proposed mathematical model can be a practical method.

The obtained correlation coefficient for velocity in the model with the data obtained in the field is around 96%.

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