J. Basic. Appl. Sci. Res., **4(5)181-185, 2014**
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ISSN 2090-4304
Journal of Basic and Applied Scientific Research
www.textroad.com

Numerical Solution for Hydromagnatic Fluid Flow between Two Horizontal Plates, Both the Plates Being Stretching Sheets

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Received: March 3, 2014
Accepted: April 20, 2014

ABSTRACT

The problem of viscous fluid flow between two horizontal parallel non-conducting plates, the lower one is a stretching sheet and the upper one is a porous stretching sheet is considered. The effects of flow parameters namely M the magnetic parameter, λ the suction parameter and R the Reynolds number have been observed on velocity profiles. Appropriate ssimilarity functions are employed to transform the equations of motion in to ordinary differential form. The resulting equations are solved by using Successive Over Relaxation(SOR) method and Simpson's (1/3) rule. Richardson extrapolation is used to get higher order accuracy in the results.

KEY WORDS: Hyderomagnetic fluid, Reynolds number, Similarity transformations.

I INTRODUCTION

The fluid flow related to stretching phenomena is important for metal industries and extrusion processes in plastic [1, 2]. This topic has been studied extensively in various situations such as porous medium, MHD flows, heat transfer and Non-Newtonian fluids. Sakiadis [3, 4] examined the boundary layer flow on a continuously stretching surface with a constant speed. An exact solution of two-dimensional Navier-Stokes equation for a stretching plate was found by Crane [5]. Chiam [6] considered steady two dimensional oblique stagnation point flow of a viscous fluid. Moreover, magnetohyderodynamic flow over a stretching surface has been studied by [7-10] with different boundaries. Flow of an electrically conducting non- Newtonian fluid past a stretching surface was studied by Able et al. [11]. Hayat et al [12] analyzed three dimensional flow over a stretching surface in a viscoelastic fluid. Kumaran et al. [13] obtained an exact solution for a boundary layer flow of an electrically conducting fluid past a quadratically stretching and linearly permeable sheet.

This study investigates hyderomagnetic fluid flow between two horizontal plates, both the plates being stretching sheets for extended ranges of the flow parameters. Numerical results have been obtained using a simple but efficient numerical scheme. Solution is readily found for extended values of parameters of the study and for very small number of iterations. Dash and Tripathy [14] considered this problem and solved it by a regular perturbation scheme.

II MATHEMATICAL ANALYSIS

The fluid flow is steady, incompressible and electrically conducting in the presence of a transverse magnetic field. Two equal and opposite forces are introduced to stretch the lower and the upper plates in a way that the position of the points (0, h), (0, -h) remains unchanged. Cartesian coordinate system is used, where, the y-axis is perpendicular to the plates located at y=h, y=-h. The fluid with constant velocity v_0 is injected through the upper porous plate. The external electric field is zero and the electric field due to polarization of charges is negligible. The induced magnetic field is neglected which is valid for small magnetic Reynolds number.

The governing equations of motion are given below:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\upsilon\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)-\frac{\sigma B_0^2}{\rho}u \tag{2}$$

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$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\upsilon\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right) \quad (3)$$

where $V=V(u,v)$ is fluid velocity and υ is kinematic viscosity.

The boundary conditions are:

$u=cx, v=0$ at y=-h ,

$u=cx$, $v=v_0$ at y=h (4)

Using similarity transformations:

$$u=cxf'(\eta), v=-chf(\eta), \eta=\frac{y}{h} \quad (5)$$

$u=cx$ represents the velocity of both the plates and c is a positive constant. Equation (1) is customarily satisfied. The equations (2) and (3) are converted as follows:

$$-\frac{1}{\rho}\frac{\partial p}{\partial x}=c^2x(f'^2-ff''-\frac{1}{R}f'''+\frac{M^2}{R}f') \quad (6)$$

$$-\frac{1}{\rho h}\frac{\partial p}{\partial \eta}=c^2h(ff'+\frac{1}{R}f'') \quad (7)$$

Now, differentiating equations (6)and(7)are respectively differentiated with respect to η and x, to yield,

$$f'''-R(f'^2-ff'')-M^2f'=\beta \quad (9)$$

where β is the constant of integration and $M=\sqrt{\sigma/\rho\nu}B_0h$, $\lambda=\frac{v_0}{ch}$ and $R=\frac{ch^2}{\nu}$. While ρ is the fluid density, σ denotes electrical conductivity, B_0 is strength of transverse magnetic field.

The boundary conditions(4) become:

$$f(-1)=0,\ f'(-1)=1,\ f(1)=\lambda,\ f'(1)=1 \quad (10)$$

III FINITE DIFFERENCE EQUATIONS

let $f'=q$ (11)

Then equation (9) becomes: $q''-R(q^2-fq')-M^2q=\beta$ (12)

The equation (12) is discretized by central difference at some point $\eta=\eta_n$ of the interval [0,∞) to get

$$(2+hRq_n)q_{n+1}-(4+2h^2M^2+2h^2Rq_n)q_n+(2-hRq_n)q_{n-1}-2h^2\beta=0 \quad (13)$$

where h is grid size and $q_n=q(\eta_n)$. For computational purposes, we shall replace the interval [0,∞) by [0, j], where j is a sufficiently large.

The finite difference equation (13) and the first order ordinary differential equations (11) are solved simultaneously by using SOR method, Smith [15, p.262] and Simpson's (1/3) rule, Gerald [16, p.293] with the formula given in Milne [17, p.48] respectively subject to the appropriate boundary conditions.

Richardson's extrapolation method, Burden [18, p.168] is used to improve the results for higher order accuracy $O(h^6)$.

IV RESULTS AND DISCUSSION

The solution of the problem has been made for some values of flow parameters for ranges $1\le M\le 4$, $1\le\lambda\le 3$ and $0.05\le R\le 0.8$. Three different grid sizes have been used to check accuracy of these results. The results are found in good agreement. This numerical scheme is easy to program and table 1 shows that the scheme is also very efficient.

The effects of the flow parameters have been studied on the velocity functions f' and f. It has been noticed that velocity field is almost symmetric about the center of the channel $(\eta=0)$ when both the plates are being stretched at the same rate but it is not the case when only,the lower plate is being stretched. It has been noted that f' increases in the lower half of the channel for increasing R (R<1.0) and decreases, in the upper half of the channel. The extrapolated results for f' are given in tables 2 to 5. It is observed that an increase in the value of *R* increases *f* at all points and transverse velocity increases with increase of η (channel width), when *M* is constant.

The effect of λ on the primary flow f' is maximum at the center of the channel for fixed values of *M*. Also this effect is same, either both the sheets are being stretched or the single sheet is being stretched. Detailed comparison, both tabular and graphical for λ =1, λ =2and λ =3 shows that the suction parameter radically changes the primary flow velocity f'. But the value of f increases with the increase of λ when *M* is constant.

When λ is constant and small (λ =1) the Lorentz force decreases the primary flow velocity f' near the lower plate and increases it near the upper plate. The results have been presented in graphical form in figs.1 to 3.

Table .1: Optimum value of relaxation parameter in SOR method when both the plates are being stretching sheets for three grid points, NI is number of iterations

M	λ	*R*	Number of Iterations in SOR method with ω_{opt}					
			h=0.1		h=0.05		h=0.025	
			NI	ω_{opt}	NI	ω_{opt}	NI	ω_{opt}
1.0	1.0	0.05	31	1.60	68	1.65	220	1.70
3.0	1.0	0.05	30	1.60	38	1.65	67	1.70
3.0	3.0	0.05	75	1.50	79	1.60	92	1.65
3.0	3.0	0.20	75	1.80	79	1.85	140	1.90
2.0	3.0	0.05	37	1.60	162	1.65	394	1.71
4.0	3.0	0.05	33	1.60	38	1.65	48	1.70
4.0	1.0	0.40	33	1.60	35	1.65	49	1.70
2.0	3.0	0.80	39	1.60	47	1.65	82	1.70
4.0	3.0	0.10	33	1.60	38	1.65	54	1.70
4.0	3.0	0.80	43	1.60	52	1.65	59	1.70

Table 2: M=1.0, $\lambda = 1.0$, R=0.05 M=3.0, λ =1.0, R=0.05

Numerical Results using Richardson Extrapolation Method									
	h=0.2	*h*=0.1	*h*=0.05	Extrapolated		*h* =0.2	*h* =0. 1	*h* =0.05	Extrapolated
η	f'	f'	f'	f'	η	f'	f'	f'	f'
0.000	1.000000	1.000000	1.000000	1.000000	0.000	1.000000	1.000000	1.000000	1.000000
0.400	0.512934	0.512044	0.511817	0.511742	0.400	0.485853	0.482648	0.481824	0.481548
0.800	0.289751	0.288500	0.288182	0.288076	0.800	0.342666	0.340097	0.339443	0.339223
1.200	0.291790	0.290538	0.290220	0.290114	1.200	0.343433	0.340849	0.340190	0.339970
1.600	0.517144	0.516253	0.516027	0.515952	1.600	0.487916	0.484681	0.483850	0.483572
2.000	1.000000	1.000000	1.000000	1.000000	2.000	1.000000	1.000000	1.000000	1.000000

Table 3: M=1.0, λ =3.0, R=0.20 M=2.0, λ =1.0,R=0.05

Numerical Results using Richardson Extrapolation Method									
	h=0.2	*h*=0.1	*h*=0.05	Extrapolated		*h*=0.2	*h*=0.1	*h*=0.05	Extrapolated
η	f'	f'	f'	f'	η	f'	f'	f'	f'
0.000	1.000000	1.000000	1.000000	1.000000	0.000	1.000000	1.000000	1.000000	1.000000
0.400	1.789670	1.790962	1.791286	1.791395	0.400	0.767668	0.767608	0.767591	0.767586
0.800	2.030021	2.031768	2.032206	2.032352	0.800	0.585113	0.585109	0.585107	0.585105
1.200	1.811893	1.813531	1.813942	1.814079	1.200	0.416443	0.416513	0.416529	0.416534
1.600	1.150124	1.151166	1.151429	1.151516	1.600	0.231219	0.231311	0.231334	0.231341
2.000	0.000000	0.000000	0.000000	0.000000	2.000	0.000000	0.000000	0.000000	0.000000

Table 4: M=4.0, λ =1.0,R=0.05 M=2.0, λ =1.0,R=0.05

Numerical Results using Richardson Extrapolation Method									
	h=0.2	*h*=0.1	*h*=0.05	Extrapolated		*h*=0.2	*h*=0.1	*h*=0.05	Extrapolated
η	f'	f'	f'	f'	η	f'	f'	f'	f'
0.000	1.000000	1.000000	1.000000	1.000000	0.000	1.000000	1.000000	1.000000	1.000000
0.400	0.473828	0.469650	0.468560	0.468193	0.400	1.527394	1.531557	1.532644	1.533010
0.800	0.368448	0.366258	0.365699	0.365511	0.800	1.632242	1.634416	1.634972	1.635159
1.200	0.368855	0.366644	0.366079	0.365890	1.200	1.631037	1.633260	1.633830	1.634021
1.600	0.475190	0.470967	0.469864	0.469493	1.600	1.523363	1.527618	1.528729	1.529104
2.000	1.000000	1.000000	1.000000	1.000000	2.000	1.000000	1.000000	1.000000	1.000000

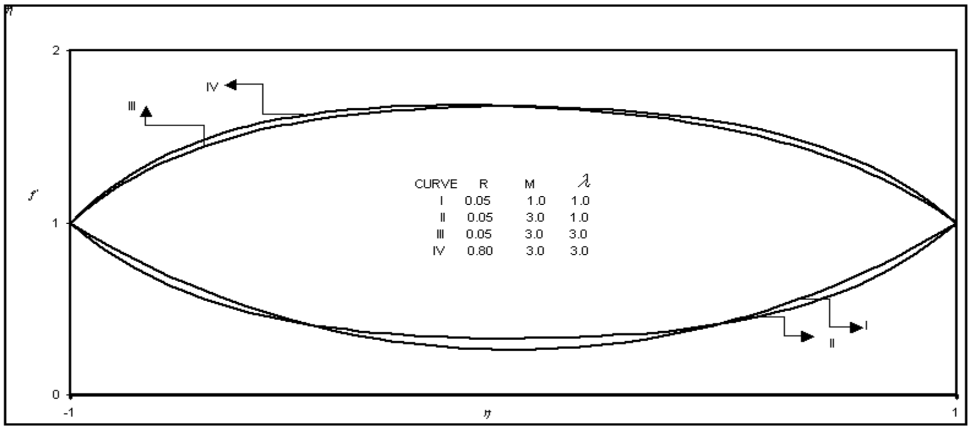

Fig.1: Graph of f' when both the plates are stretching sheets

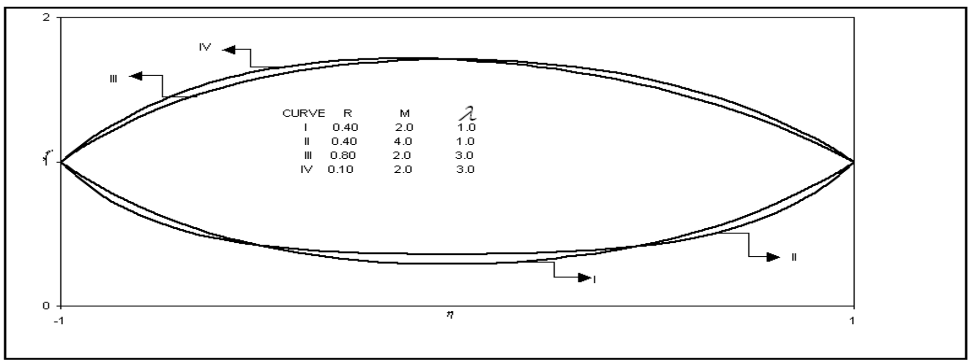

Fig.2: Graph of f' when both the plates are stretching sheets.

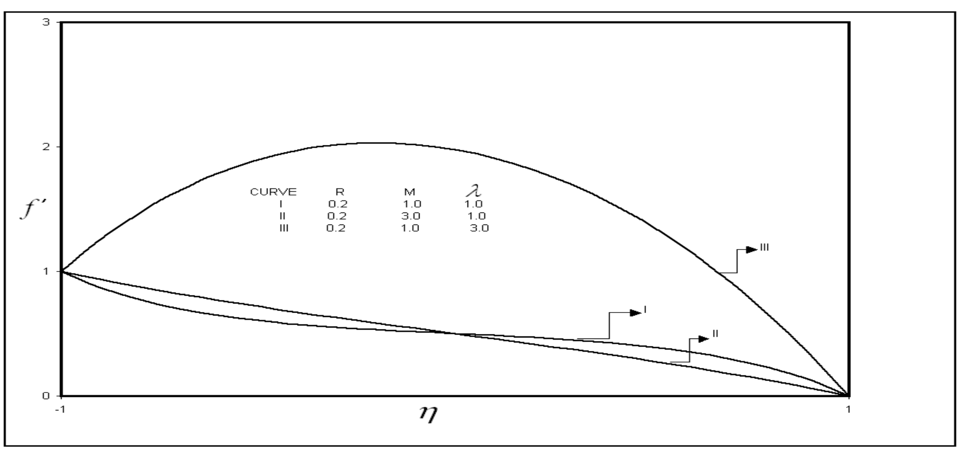

Fig.3: Graph of f' when lower Plate being a stretching sheet.

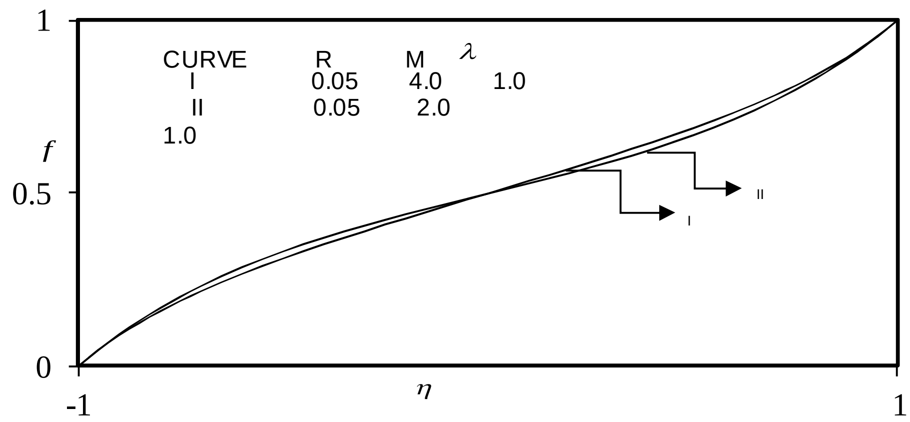

Fig.4: graph of f for different values of M, R and λ

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