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Exact Solution of Two Thin Film Non-Newtonian Immiscible Fluids on a Vertical Belt

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ABSTRACT

This investigation deals with the analytic study on the behavior of thin film non-Newtonian double layer liquid flow on a vertical moving belt. The exact solutions for the velocity fields and the interface velocity distributions are based on fully developed Laminar, Incompressible flow and the power law model of non-Newtonian fluids. The results include the profile of velocity, volume flux, average velocity and Vorticity function. The characteristics of various Model parameters on the velocity fields and skin friction involve in the solution shown through graphs and has been discussed.
KEYWORDS: Thin film flows, Lifting, Drainage, non-Newtonian fluids and Power law model.

1. INTRODUCTION

In recent years, the study of non-Newtonian fluids in Engineering Sciences such as manufacturing process, greases, polymer solution, oils, etc., play an essential role. On the other side, the applications of non-Newtonian fluids are seen in drilling mud's and oil wills, paper production, polymer sheets and different food stuffs like ketchup, sauce and honey. Further, the use of double layer in thenon-Newtonian fluids is of particular interest like double layer coating, double layer paints in various chemical processing. Relevant and interesting work may be found in [1]. One of the widely established models amongst non-Newtonian fluids is class of power law model fluids which has its constitutive equations based on strong theoretical foundations.

Relevant and attractive work may be found in the following published articles. The exact solution of an MHD fluid flow over a stretching sheet with porosity discussed by Ahmad et al [1]. The effect of various physical parameters has been discussed on the velocity profile. Husain and Ahmad et al [2, 3] discussed different numerical techniques for the MHD flow on a stretching sheet in the presence of porosity. They have shown the effect of magnetic field on the velocity as well as on pressure field.

The velocity of two immiscible and incompressible fluids between two parallel plates is discussed by Bird et al in [4]. It was shown there that when the heights of two fluids are equal, then the velocity of the less viscous fluid becomes maximum as compare to the more viscous fluid Kapur and Shukla [5] discussed the maximum velocity of both fluids at different points and their interface.

Later on Kapur and shukla [6] discuss the flow of n-immiscible fluids between two plates of different heights. They have shown that whatever the number of fluids and whatever their heights are, a unique velocity maximum always exists. The flow of two immiscible fluids with uniform suction at the stationary plate was discussed by Sacheti in [7].He notified that the suction is to supply an adverse pressure gradient causing back flow near the stationary plate. Anne Juel et al [8] studied the experimental observations of two liquid layers in reasonable agreement with linear stability analyses. The comparisons between classical non-Newtonian EHL and non-Newtonian TFEHL are discussed by Hsiao-Ming Chu et al [9]. Recently Kim and Kwak [10] studied double layer coating liquid flows. Their approximations are based on Laminar flow and Power Law Model of non-Newtonian fluids. They discussed the coating liquid flow of immiscible resin in model of capillary annuls, where the surface of glass fiber moves at high fiber drawing speed. Dandapat and Singh [11] discussed unsteady two-layer liquid film of uniform thickness on a horizontal rotating disk for small values of Reynolds number. Zeeshan et al [12] studied on the behavior of an incompressible viscoelastic PTT fluid in the double layer coating liquid flow inside a secondary coating die of the optical fiber coating applicator. The work under various configurations on the thin film flows is discussed by S. Mildinova et al in [13]. They have shown in Power Law Model that the non-linear interaction in a falling film of a non-Newtonian liquid exhibits a tendency towards permanent two dimensional waves as in Newtonian liquids. Alam et al. [14] investigated the thin-film flow of Johnson–Segalman fluids for lifting and drainage problems. Taza Gul et al. [15] investigated effects of slip condition on thin film flow of third grade fluids for lifting and drainage problem under the condition of constant viscosity. The effects of various parameters on the lift and drainage velocity profiles are also studied. Some more relevant references [16-20]

The main aim of the present work is to study thin film fluid flows of two immiscible non-Newtonian power law model fluids on a vertical belt.

2. Basic equations:

Consider thin film layers of Power Law Model fluids over a vertical moving belt.
The governing equations are

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$$\nabla . \boldsymbol{u_i} = \boldsymbol{0}, \quad (1)$$

$$\rho_i \frac{D\boldsymbol{u_i}}{Dt} = \nabla . \boldsymbol{T_i} + \rho_i \, \boldsymbol{g}, \; i = 1,2 \quad (2)$$

where $\rho_i, i = 1,2$ are densities of two non-Newtonian fluids, $\mathbf{g}$ is body force per unit mass, $\boldsymbol{u_i}, i = 1,2$ are velocity vectors of the fluids, $\frac{D}{Dt} = \frac{\partial}{\partial t} + (\boldsymbol{u}.\nabla)$ denotes material time derivative and $\boldsymbol{T_i}$ are the shear stress.

3. Power Law Model:

A Power-Law fluid, is the simplest type of Newtonian and non-Newtonian fluids for which the shear stress τ_{xy} is given by

$$\tau_{xy} = k \left|\frac{d\boldsymbol{u}}{dx}\right|^{n-1} \frac{d\boldsymbol{u}}{dx}. \quad (3)$$

Here k is known as flow consistency index, $\frac{du}{dx}$ is the shear rate and n, is the flow behavior index.

$$\eta = k \left|\frac{d\boldsymbol{u}}{dx}\right|^{n-1}, \quad (4)$$

η, is known as apparent viscosity. η, is also known as the Ostwald–de Waele power law, this mathematical connection is constructive because of its simplicity, but only approximately describes the form of a real generalized Newtonian fluid. For example, If $n < 1$, the power law predicts that with increasing shear rate would decrease the apparent viscosity η. The viscosity of fluid become zero as the shear rate approaches infinity and become infinity at rest, but a real fluid has both a minimum and a maximum apparent viscosity that depend on the concentration and molecular weight of the polymer, etc. If $n = 1$ it will show the Newtonian behavior and the consistency is just the viscosity. However, if $n > 1$, the viscosity increases with the amount of shearing, and the fluid has a shear-thickening (also known as dilatant) behavior. Therefore, the Power Law is only a good description of fluid behavior across the range of shear rates to which the coefficients were built-in. In these above equations, k and n are two testing parameters.

4. Formulation of the lift problem:

Consider two immiscible thin films and incompressible fluid layers; we assume that a wide flat belt moves vertically upward at constant speed U through a large bath of Power law model liquid. The belt carries with itself a layer of liquid of constant thickness δ_1 taking as first thin liquid layer. The initial total film thickness is δ and thickness of the second immiscible layer is $\delta - \delta_1$ on the free surface of the belt discussed in [11]. For analysis Cartesian coordinate has been used in which the y-axis is taken parallel to the belt and x-axis perpendicular to the belt.

Assuming the fluid layers are steady, laminar such that these layers satisfy the constitutive equation of Power Law Model and the thicknesses $\delta_1, \delta - \delta_1$ of both thin-films are uniform. The external pressure is atmospheric everywhere.

Velocity fields for both fluid layers:

$$\boldsymbol{U} = (0, u_i(x), 0) \, , i = 1,2 \quad (5)$$

The subscripts $i = 1,2$ represent first thin film liquid on the belt and second thin film liquid on the interface mentioned in geometry.

Boundary conditions are:

$$u_1 = U \quad \text{at } x = 0 \quad (6)$$

$$\frac{du_2}{dx} = 0, \quad \text{at } x = \delta \quad (7)$$

$$u_1 = u_2 \text{ at } x = h, \quad (8)$$

$$k_1 \left(\frac{du_1}{dx}\right)^n = k_2 \left(\frac{du_2}{dx}\right)^n, \quad \text{at } x = h, \quad (9)$$

Here h shows interface between thin films, U is the velocity of belt and δ is the total thickness of films which is uniform. $\frac{du_1}{dx}$ and $\frac{du_2}{dx}$ are the shear rates of both thin films. n is the flow behavior index and k_1, k_2 are known as flow consistency indexes of Power Law Model.

Inserting the velocity field given from equation (5) in equations (1) and (2), the continuity equation (1) satisfies identically and equation (2) reduces to:

$$\frac{d}{dx}\left[k_i \left(\frac{du_i}{dx}\right)^n\right] = \left(\frac{\partial p}{\partial y} - \rho_i g\right), i = 1,2. \quad (10)$$

suppose constant pressure gradient $\frac{\partial p}{\partial y} = \lambda$.

Using boundary conditions (6-9) the shear stresses of both fluid layers are:

$$k_1 \left(\frac{du_1}{dx}\right)^n = (\lambda - \rho_1 \, g)(x - h) + (\lambda - \rho_2 \, g)(h - \delta), \quad (11)$$

$$k_2 \left(\frac{du_2}{dx}\right)^n = (\lambda - \rho_2 \, g)(x - \delta), \quad (12)$$

Integrating Eq. (10) twice and using boundary conditions from Eqs. (6-9), the velocity profiles of both fluid layers are :

$$u_1 = U + \frac{n\,k_1^{-\left(\frac{1}{n}\right)}}{(\rho_1 g-\lambda)(1+n)}\left[[(\rho_2 g-\lambda)(\delta-h)+h(\rho_1 g-\lambda)]^{\frac{1+n}{n}} - [(\rho_2 g-\lambda)(\delta-h)-(\rho_1 g-\lambda)(x-h)]^{\frac{1+n}{n}}\right], \quad (13)$$

$$u_2 = U + \frac{nk_2^{-\left(\frac{1}{n}\right)}}{(\rho_2 g-\lambda)(1+n)}\left[[(\rho_2 g-\lambda)(\delta-h)]^{\frac{1+n}{n}} - [(\rho_2 g-\lambda)(\delta-x)]^{\frac{1+n}{n}}\right] + \frac{nk_1^{-\left(\frac{1}{n}\right)}}{(\rho_1 g-\lambda)(1+n)}\left[[(\rho_2 g-\lambda)(\delta-h)+h(\rho_1 g-\lambda)]^{\frac{1+n}{n}} - [(\rho_2 g-\lambda)(\delta-h)]^{\frac{1+n}{n}}\right], \quad (14)$$

u_1 is the velocity of fluid layer on moving belt and u_2 is the velocity of fluid layer on interface.

4.1.1 Shear stress at the interface:

Inserting $x = 0$ in Eq.(12), the shear stress of second layer become:

$$\tau_{2_{xy}}\Big|_{x=0} = \delta(\lambda - \rho_2\, g), \quad (15)$$

4.1.2 Flow rate and average velocity of first layer:

The flow rate per unit width is given by the formula:

$$Q = \int_0^\delta u_1(x)dx \quad (16)$$

Inserting Eq. (13) in Eq. (16) and integrating we obtain.

$$Q = U\delta + \frac{n\delta k_1^{-1/n}(-\delta\lambda+gh\rho_1+g(-h+\delta)\rho_2)^{1+\frac{1}{n}}}{(1+n)(-\lambda+g\rho_1)} + \frac{n^2k_1^{-1/n}(g(h-\delta)\rho_1+g(-h+\delta)\rho_2)^{2+\frac{1}{n}}}{(1+n)(1+2n)(\lambda-g\rho_1)^2}, \quad (17)$$

The average velocity $\overline{U}$ is given by:

$$\overline{U} = \frac{Q}{h} = \frac{U\delta}{h} + \frac{n\delta k_1^{-1/n}(-\delta\lambda+gh\rho_1+g(-h+\delta)\rho_2)^{1+\frac{1}{n}}}{h(1+n)(-\lambda+g\rho_1)} + \frac{n^2k_1^{-1/n}(g(h-\delta)\rho_1+g(-h+\delta)\rho_2)^{2+\frac{1}{n}}}{h(1+n)(1+2n)(\lambda-g\rho_1)^2}, \quad (18)$$

4.1.3 Vorticity of the second fluid layer:

The vorticity function $\vec{\omega}$ (z-component of the vorticity vector) of the flow is given by:

$$\vec{\omega} = \nabla \times U = \frac{du_2}{dx}, \quad (19)$$

$$\vec{\omega} = k_2^{-\frac{1}{n}}(\lambda - g\rho_2)^{\frac{1}{n}}(x-\delta)^{\frac{1}{n}}.$$

5. Drainage problem for non-Newtonian Power Law Model:

For drainage, we consider two immiscible thin layers of non-Newtonians Power Law Model fluids. First fluid layer falls on the stationary vertical belt and second fluid layer falls on the free surface of the belt. The gravity causes the fluid motion. We assume that the flows are incompressible, uniform, steady and laminar. The thicknesses of both layers are same as in previous problem. The fluid moves downward due to gravity with standard atmospheric pressure.

The governing equation from (2) and the boundary conditions for drainage problem are:

$$\frac{d}{dx}k_i\left(\frac{du_i}{dx}\right)^n = \left(\frac{\partial p}{\partial y} + \rho_i g\right), i = 1,2. \quad (20)$$

$$u_1 = 0 \quad \text{at} x = 0 \quad (21)$$

$$\frac{du_2}{dx} = 0, \quad \text{at} x = \delta \quad (22)$$

$$u_1 = u_2 \text{at} x = h, \quad (23)$$

$$k_1\left(\frac{du_1}{dx}\right)^n = k_2\left(\frac{du_2}{dx}\right)^n, \quad \text{at} x = h, \quad (24)$$

Using boundary conditions (21-24) in the governing Eq. (20),the velocity profiles of both fluid layers are:

$$u_1 = \frac{-n\,k_1^{-\left(\frac{1}{n}\right)}}{(\rho_1 g+\lambda)(1+n)}\left[[(\rho_2 g+\lambda)(h-\delta)-h(\rho_1 g+\lambda)]^{\frac{1+n}{n}} - [(\rho_2 g+\lambda)(h-\delta)-(\rho_1 g+\lambda)(h-x)]^{\frac{1+n}{n}}\right], \quad (25)$$

$$u_2 = \frac{-nk_2^{-\left(\frac{1}{n}\right)}}{(\rho_2 g+\lambda)(1+n)}\left[[(\rho_2 g+\lambda)(h-\delta)]^{\frac{1+n}{n}} - [(\rho_2 g+\lambda)(x-\delta)]^{\frac{1+n}{n}}\right] - \frac{nk_1^{-\left(\frac{1}{n}\right)}}{(\rho_1 g+\lambda)(1+n)}\left[[(\rho_2 g+\lambda)(h-\delta)-h(\rho_1 g+\lambda)]^{\frac{1+n}{n}} - [(\rho_2 g+\lambda)(h-\delta)]^{\frac{1+n}{n}}\right], \quad (26)$$

u_1 is the velocity of fluid on the stationary belt and u_2 is the velocity of fluid on interface.

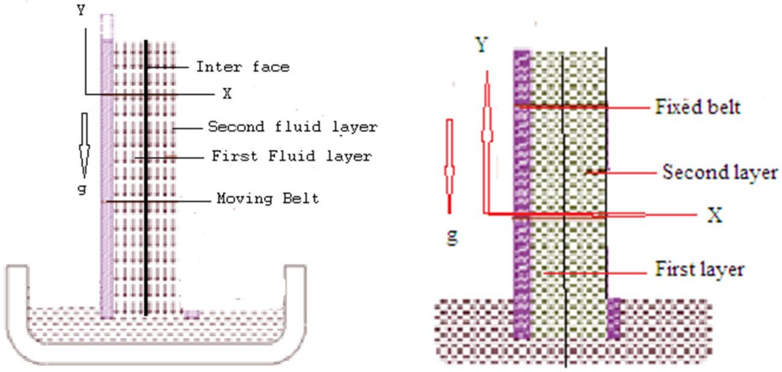


Fig.1&2. Geometry of Lifting problem when the belt is moving up (on the left) and geometry of drainage problem when the belt is stationary (on the right).

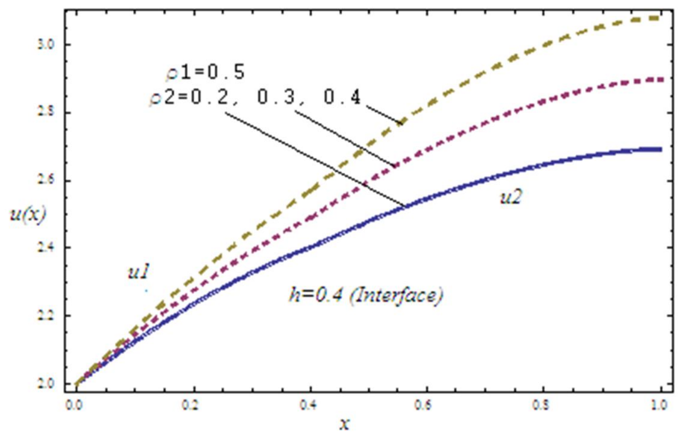
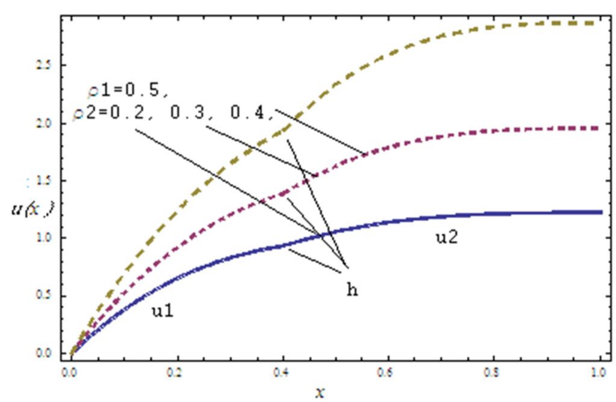


Fig.3: The effect of the second fluid layer density ρ_2. When $\delta = 1, n = 1.5, g = 4, \lambda = 0.2, U = 2, k_2 = 0.5, k_1 = 0.7$.



□=1;n=0.5;g=4;□=0.2;U=2;k_2=0.5;k_1=0.7;h=.4;

Fig.4: The effect of ρ_2, in drainage problem. When $\delta = 1, n = 0.5, g = 4, \lambda = 0.2, U = 2, k_2 = 0.5, k_1 = 0.7, h = 0.4$,

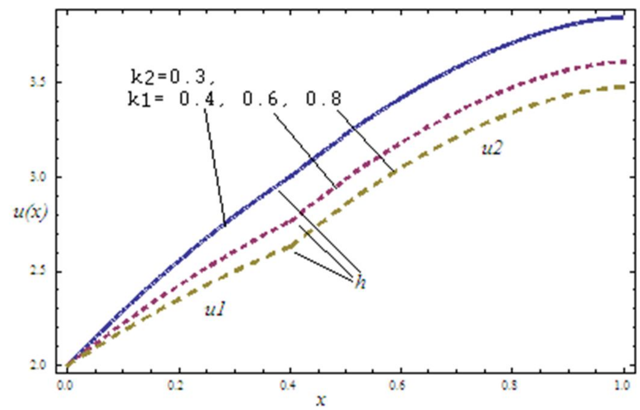


Fig.5: Variation in viscosity parametersk_1, for shear thickening fluids in lift problem. When $\delta = 1, n = 1.5, g = 4, \lambda = 0.2, U = 2, h = 0.4, \rho_1 = 0.7, \rho_2 = 0.5$.

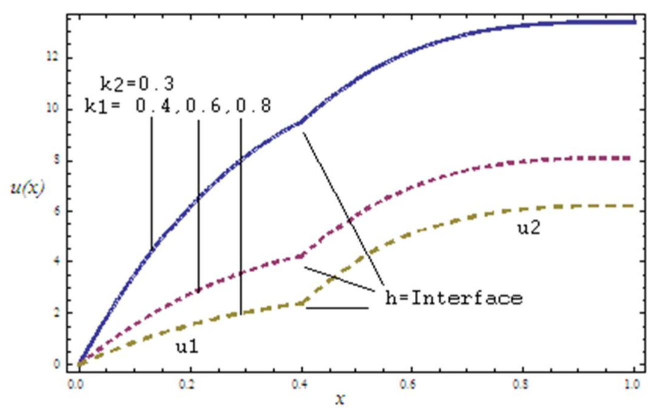


Fig.6: Variation in viscosity parameter k_1, for shear thinning fluids in drainage problem. When $\delta = 1, n = 0.5, g = 4, \lambda = 0.2, U = 2, h = 0.4, \rho_1 = 0.7, \rho_2 = 0.5$.

6. RESULTS AND DISCUSSION

The effect of Densitiesρ_1, ρ_2 and viscosities $k_1, k_2$of the fluid layersu_1, on the belt and u_2, on the inter face h, is shown for both lift and drainage problems in Figs. 3-6.Figs 1 and 2 show the geometry of lift and drainage velocity profiles. When the apparent viscosity decreases with the stress rate then the fluid is known as shear-thinning fluid. This is also known as pseudo plastic. When the viscosity is independent of shear rate, the apparent viscosity approaches to Newtonian plateau in polymeric systems (melts and solutions) at low shear rate.

The effect of the densitiesρ_1, ρ_2, for lift velocity profile has been shown in Fig 3 and the same effect for drainage velocity profile has been shown in Fig 4. In lift and drainage velocity profiles increase in densities of fluid layers increases velocity profiles of both layers gradually. In both cases, the velocities are very smooth and continuous. The velocity of fluid layer $u_2$increases rapidly if$\rho_2 > \rho_1$and increases slowly if $\rho_2 < \rho_1$. This effect is generally encountered in suspension, blood, ice and latex paint etc. This class is similar to shear thinning systems. The only difference is that the increase in shear rate increases the apparent viscosity. Common examples of such fluids are Corn flour in water and thick suspension. The effect of the viscosities of both fluid layers u_1 and u_2 for lift and drainage velocity profiles are shown in Figs 5and 6. When $k_2 < k_1$, Increase in viscosity k_2 decreases velocity of both fluid layers and it happens in lift as well as in drainage velocity profiles.

7. Conclusion

The constitutive equation governing the flow of a power law model for lifting and drainage of fluid layers are solved exactly. It is concluded that velocity decreases as the density of fluid layers increases in lifting as well as in drainage. For small values of $n = 1$, the velocity profile tend to Newtonian one, however when $n < 1$, these profiles become more flattened showing the shear-thinning effect. But when $n > 1$, then it can be seen that the speed of boundary layers are relatively small to that of shear thinning and Newtonian fluids. Increase in viscosity of first fluid layer increases the velocity of lift slowly and drainage velocity profile rapidly. Reason is that gravity cause the fluid motion.

According to the best of our knowledge there is no previous literature about discussed problem, this is our first attempt to handle this problem.

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