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On Magnatohydrodynamic Viscous Flow over a Stretching Sheet with Prescribed Heat Flux

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ABSTRACT

Newtonian fluids flow due to a permeable stretching sheet with prescribed surface heat flux is studied. A uniform magnetic field is applied transverse to the sheet. The nonlinear partial differential equations are changed in to their ordinary differential form by using similarity transformations. Method of Stretching Variables is employed to obtain the numerical solution of the resulting equations of motion. The velocity, temperature profiles and skin coefficient are discussed for various parameters namely Grash of number G_r , Prandtel number P_r , Hartmann number M and Suction parameter S .

KEYWORDS: Heat Transfer, Magnatohydrodynamic (Mhd), Viscous Flow, Stretching Sheet, Heat Flux

I INTRODUCTION

Boundary-layer behavior of magnatohydrodynamic viscous fluids flow due to a permeable stretching sheet with prescribed surface heat flux is important, for example, materials which are manufactured by extrusion processes and heat treated materials. The problem of boundary layer flow and heat transfer in a liquid film on an unsteady stretching surface was discussed by Andersson et al. [52]. Bidin et al. [51] considered the numerical solution of the boundary layer flow over an exponentially stretching sheet with thermal radiation. Sakiadis [55] was premier to study the boundary layer flow over a stretching sheet which was extended by Crane [57] for two dimensional phenomenon. Magyari et al. [63] considered fluids flow over a stretching sheet with suction or injection. Cheng [64] studied the problem of mixed convection along a vertical surface in the presence of a uniform transverse magnetic field in a porous medium. Ramachandran et al. [66] investigated two-dimensional stagnation flows adjacent to a vertical heated surface with both the prescribed wall temperature and prescribed wall heat flux. Using the homotopy analysis method (HAM), Anura [58] studied thermal boundary layer flow over a stretching sheet in a micropolar fluid with radiation effect. Heat transfer of a continuous stretching surface with suction or blowing was investigated by Chen et al. [56]. Dutta et al. [53] analyzed the temperature field in the flow over a stretching sheet with uniform heat flux. The effect of thermal radiation and magnetic field on unsteady mixed convection flow and heat transfer over a porous stretching surface was discussed by Elbashbeshy et al.[61]. The effects of variable thermal conductivity and heat source /sink on MHD flow near a stagnation point on a linearly stretching was investigated by Sharma et al. [62]. Tsou et al. [60] studied the flow and heat transfer in the boundary layer on a continuous moving surface. Turkyilmazoglu et al. [59] established the exact analytical solutions for heat and mass transfer of MHD slip flow in Nano fluids. Sajid et al. [65] extended the above problem for MHD viscous flow due to shrinking sheet. Ali et al. [54] analyzed MHD viscous fluids flow and heat transfer due to permeable shrinking sheet.

II THE GOVERNING EQUATION OF FLUID MOTION

Consider the steady MHD viscous, incompressible, electrically conducting fluid flow over a stretching sheet with prescribed heat flux which coincides with plane $y = 0$ and the flow coffined $y > 0$, in the presence of uniform magnetic field perpendicular to x-axis. For stretching the sheet, two equal and opposite forces are applied along x-axis. Keeping the origin fixed in the fluid of ambient temperature.

The momentum equation and energy equation with associated boundary conditions are

$$\partial u/\partial x + \partial v/\partial y + \partial w/\partial z = 0. \tag{1}$$

$$\rho(u\partial u/\partial x + v\partial u/\partial y + w\partial u/\partial z) = -\frac{\partial P}{\partial x} + \mu(\partial^2 u/\partial x^2 + \partial^2 u/\partial y^2 + \partial^2 u/\partial z^2) + g\rho\beta(\partial T/\partial z) - \sigma B_0^2 u, \tag{2}$$

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$$u\,(\partial T/\partial x)+v\,(\partial T/\partial y)+w(\partial T/\partial z)=\alpha\,(\partial^2 T/\partial x^2+\partial^2 T/\partial y^2+\partial^2 T/\partial z^2), \tag{3}$$

where $\underline{V}=V\ \ (u,v,w)$ is the velocity, B_0 is strength of magnetic field, P is the pressure independent of x, g is acceleration due to gravity and β is the volumetric coefficient of thermal expansion, ρ is fluid density, μ is the coefficient of viscosity, σ is electrical charge density, T is the temperature inside the boundary layer and α is the thermal diffusivity.

The associated boundary conditions are:

$$u=cx,\ v=c(m-1)y,\ w=-\text{ and }\partial T/\partial z=-\frac{q_0}{K}\ \text{ as } z=0, \tag{4}$$

$$u\to 0,\ T\to T_\infty,\quad \text{as } z\to\infty, \tag{5}$$

where c is positive constant for stretching sheet, v_0 the suction velocity, q_0 is prescribed heat flux. K is thermal conductivity.

Using similarity transformations

$$u=cxf'(\eta),\ v=c(m-1)f'(\eta),\ w=-\sqrt{c\upsilon}mf(\eta) \tag{6}$$

$$\theta(\eta)=K(T-T_\infty)/q_0\sqrt{\frac{c}{\upsilon}},\quad \eta=\sqrt{\frac{c}{\upsilon}}z \tag{7}$$

The equations (2) and (3) are respectively transformed to ODE's :

$$f'''-(f')^2+mff''+G_r\theta'-M^2f'=0 \tag{8}$$

$$\theta''+mP_r f\,\theta'=0 \tag{9}$$

$G_r=\frac{g\beta q_w}{Kc^2x}$, $\upsilon=\frac{\mu}{\rho}$ the kinematic viscosity, $M^2=\frac{\sigma B_0^2}{\rho c}$, $P_r=\frac{\upsilon}{\alpha}$. When sheet stretches in x-direction , $m=1$ and for sheet stretchers axisymmetrically , $m=2$.

The boundary conditions in equation (4) become:

$$f'(0)=1,\ f(0)=v_0/m\sqrt{c\upsilon}=S\ (say), \tag{10}$$

$$f'(\eta)=0 \text{ as } \eta\to\infty. \tag{11}$$

$$\theta'(0)=-1 \text{ as } \eta\to 0 \text{ and } \theta(\eta)=0 \text{ as } \eta\to\infty. \tag{12}$$

where S is mass suction parameter.

III NUMERICAL SOLUTION BY USING METHOD OF STRETCHING VARIABLES

For the numerical solution of MHD flow over a stretching sheet with mass suction, the governing equation (8) and (9) along with boundary conditions (10) to (12), the method of stretching variables is harnessed with ζ , F and Ψ as follows:

$$\zeta=\gamma\eta \text{ and } F=\gamma f,\ \Psi=\gamma\theta(\eta) \tag{13}$$

where $\gamma>0$, ξ is an amplification factor, $S=f(\eta)$ at $\eta\to 0$.

By using relation (13), the equation (8) becomes:

$$\gamma^2(d^3F/d\zeta^3)-(dF/d\zeta)^2+mF\times(d^2F/d\zeta^2)+G_r\frac{\Psi'}{\gamma}-M^2(dF/d\zeta)=0. \tag{14}$$

The relations (10), (11) and (13) yield,

$$F=\gamma S,\ (dF/d\zeta)=1 \text{ and } (dF/d\zeta)=0,\ \text{when } \zeta\to\infty \tag{15}$$

$$\Psi'=-\gamma \text{ at } \zeta=0 \text{ and } \Psi=0,\ \text{when } \zeta\to\infty. \tag{16}$$

We assume the functions,

$$F=(\gamma S+2/3)-(2/3)(e^{-\frac{3}{2}\zeta}), \tag{17}$$

and

$$\Psi' = -\gamma e^{-\frac{3}{2}\zeta}, \tag{18}$$

which satisfy equations (15) and (16).

The equation (17) and equation (14) together yield:

$$\begin{aligned}&(9/4)\gamma^2 e^{-\frac{3}{2}\zeta} - (e^{-\frac{3}{2}\zeta})^2 - m((3/2)e^{-\frac{3}{2}\zeta})((\gamma s + 2/3)\\ &-(2/3)e^{-\frac{3}{2}\zeta} - G_r e^{-\frac{3}{2}\zeta} - M^2(e^{-\frac{3}{2}\zeta}) = 0.\end{aligned} \tag{19}$$

Thus the residual function $R(\zeta, \gamma)$, (defect function) is obtained:

$$R(\zeta,\gamma) = [(9/4)\gamma^2 - m(3/2)((\gamma S + 2/3) - G_r - M^2]e^{-\frac{3}{2}\zeta} + (m-1)e^{-3\zeta}, \tag{20}$$

$$R(\zeta,\gamma) = K\, e^{-\frac{3}{2}\zeta} + Qe^{-3\zeta}, \tag{21}$$

where

$$K = (9/4)\gamma^2 - m(3/2)((\gamma S + 2/3) - G_r - M^2 \text{ and } Q = m - 1 \tag{22}$$

$R(\zeta,\gamma)$ is defect function which is minimized by using Least Square method,

$$\frac{\partial}{\partial\gamma}\left(\int_0^\infty R^2 d\zeta\right) = 0,$$

it implies that $K + \frac{2}{3}Q = 0$ and hence

$$(9/4)\gamma^2 - m(3/2)(\gamma S + 2/3) - G_r - M^2 + (2/3)(m-1) = 0, \tag{23}$$

$$\gamma = (mS \pm \sqrt{(mS)^2 + 4(\frac{2}{3} + G_r + M^2 + \frac{m}{3}})/3. \tag{24}$$

The valid relation of ζ, M^2, S and G_r is

$$\gamma = (mS + \sqrt{(mS)^2 + 4(\frac{2}{3} + G_r + M^2 + \frac{m}{3}})/3 \tag{25}$$

Using (9), (18) and (25), we obtained

$$\gamma = \frac{1}{2}\left[\frac{4mP_r}{9(3 - 2mP_r S)} + \frac{mS + \sqrt{(mS)^2 + 4(\frac{m+2}{3} + G_r + M^2)}}{3}\right] \tag{26}$$

IV RESULTS AND DISCUSSION

The equations (8) and (9) are solved numerically. The results have been obtained and presented in tabular and graphical forms. Table 1 and table. 2 show the effect of G_r on skin friction coefficient $-f''(0)$. It is seen that $-f''(0)$ increases with increasing the values of G_r for both the cases : $m=1$, $m=2$. In tables 3, 4 it is observed that $-f''(0)$ increases with increasing values of P_r in both cases. The effect of magnetic parameter M^2 on $-f''(0)$ is shown in table 5 and table 6. The magnitude of skin friction coefficient $-f''(0)$ also increases with increase in the values of M^2. The effects of suction parameter S on velocity profiles is shown in figure 1, when the value of S increases, the velocity decreases. The effect of parameter G_r on velocity can be observed from the figure 2, when $m=1$ and $m=2$. It is shown that velocity decreases with increase in the values of G_r. Fig. 3 indicates that by increasing the values of M^2, the velocity profile decreases when $m=1$ and $m=2$. The temperature increases with increasing values of S when m=1 but decreases when

$m = 2$ as depicted in Fig. 4. The temperature decrease with the increase of G_r as shown in Fig.5, for both the cases. The effect of P_r has been observed in Fig.6. The effect of M^2 on temperature profile is presented in Fig.7.

V CONCLUSION

A mathematical analysis has been carried out on momentum and heat transfer characteristics for an incompressible viscous MHD fluid flow over a stretching sheet with prescribed heat flux in the presence of magnetic field. The numerical results have been obtained using a very easy method. It is observed that by increasing the flow parameters namely magnetic field strength M^2, the suction parameter S or Grashof number G_r the velocity decreases. The temperature profiles increases with increasing values of S when m=1 but decreases when $m = 2$. The temperature profiles decrease with the increase of G_r.

Table 1: $-f''(0)$ and G_r when $P_r = 1,\ m = 1,\ S = 1,\ M^2 = 0.5$

Gr	0.0	0.5	1.0	1.5
$-f''(0)$	1.2447	1.3333	1.4124	1.4847

Table 2: $-f''(0)$ and G_r when $P_r = 1,\ m = 2,\ S = 1,\ M^2 = 0.5$

Gr	0.0	0.5	1.0	1.5
$-f''(0)$	0.6749	0.7462	0.8122	0.8741

Table 3: $-f''(0)$ and P_r when $G_r = 0.5,\ m = 1,\ S = 1,\ M^2 = 0.5$

P_r	0.1	0.3	0.5	0.7
$-f''(0)$	1.0119	1.0416	1.0833	1.1458

Table 4: $-f''(0)$ and P_r when $G_r = 0.5,\ m = 2,\ S = 1,\ M^2 = 0.5$

P_r	0.1	0.3	0.5	0.7
$-f''(0)$	1.4384	1.5239	1.7461	3.7460

Table 5: $-f''(0)$ and M^2 when $G_r = 0.5,\ P_r = 1,\ m = 1,\ S = 1.$

M^2	1.0	1.5	2.0
$-f''(0)$	1.4124	1.4847	1.5515

Table 6: $-f''(0)$ and M^2 when $G_r = 0.5,\ P_r = 1,\ m = 2,\ S = 1.$

M^2	1.0	1.5	2.0
$-f''(0)$	0.8122	0.8741	0.9325

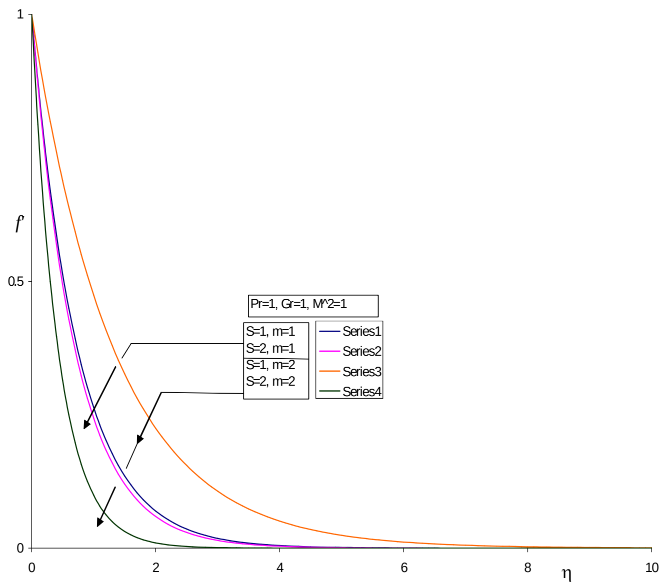

Fig: 1. Velocity profiles of f' under the effect of S.

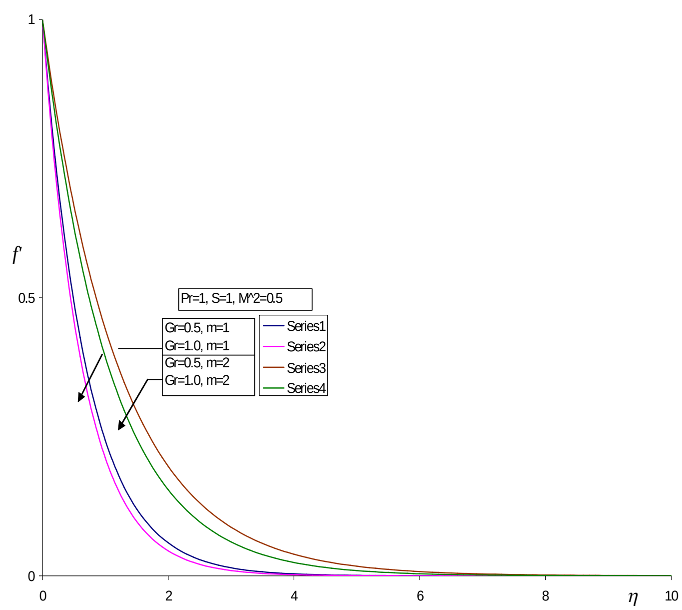

Fig: 2. Velocity profiles of f' under the effect of G_r.

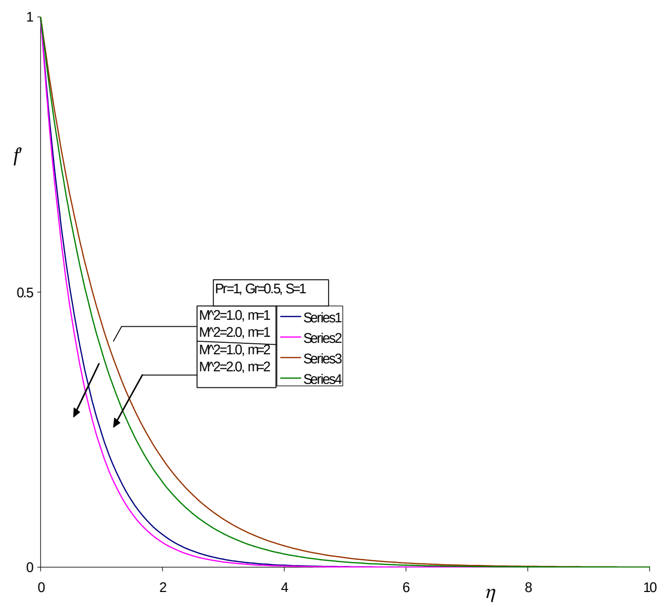

Fig: 3. Velocity profiles of f' under the effect of M^2

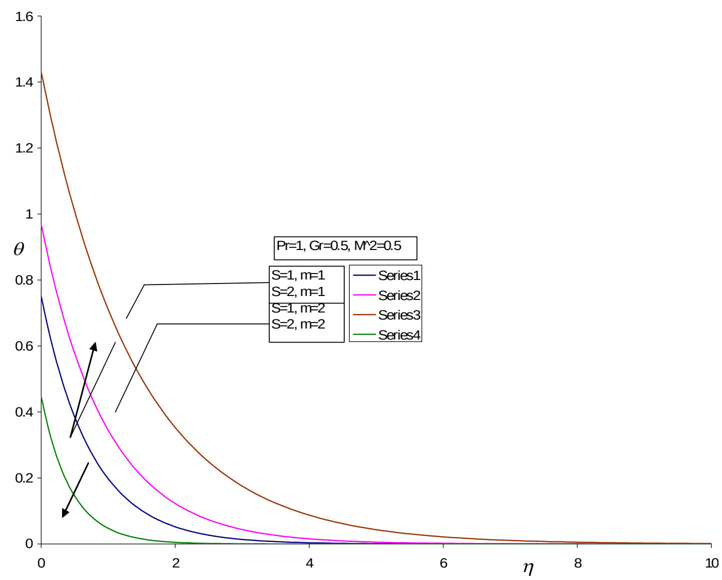

Fig: 4. Temperature profiles of θ under the effect of S.

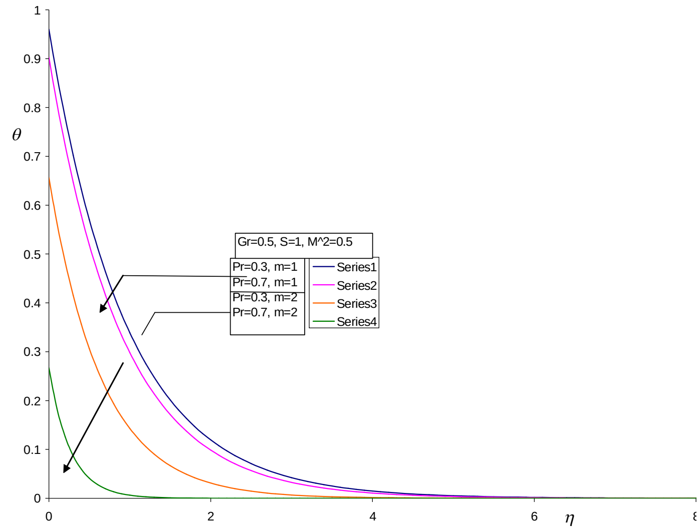

Fig: 5. Temperature profiles of θ under the effect of G_r

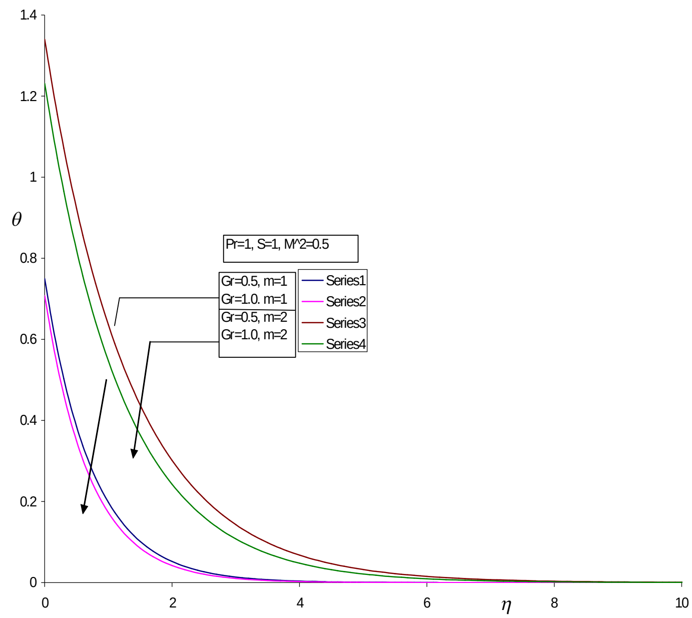

Fig: 6. Temperature profiles of θ under the effect of P_r

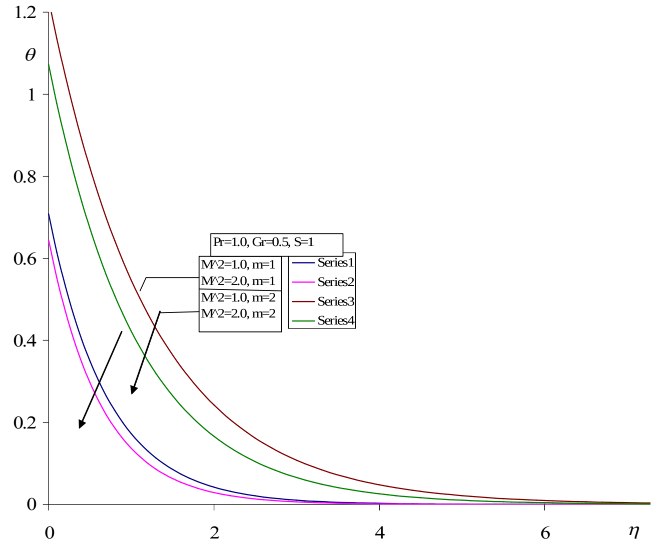

Fig: 7. Temperature profiles of θ under the effect of M^2.

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