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Unsteady Thin Film Third Grade Fluid on a Vertical Oscillating Belt using Adomian Decomposition Method

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ABSTRACT

This work is an analytical study of unsteady thin film third grade fluid. The governing non linear partial differential equation has been solved analytically by Adomian Decomposition Method (ADM). The effect of model parameters on velocity profile have been plotted and discussed numerically as well as graphically.
KEYWORDS: unsteady thin film fluid flow, lifting, drainage, Third grade, ADM.

I INTRODUCTION

Thin film fluid flows have large applications in engineering and industries. Like wire coating and fiber optics. The studies of Newtonian and non-Newtonian fluid films have been investigated by various researchers. The purpose of the present is to solve the non-linear differential equation of thin film flow on vertical moving and oscillating belt. In the best of author knowledge no attempt has been made to investigate thin film of third grade fluid on vertical moving and oscillating belt. Siddiqui et al. [1] investigated Sisko and Oldroyd 6- constant fluids on a vertical moving belt. They discussed the velocity field as well as volume flux and average velocity for different physical parameters. Mahmood et al. [2] discussed thin film of third grade fluid on an inclined plane the governing nonlinear equation for velocity are solved by perturbation technique and homotopy perturbation Method. Alam et al. [3- 4] investigated Johnson-Segalman thin film fluids on a vertical moving belt using Adomian Decomposition method (ADM). Shah et al. [5-6] discussed the exact solution of wire coating unsteady second grade fluid in a canonical die. They analyzed the motion of the fluid for small and large time levels. They have also discussed the effect of model parameters involved in the velocity profile. Aiyesimi et al. [7] investigated unsteady thin film flow of an electrically conducting third grade fluid down an inclined plane. They analyzed the effect of model parameters on velocity profile and temperature distributions. Ali et al. [8] investigated unsteady second grade fluid on oscillating vertical plate. They have been shown numerical results of skin friction and Nusselt number. Munson and Young [9] also investigated the thin-film flow of Newtonian fluids. Gul et al. [10] investigated the influence of a magnetic field on a vertical belt on the flow of an incompressible third grade electrically conducting fluid with slip boundary conditions. He studied the effect of various physical parameters.

Ming Chu et al. [11] discussed lubricating thin film between two solid surfaces as three fixed layer. They discussed the non- Newtonian thin film elasto hydrodynamic lubrication (TFEHL) and the classical non-Newtonian elasto hydrodynamic lubrication (EHL).The effect of viscous dissipation and the temperature dependent thermal conductivity of thin film liquid of a non-Newtonian Ostwald-de Waele fluid over a porous stretching and horizontal surface is investigated by Chiu et al. [12]. Asgar et al. [13] studied rotating fluid flow of third grade past a porous plate. They analyzed the effect of partial slip on the rotation of flow. Farooq et al. [14] discussed the exact solution of MHD flow over the porous sheet. The governing non linear partial differential equations are solved by using undetermined coefficient method and study the effect of different parameter on velocity profile and pressure. Ali et al. [15] studied the numerical solution of incompressible, viscous fluid flow and heat transfer over porous sheet. Sajid et al. [16] investigated thin film flow of fourth grade fluid on a vertical cylinder using Homotopy Analysis Method (HAM) to obtain the solution for the velocity profile and compare it with the exact solution. Ghanbarpour et al. [17] studied thin film flow of Sisko and Oldroyd 6 constant fluid on vertical moving belt and the governing nonlinear differential equations are solved by using (HAM). The related work can also be seen in [18-21]. Husain and Ahmad et al [22-23] discussed different numerical techniques for the MHD flow on a stretching sheet in the presence of porosity. They show the effects of various parameters, Hartmann number, mass suction parameter, Prandtl number Pr and sink parameter on the velocity and temperature profiles. In 1992, Adomian [24-25] introduced the ADM for the approximate solutions for linear and non linear problems. Wazwaz [26-27] used ADM for the reliable treatment of Bratu-type and Emden-Fowler equations.

II BASIC EQUATIONS

For incompressible third grade fluid the governing equations are:

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Continuity Equation

$$\nabla . \boldsymbol{V} = 0 \tag{1}$$

Momentum Equation

$$\rho \frac{DV}{Dt} = \nabla . \boldsymbol{T} + \rho g \tag{2}$$

Where $\boldsymbol{V}$ is the velocity of the fluid, ρ is the density, g denotes gravity and $\frac{D}{Dt}$ is the material derivative. The Cauchy's stress tensor $\boldsymbol{T}$ for non-Newtonian third grade fluid is

$$\boldsymbol{T} = -p\boldsymbol{I} + \mu \boldsymbol{A_1} + \alpha_1 \boldsymbol{A_2} + \alpha_2 \boldsymbol{A_2}^2 + \beta_1 \boldsymbol{A_3} + \beta_2(\boldsymbol{A_1}\boldsymbol{A_2} + \boldsymbol{A_2}\boldsymbol{A_1}) + \beta_3(t_r \boldsymbol{A_1}^2)\, \boldsymbol{A_1} \tag{3}$$

Where p is the pressure, $\boldsymbol{I}$ is the identity tensor, μ is the dynamic viscosity α_1, α_2 are normal stress material and β_1, β_2 and β_3 are material constant and $\boldsymbol{A}_n$ is Rivilin and Ericksen tensor given by

$$\boldsymbol{A_1} = (\nabla \boldsymbol{V}) + (\nabla \boldsymbol{V})^T,$$

$$\boldsymbol{A_{n+1}} = \frac{\partial}{\partial t}\boldsymbol{A_n} + \boldsymbol{A_n}(\nabla \boldsymbol{V}) + (\nabla \boldsymbol{V})^T \boldsymbol{A_n}, n = 1,2,3 \dots, \tag{4}$$

III FORMULATION OF THE LIFT PROBLEM

In this problem we consider a thin film flow of a third grade fluid on a vertically moving and oscillating belt. V is considered as the uniform velocity of the moving belt. When the belt is moving vertically with velocity V then it carries with itself a fluid layer of uniform thickness δ. We assume that the fluid is incompressible, the flow is unsteady, laminar and the pressure is standard atmospheric.

The velocity profile for the problem is

$$\boldsymbol{V} = (0, v(x), 0), \tag{5}$$

Boundary Conditions for the problem

$$v(0,t) = \boldsymbol{V} + \boldsymbol{V}\Omega \cos \omega t \tag{6}$$

$$\frac{\partial v(\delta, t)}{\partial x} = 0 \tag{7}$$

For the velocity profile $\boldsymbol{V}$= *(0, v(x), 0)* the continuity equation Eq. (1) is identically satisfied and the momentum equation yields to the following form

$$\rho \frac{\partial v}{\partial t} = -\frac{\partial p}{\partial y} + \frac{\partial}{\partial x}\boldsymbol{T_{yx}} + \rho g \tag{8}$$

Since we assume that the pressure is standard atmospheric pressure so Eq. (8) reduces to

$$\rho \frac{\partial v}{\partial t} = \frac{\partial}{\partial x}\boldsymbol{T_{yx}} + \rho g \tag{10}$$

Now using Eq. (4) and Eq. (10) in Eq. (3) we get

$$\boldsymbol{T_{xy}} = \mu \frac{\partial v}{\partial x} + \alpha_1 \frac{\partial}{\partial t}\left(\frac{\partial v}{\partial x}\right) + \beta_1 \frac{\partial^2}{\partial t^2}\left(\frac{\partial v}{\partial x}\right) + 6(\beta_2 + \beta_3)\left(\frac{\partial v}{\partial x}\right)^2 = \boldsymbol{T_{yx}} \tag{11}$$

Making use of Eq. (11) in to Eq. (10) we obtain the following equation

$$\rho \frac{\partial v(x)}{\partial t} = \frac{\partial}{\partial x}\left[\mu \frac{\partial v}{\partial x} + \alpha_1 \frac{\partial}{\partial t}\left(\frac{\partial v}{\partial x}\right) + \beta_1 \frac{\partial^2}{\partial t^2}\left(\frac{\partial v}{\partial x}\right) + 6(\beta_2 + \beta_3)\left(\frac{\partial v}{\partial x}\right)^3\right] + \rho g \tag{12}$$

Now introducing the non dimensional parameters

$$\tilde{v} = \frac{v}{V}, \quad \tilde{x} = \frac{x}{\delta}, \quad \tilde{t} = \frac{tv}{\delta^2 \rho}, \quad s_t = \frac{\rho \delta^2 g}{\mu V}, \alpha = \frac{\alpha_1}{\rho \delta^2}, \beta = \frac{\beta_3 V^2}{\mu \delta^2} \tag{13}$$

After dropping the `~`notation the non dimensional form of Eq. (12) is

$$\frac{\partial v}{\partial t} = \left(\frac{\partial^2 v}{\partial x^2}\right) + \alpha \frac{\partial}{\partial t}\left(\frac{\partial^2 v}{\partial x^2}\right) + 6\beta \left(\frac{\partial v}{\partial x}\right)^2 \left(\frac{\partial^2 v}{\partial x^2}\right) - s_t \tag{14}$$

The non dimensional boundary conditions

$$v(0,t) = 1 + \Omega \cos(\omega t), \quad \frac{\partial v(1,t)}{\partial x} = 0 \tag{15}$$

IV THE ADOMIAN DECOMPOSITION METHOD

The Adomian Decomposition Method for non linear PDE's Consider the nonlinear problem

$$L_x v + L_t v + Rv + \mathrm{N}v = G(x,t) \tag{16}$$

Where L_x is the highest order derivative in x, L_t is highest order derivative in t, R containing the remaining linear terms of lower derivative, N is the non linear term and $G(x,t)$ is forcing or non-homogenous term

Solving Eq. (16) for $L_x v$

$$L_x v = G(x,t) - L_t v - Rv - \mathrm{N}v \tag{17}$$

Applying $L_x{}^{-1}$ on Eq.(17)

$$L_x{}^{-1} L_x v = L_x{}^{-1} G(x,t) - L_x{}^{-1}(L_t v) - L_x{}^{-1}(Rv) - L_x{}^{-1}(\mathrm{N}v) \tag{18}$$

$$v = L_x{}^{-1} G(x,t) - L_x{}^{-1}(L_t v) - L_x{}^{-1}(Rv) - L_x{}^{-1}(\mathrm{N}v) \tag{19}$$

$$L_x{}^{-1} G(x,t) = \emptyset_0 \tag{20}$$

$\emptyset_0$ is the solution of source term

Using Eq. (20) in Eq. (19)

$$v = \emptyset_0 - L_x^{-1}(L_t v) - L_x^{-1}(Rv) - L_x^{-1}(\mathrm{N}v) \quad (21)$$

$$v(x,t) = \sum_{n=0}^{\infty} v_n(x,t) \text{ And } \mathrm{N}(v) = \sum_{n=0}^{\infty} A_n \quad (22)$$

Where A_n are Adomianpolynomials

Using Eq. (22)in Eq. (21)

$$\sum_{n=0}^{\infty} v_n(x,t) = \emptyset_0 - L_x^{-1}\left(\sum_{n=0}^{\infty} A_n\right) - L_x^{-1}\left(\sum_{n=0}^{\infty} B_n\right) - L_x^{-1}\left(R\left(\sum_{n=0}^{\infty} v_n(x,t)\right)\right), \quad (23)$$

The component solution of $v_n(x,t)$ for $n \geq 0$

$$v_0(x,t) = \emptyset_0 \quad (24)$$

$$v_1(x,t) = L_x^{-1}A_0 - L_x^{-1}Rv_0(x,t) - L_x^{-1}B_0, \quad (25)$$

$$v_2(x,t) = L_x^{-1}A_1 - L_x^{-1}Rv_1(x,t) - L_x^{-1}B_1, \quad (26)$$

V SOLUTION OF LIFTING PROBLEM

In operator form Eq.(14) can be written as

$$L_x v(x,t) = \frac{\partial v}{\partial t} + S_t - \alpha \frac{\partial}{\partial t}\left(\frac{\partial^2 v}{\partial x^2}\right) - 6\beta \left(\frac{\partial v}{\partial x}\right)^2 \left(\frac{\partial^2 v}{\partial x^2}\right), \quad (27)$$

The series solution of equation (27) can be written as

$$\sum_{n=0}^{\infty} v_n(x,t) = L_x^{-1}S_t - \alpha L_x^{-1}\sum_{n=0}^{\infty} A_n - 6\beta L_x^{-1}\left(\sum_{n=0}^{\infty} B_n\right), \quad (28)$$

Here A_n,B_n is used for Adomian polynomials and defined as

$\sum_{n=0}^{\infty} A_n = \frac{\partial}{\partial t}\left(\frac{\partial^2 v}{\partial x^2}\right)$ and $\sum_{n=0}^{\infty} B_n = \left(\frac{\partial v}{\partial x}\right)^2 \left(\frac{\partial^2 v}{\partial x^2}\right)$

In series form equation (28) reduced as

$$v_0(x,t) + v_1(x,t) + v_2(x,t) \dots = L_x^{-1}S_t - \alpha L_x^{-1}(A_0 + A_1 + A_2 \dots.) - 6\beta L_x^{-1}(B_0 + B_1 + B_2 \dots.), \quad (29)$$

In components form the Adomian polynomials are derived as

$$A_0 = \frac{\partial}{\partial t}\left(\frac{\partial^2 v_0}{\partial x^2}\right), \quad B_0 = \left(\frac{\partial v_0}{\partial x}\right)^2 \left(\frac{\partial^2 v_0}{\partial x^2}\right) \quad (30)$$

$$A_1 = \frac{\partial}{\partial t}\left(\frac{\partial^2 v_1}{\partial x^2}\right), \quad B_1 = 2\left(\frac{\partial v_0}{\partial x}\right)\left(\frac{\partial v_1}{\partial x}\right)\left(\frac{\partial^2 v_0}{\partial x^2}\right) + \left(\frac{\partial v_0}{\partial x}\right)^2 \left(\frac{\partial^2 v_0}{\partial x^2}\right) \quad (31)$$

$$A_2 = \frac{\partial}{\partial t}\left(\frac{\partial^2 v_2}{\partial x^2}\right), B_2 = \left(\frac{\partial v_1}{\partial x}\right)^2 \left(\frac{\partial^2 v_0}{\partial x^2}\right) + 2\left(\frac{\partial v_0}{\partial x}\right)\left(\frac{\partial v_2}{\partial x}\right)\left(\frac{\partial^2 v_0}{\partial x^2}\right) + 2\left(\frac{\partial v_0}{\partial x}\right)\left(\frac{\partial v_1}{\partial x}\right)\left(\frac{\partial^2 v_1}{\partial x^2}\right) + \left(\frac{\partial v_0}{\partial x}\right)^2 \left(\frac{\partial^2 v_2}{\partial x^2}\right) \quad (32)$$

For the component solution of the problem comparing both sides of equation (29)

$$v_0(x,t) = L_x^{-1}S_t, \quad (33)$$

$$v_1(x,t) = -\alpha L_x^{-1}(A_0) - 6\beta L_x^{-1}(B_0), \quad (34)$$

$$v_2(x,t) = -\alpha L_x^{-1}(A_1) - 6\beta L_x^{-1}(B_1), \quad (35)$$

Inserting boundary conditions (19) into Equation (33-35) the component solutions are

$$v_0(x,t) = 1 + \Omega \mathrm{Cos}[t\omega] - \left(1 + \Omega \mathrm{Cos}[t\omega] + \frac{S_t}{2}\right)x + \left(\frac{S_t}{2}\right)x^2 \quad (36)$$

$$v_1(x,t) = \left(\frac{1}{3}\Omega\omega \mathrm{Sin}[t\omega] + \beta S_t\left(3 + S_t + \frac{S_t^2}{4}\right) + \beta\Omega S_t(6 + S_t)\mathrm{Cos}[t\omega] + 3\beta S_t \Omega^2 \mathrm{Cos}[t\omega]^2\right)x - \left(\frac{1}{2}\Omega\omega \mathrm{Sin}[t\omega] + 3\beta S_t + 3S_t\beta\Omega(2 + S_t)\mathrm{Cos}[t\omega] + 3\beta S_t\Omega^2\mathrm{Cos}[t\omega]^2 + 3\beta S_t^2 + \frac{3\beta S_t^3}{4}\right)x^2 + \left(\frac{1}{6}\Omega\omega \mathrm{Sin}[t\omega] + 2\beta S_t^2 + 2S_t^2\beta\Omega \mathrm{Cos}[t\omega] + \beta S_t^3\right)x^3 - \left(\frac{1}{2}\beta S_t^3\right)x^4 \quad (37)$$

Inserting the boundary conditions from equation (19) into Eq. (35) the second component solution is obtained but the solution of the second component problem is too large to derive and we use only its graphical solution.

VI FORMULATION OF THE DRAINAGE PROBLEM

In drainage problem we consider the thin film third grade fluid on vertically oscillating belt. In this problem belt is not moving vertically but only oscillating and fluid is draining down the belt due to gravity. Therefore, the stock number is positively mentioned in Eq. (18). We assume that the flow is unsteady laminar and is of uniform thickness δ, the pressure is standard atmospheric everywhere.

$$v(0,t) = \boldsymbol{V}\Omega \cos \omega t \quad (38)$$

$$\frac{\partial(\delta,t)}{\partial x} = 0 \quad (39)$$

The non dimensional boundary conditions for drainage problem are

$$v(0,t) = \Omega\cos(\omega t), \ \frac{\partial(1,t)}{\partial x} = 0 \quad (40)$$

VII SOLUTION OF THE DRAINAGE PROBLEM

The Adomian polynomials $A_n$and $B_n$for both problems are same. But the boundary/initial conditions are different derived in equation (40).

$$v_0(x,t) = -L_x{}^{-1}S_t, (41)$$

Inserting the boundary conditions (40) into equation (41) and into equations (34,35) the component solution obtained as

$$v_0(x,t) = \Omega\mathrm{Cos}[t\omega] - \left(\Omega\mathrm{Cos}[t\omega] - \frac{S_t}{2}\right)x - \left(\frac{S_t}{2}\right)x^2 (42)$$

$$v_1(x,t) = \left(\frac{1}{3}\Omega\omega Sin[t\omega] - 3\beta S_t\Omega^2 Cos[t\omega]^2 + \beta S_t{}^2\Omega Cos[t\omega] - \frac{\beta S_t{}^3}{4}\right)x - \left(\frac{1}{2}\Omega\omega\mathrm{Sin}[t\omega] - 3\beta S_t\Omega^2\mathrm{Cos}[t\omega]^2 + 3\beta S_t{}^2\Omega\mathrm{Cos}[t\omega] - \frac{3\beta S_t{}^3}{4}\right)x^2 + \left(\frac{1}{6}\Omega\omega\mathrm{Sin}[t\omega] + 2S_t{}^2\beta\Omega\mathrm{Cos}[t\omega] - \beta S_t{}^3\right)x^3 + \left(\frac{\beta S_t{}^3}{2}\right)x^4 \quad (43)$$

The solution of the second component problem is too large we use only its graphical solution.

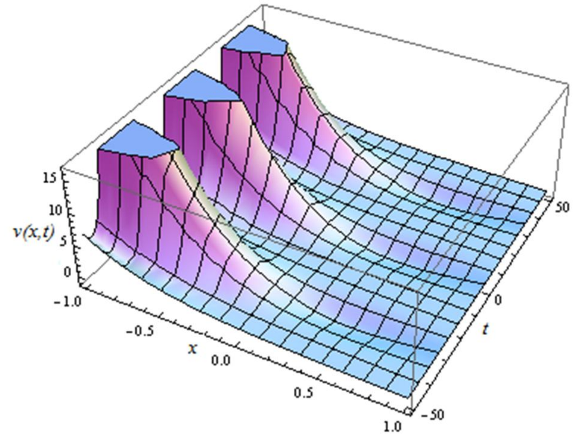

Figure 1: The influence of lifting velocity on different time level by taking $\omega = 0.2, \alpha = 0.02, S_t = 0.5, M = 0.5, \Omega = 0.4, \beta = 0.5$

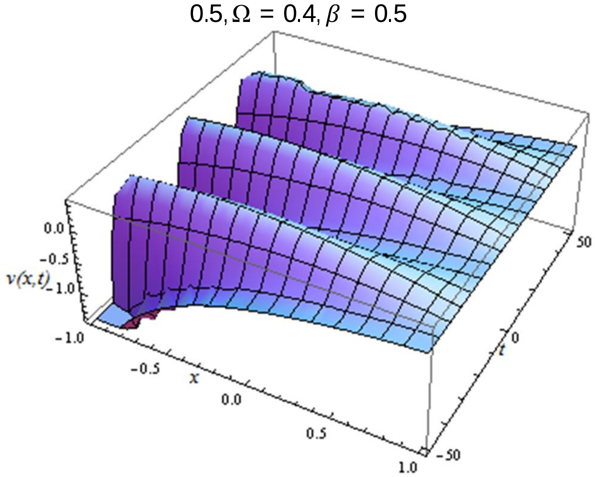

Figure 2:the influence of drainage velocity on different time level by taking $\omega = 0.2, \alpha = 0.02, S_t = 0.5, M = 0.5, \Omega = 0.4, \beta = 0.5$

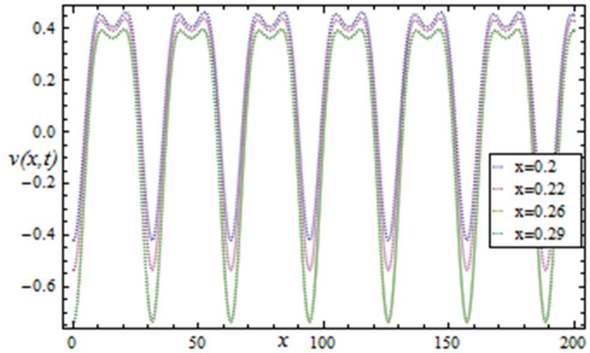

Figure 3: Lift velocity distribution of fluid at different time level. When $\omega = 0.2, \alpha = 0.02, S_t = 0.5, M = 0.5, \Omega = 0.4, \beta = 0.5$

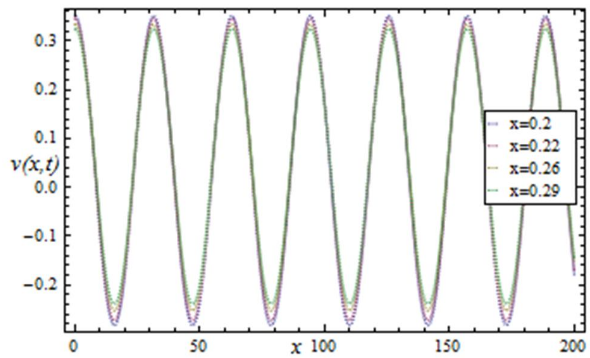

Figure 4: Drainage velocity distribution of fluid at different time level. When $\omega = 0.2, \alpha = 0.02, S_t = 0.5, M = 0.5, \Omega = 0.4, \beta = 0.5$

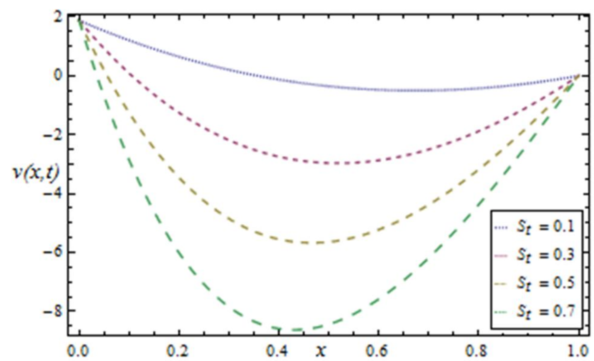

Figure 5: Effect of stoke number on lift velocity distribution of fluid where $M = 0.4, \alpha = 0.02, \beta = 0.5, \omega = 0.02, t = 10, \Omega = 0.2$

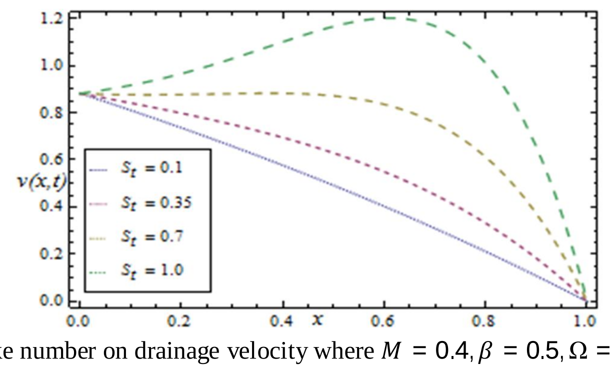

Figure 6: Effect of stoke number on drainage velocity where $M = 0.4, \beta = 0.5, \Omega = 0.4, t = 10, \omega = 0.02, \alpha = 0.2$

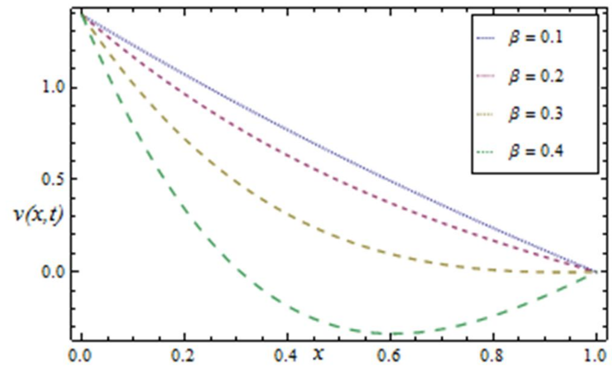

Figure 7: Effect of non-Newtonian parameter β on lift velocity where $M = 0.6, S_t = 0.5, \Omega = 0.4$, t = 10, $\omega = 0.02$, $\alpha = 0.2$

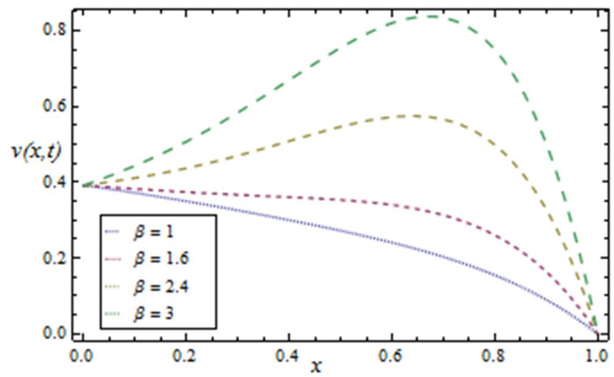

Figure 8: Effect of non Newtonian parameter β in drainage velocity $M = 0.5, S_t = 0.5, \Omega = 0.4$, t = 10, $\omega = 0.02$, $\alpha = 0.2$

VIII RESULTS AND DISCUSSION

The analytical solutions for velocity field of thin film third grade fluid are solved by Adomian Decomposition Method (ADM) for both lifting and drainage problems. The effect of stoke number s_t , non-Newtonian peramater β and other parametrs involved in the problem are discussed and result are shown graphically and numerically to see the effect of these for both lifting and drainge velocity fields. Figs 1-2 are plotted to see the influence of different time level on both velocity fields. Figs 3-4 shows the influence of different values of (*x=0.2, 0.22, 0.26, 0.29*) and by taking (ω*=0.2,* α*=0.02,*$s_t = 0.5$*,* β*=0.5 and* Ω*=0.4*). In fig.3 the lifiting velocity decreases with the increase of *x.* In fig.4 the drianage velocity is also having inverse relation to *x*. Fig.5showes the influence of stoke numberss_t on liftingvelocity. The lifting velocity field decreases by increasing stoke number $s_t$while in fig.6showes the influence of stoke numberss_t on drainage velocity. The drainge velocity increases by increasing stoke numer s_t for different values*(* s_t*= 0.1,0.3,0.5,0.7).* Fig.7shows the influence of non-Newtonian parameter β on lifting velocity. The lifting velocity field decreases by increasing non-Newtonian peramater β for different values(β*=0.1,0.2,0.3,0.4*) while in fig.8 show the influence of non-Newtonian parameter β on drainage velocity.The drainge velocity increases by increasing the non-Newtonian peramater β for different values (β = 1,1.6,2.4,3.0). Table.1showes the compnent solution of liftinfg problem while table.2 showes the component solutionof drainage problem for different values of *x,(x=0.2,0.24,0.26,0.29)* and for fixed values of ω*=0.2,* α*=0.02,*$s_t = 0.5$*,* β*=0.5 and* Ω*=0.4*

IX CONCLUSION

In the present work we studied unsteady thin film third grade fluid on vertical oscillating belt. The governing nonlinear partial differential equations for both lifting and drainage problems are solved by Adomian decomposition Method (ADM). The effect of model parameters on velocity profile have been plotted and discussed numerically as well as graphically. It is concluded that the lifting velocity decreases by increasing stoke number $s_t$and the drainge velocity increases by increasing stoke numer s_t, while the lifting velocity field decreases by increasing non-Newtonian peramater β and the drainge velocity increases by increasing non-Newtonian peramater β.

Table. 1
Component solution of ADM for the lifting velocity profile at different values of *x*.

x	*x=0.2*	*x=0.22*	*x=0.26*	*x=0.29*
0.	-0.419735	-0.535392	-0.735831	-0.735831
1.	-0.395685	-0.509266	-0.706109	-0.706109
2.	-0.319763	-0.427057	-0.613094	-0.613094
3.	-0.204008	-0.301621	-0.471084	-0.471084
4.	-0.0656294	-0.151391	-0.300579	-0.300579
5.	0.0755788	0.00363542	-0.123838	-0.123838
6.	0.206428	0.145777	0.394307	0.394307
7.	0.311977	0.262507	0.175196	0.175196
8.	0.387681	0.347068	0.27644	0.27644
9.	0.433553	0.401112	0.342761	0.342761
10.	0.453883	0.42723	0.378733	0.378733

Table. 2
Component solution of ADM for the Drainage velocity profile at different values of *x*.

x	*x=0.2*	*x=0.22*	*x=0.26*	*x=0.29*
0.	0.351192	0.345463	0.333571	0.324281
1.	0.345327	0.339765	0.328185	0.319112
2.	0.327393	0.322323	0.311692	0.303299
3.	0.327325	0.294085	0.311692	0.277836
4.	0.259345	0.256263	0.249504	0.043924
5.	0.211768	0.210120	0.206184	0.202679
6.	0.156956	0.156927	0.156204	0.155082
7.	0.096501	0.098159	0.100811	0.102220
8.	0.032475	0.035773	0.041738	0.045656
9.	-0.032433	-0.027619	-0.018574	-0.012307
10.	-0.095028	-0.088861	-0.077066	-0.068605

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