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Reconstruction of a distributed force impacting an elastic rectangular plate

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ABSTRACT

This work deals with reconstruction of distributed force signal resulting from a non punctual object impacting perpendicularly an elastic homogeneous and isotropic rectangular plate. The impacting force is assumed to be uniformly distributed over a rectangular patch of the plate. The direct problem was solved by using modal decomposition method with explicit analytical modes. A discrete problem was written for that by sampling the obtained convolution integral. To extract the pressure signal by deconvolution of the dynamic response measured at a given point of the plate, solution of an inverse problem had been considered. Since this type of problem is known to be ill-posed due to bad conditioning of the involved Toeplitz like matrix, regularization is needed to obtain a physically meaningful solution. A new regularization technique based on truncation filtering was examined. This technique uses as a first step the generalized decomposition of Toeplitz matrix on singular values. Then, regularization of the decomposed form through a truncation filter is performed. The truncation consists in eliminating the first low index terms up to an optimal rank representing the contribution of low amplitude generalized singular values. If the impact force signal has a half sine like standard form, the index corresponding to time instant where the maximum displacement response is obtained was found to be the optimal order of truncation. This technique has proved to be effective in reconstruction of impact pressures through various cases of study and the computational cost was found to be much lower than that of the classical truncation method based on L-curve criterion.

KEY WORDS: Inverse problem, Identification, Impact, Regularization, Plates, Dynamic response, Transfer function.

INTRODUCTION

To perform structural health monitoring or reliability analysis of structures, it is essential to provide accurate characterization of input forces experienced during service operation. In common practice, the input force is measured by using a force transducer that is positioned in the load path. On many circumstances, such as a high-speed impact of an object onto a

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structure, it is difficult to apply this technique: a bird impacting an aeroplane fuselage. Another technique that has been widely employed for the impact-force signal reconstruction is based on analysis of the inverse problem. This means that the dynamic force is recovered from the data of the measured elastic response. When the impact point is known, the problem is equivalent to deconvolution of two signals: the measured response and the transfer function characterizing the dynamics of the structure. In many cases, the deconvolution results in an ill-posed problem in which the data noise strongly affects the solution accuracy. Therefore, it is difficult to obtain an accurate solution for such problems, so that, regularization is needed to obtain a physically meaningful solution.

There is a number of publications which deals with the impact-force reconstruction. Doyle (1984; 1987), Martin and Doyle (1996) reconstructed the impact force profile by using spectral analysis. The proposed method had utilized the convolution theorem that expresses the time domain deconvolution as a simple division in the frequency domain. Later on, Adams and Doyle (2000), Hansen (1998), Jacquelin et al. (2003) and Liu et al. (2005) adopted a more systematic approach to regularize the deconvolution problem by using either the singular value decomposition method (SVD) or the generalized singular value decomposition method (GSVD). These authors had considered the problem of a localized impact where the object could be approximated as a single point. In many cases, however, the impacting object is massive and the impact zone could not be approximated as a single point and the impacting force takes the form of a distributed pressure over this zone. Liu and Shepard (2006), Djamaa et al. (2007), Jiang and Hu (2009) have tackled the problem of reconstruction of distributed dynamic loads on structures like Euler beam, thin plates or cylindrical shells. They have used either the modified modal method or the mode-selection method.

Certain researchers have indicated that a major drawback of the Tikhonov-GSVD method is the expensive computational cost of the GSVD and consequently the method is only suitable for small scale inverse problems.

In the present study, we consider the impact force reconstruction problem in the case where a distributed force is applied onto an elastic rectangular plate. For this purpose, the direct solution has been computed by using an analytical formula. The proposed regularization method is based on GSVD method. Truncation regularization (1994), which looks a lot like Tikhonov regularization (1977), is investigated. This method is a particular case of the general filter factor regularization method. A new regularization strategy is examined. It is based on a priori defined truncation order which reduces considerably the computational cost.

MATERIALS AND METHODS

We consider a rectangular plate as shown in figure 1 which has the dimensions a, b and e representing respectively length, width and thickness. It is assumed to be simply supported on its ends. The plate is assumed to be made of a homogeneous and isotropic elastic material with Young's modulus E, Poisson's ratio υ and density p. The applied force modeling impact is assumed to be uniformly distributed over a rectangular patch of the plate. The dynamic response in terms of displacement, velocity, acceleration or strains is considered at a point which is located at a given distance away from the centre of the loading rectangle, figure 1.

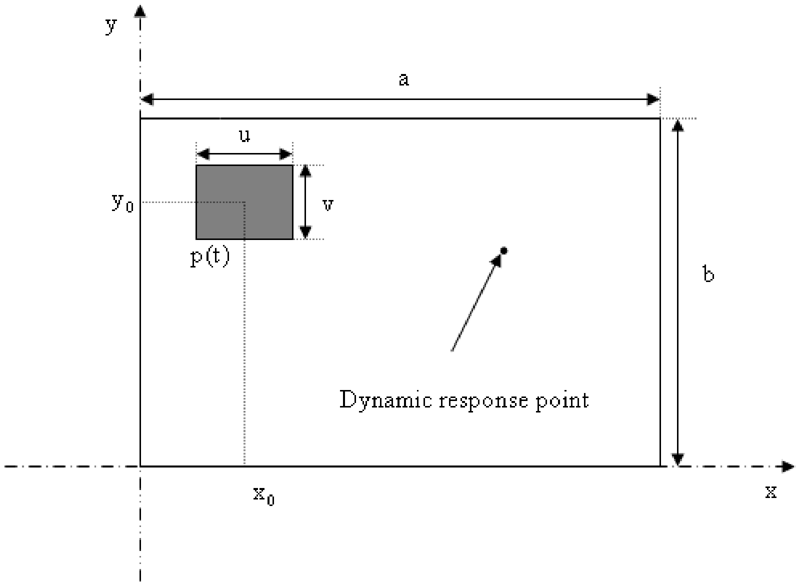

Figure 1: Simply supported rectangular plate showing the impacted zone with a uniform distributed pressure and the measurement point

The equation of motion of a simply supported rectangular Timoshenko plate (Timoshenko and Woinovsky-Krieger, 1940) can be expressed under the following form:

$$D\Delta\Delta w(x,y,t) + c\dot{w}(x,y,t) + \rho h\ddot{w}(x,y,t) = q(x,y,t) \quad (1)$$

where x is the horizontal coordinate, y the vertical coordinate, t the time, $w(x,y,t)$ the transverse displacement, $q(x,y,t) = p(t)\Im_{[x_0-u/2,x_0+u/2]\times[y_0-v/2,y_0+v/2]}(x,y)$ the applied loading with $\Im$ the indicative distribution taking the value one on the domain shown in subscript and zero elsewhere, c the damping coefficient, $D = Ee^3/(12(1-\upsilon^2))$ the plate flexural rigidity modulus and $\Delta\Delta w(x,y,t) = \dfrac{\partial^4 w}{\partial x^4} + 2\dfrac{\partial^2 w}{\partial x^2 \partial y^2} + \dfrac{\partial^4 w}{\partial y^4}$.

The above governing equation is subjected to the following boundary conditions:

$$w = 0 \text{ and } \frac{\partial^2 w}{\partial x^2} = 0 \text{ for } x = 0 \text{ and } x = a$$

$$(2)$$

$$w = 0 \text{ and } \frac{\partial^2 w}{\partial y^2} = 0 \text{ for } y = 0 \text{ and } y = b$$

By applying the modal superposition technique, the displacement $w(x,y,t)$ can be shown to be expressed under the following form:

$$w(x,y,t)=\int_0^t h(x_0,y_0,u,v,x,y,t-\tau)p(\tau)d\tau \tag{3}$$

with the transfer function

$$\begin{aligned}h(x_0,y_0,u,v,x,y,\tau)=\frac{16}{\rho h\pi^2}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{1}{mn\gamma_{mn}}\sin(\frac{m\pi x_0}{a})\sin(\frac{n\pi y_0}{b})\sin(\frac{m\pi u}{a})\\ \sin(\frac{n\pi v}{b})\sin(\frac{m\pi x}{a})\sin(\frac{n\pi y}{b})\sin(\gamma_{mn}\tau)e^{-\xi_{mn}\omega_{mn}\tau}\end{aligned} \tag{4}$$

in which ω_{mn}, γ_{mn} and ξ_{mn} are respectively the circular eigenfrequency, the damped circular eigenfrequency ($\gamma_{mn}=\omega_{mn}\sqrt{1-\xi_{mn}^2}$) and the damping ratio for eigenmode (m,n).

In order to represent realistically the impact-force, it is assumed subsequently that the pressure takes the form of a half-sine function, figure 2.

The elastic response $w(x,y,t)$ can be computed over the considered time interval $[0,T_c]$ by integrating explicitly equation (3) where x_0, y_0, u, v, x, y take fixed values. In this work, the selected configuration of the impacted plate is defined by the following value parameters: $a=2.05\text{ m}$, $b=2.05\text{ m}$, $e=5\times10^{-3}\text{ m}$, $\xi_{mn}=0$, $x_0=y_0=0.1025\text{ m}$, $u=v=0.0342\text{ m}$, $x=y=0.041\text{m}$ and $T_c=0.012\text{ s}$.

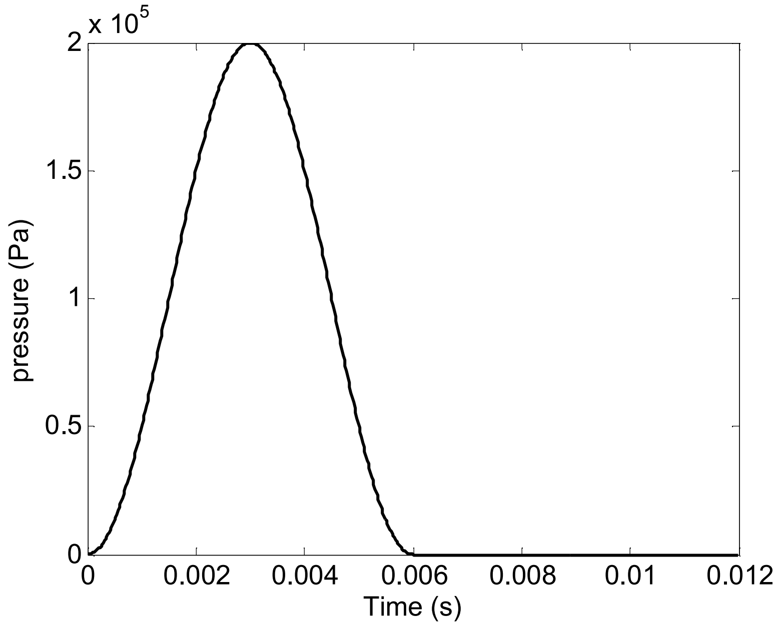

Figure 2: Input pressure signal profile having the form of half sine defined over the interval time $[0,12\text{ ms}]$ with an effective period $T=0.006\text{s}$

Figure 3 gives the displacement calculated at the point $(x = 0.041\,m, y = 0.041\,m)$. This direct elastic displacement response is stored and will be used in the following to reconstruct the impact-force signal.

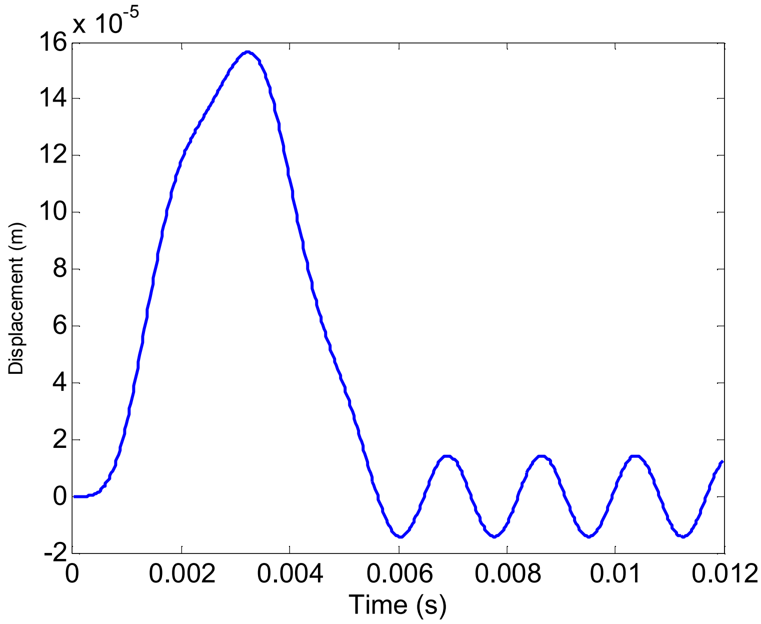

Figure 3: Direct problem solution; calculated transverse displacement at point $(x = 0.041\,m, y = 0.041\,m)$

To identify the impact-force acting on the plate over the rectangular domain of impact, the transfer function based approach is used. In more general problems, transfer functions can be determined analytically (Meirovitch, 1990), experimentally (Hansen, 1998), or numerically. Here, the transfer function is evaluated analytically through time integration of equation (3) by using equation (4).

To solve the deconvolution problem associated to equation (3), a discrete problem must be written by sampling the convolution integral. This leads in the time domain to the following system of algebraic equations

$$W = HP \tag{5}$$

With:

$$H = \begin{pmatrix} H(\Delta t) & 0 & & & 0 \\ H(2\Delta t) & H(\Delta t) & \ddots & & \\ H(3\Delta t) & H(2\Delta t) & \ddots & \ddots & \\ \vdots & \vdots & \ddots & \ddots & 0 \\ H(n\Delta t) & H((n-1)\Delta t) & \dots & \dots & H(\Delta t) \end{pmatrix} \tag{6}$$

$$W = \left[w(\Delta t) \quad w(2\Delta t) \quad \cdots \quad w(n\Delta t) \right]^t$$
$$P = \left[p(\Delta t) \quad p(2\Delta t) \quad \cdots \quad p(n\Delta t) \right]^t$$

Where the transfer function H is a square lower triangular Toeplitz matrix, Δt is the sampling rate and n the total number of samples.

The sampling rate must be selected in order to recover a predefined cut-off frequency in the pressure signal. The matrix H is ill-conditioned. This means that it can lead to an unstable solution which has no physical meaning. Therefore, to find a physically acceptable solution the deconvolution problem defined by equations (5) and (6) should be regularized.

Here, the regularization technique based on the generalized singular value decomposition (GSVD) is considered. It should be mentioned that the simpler SVD method has failed to regularize the actual problem. The GSVD-regularized solution of problem defined by equations (5) and (6) can be written as follows

$$[P] = [X][\Phi][\Delta]^{-1}[U]^t[W] = \left[H^*\right][W] \tag{7}$$

Where (X, Δ, U) being the singular factors of H, $[\Phi]$ is the filter factor and $\left[H^*\right] = [X][\Phi][\Delta]^{-1}[U]^t$ is the regularized pseudo-inverse of H.

The filter factors goal is to minimize the influence of the low amplitude generalized singular values. Many techniques have been considered in the literature for that. Among them, one finds the regularization techniques: Tikhonov method (Tikhonov and Arsenin, 1977) and GSVD truncation method (Hansen, 1998). Here truncation based regularisation technique is used.

The truncation consists of eliminating the first low index terms up to the rank k. This index is called the regularization parameter. The index k should be selected in order to eliminate the small generalized singular values as well as the oscillating singular vectors.

The filter Φ defined by the truncation method writes:

$$\Phi_{ij} = f_i \, \delta_{ij} \quad i, j = 1, \ldots, n \tag{8}$$

Where $f_i = 0$ if $i \leq k$ and $f_i = 1$ otherwise.

To build the filter Φ within the framework of truncation method, the rank k should be specified

The optimal rank should minimize the error between the identified pressure and the real pressure.

Classically, the L-Curve method has been applied in order to determine the regularization parameter k by means of a graphical based method. This technique was developed by Hansen (Hansen, 1999). It is based on searching the optimum of a functional composed of two terms, a residue called $RN = \|W - HP\|_2$ (Residual Norm) and the norm of the solution, designated by $SN = \|P\|_2$ (Semi-Norm). When the k parametric curve defining SN versus RN is plotted, the optimal regularization parameter corresponds to the point of maximum curvature, the corner. However, in practice, this method is often problematic; for measured data the L-curve is discrete and the determination of such point is delicate, because it is not distinguishable on the graph.

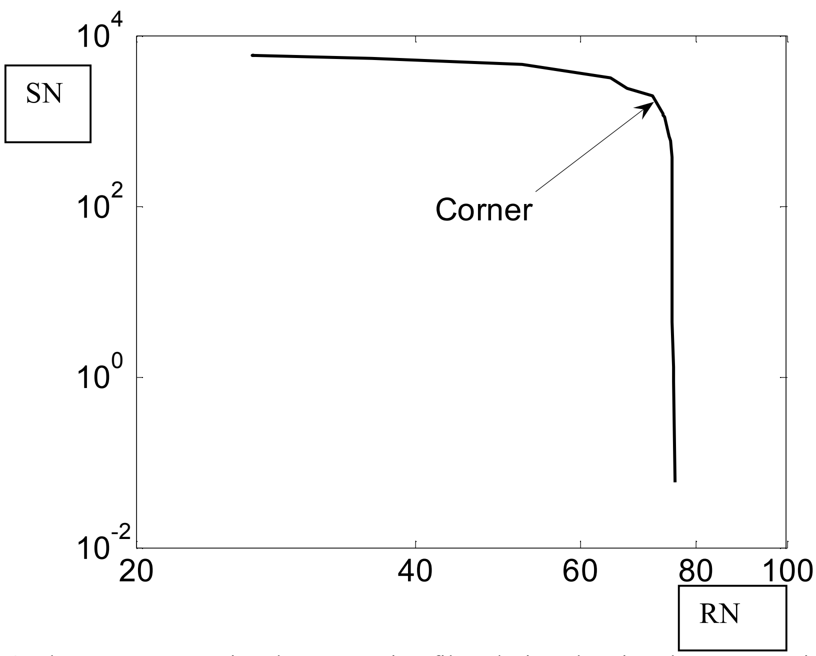

Figure 4. The L-curve associated to truncation filter design showing the corner point defining the optimal truncation order

Here a new method is proposed to determine the optimal rank truncation. Through various numerical tests conducted on pressure signals having half sine form, the rank defined as the index of the maximum value of the calculated displacement was found to yield the closer form of the real pressure signal.

RESULTS

Figure 5 presents the superposition of the real impact pressure with the pressure profile as obtained by the inverse problem solution: $(x_0 = 0.0683\,m, y_0 = 0.0683\,m)$ and $(x = 0.041\,m, y = 0.041\,m)$ and pulse period $T = 6ms$. The truncation order is 160, which is defined as the index of the maximum value of the calculated displacement. The associated CPU time is 68.94.

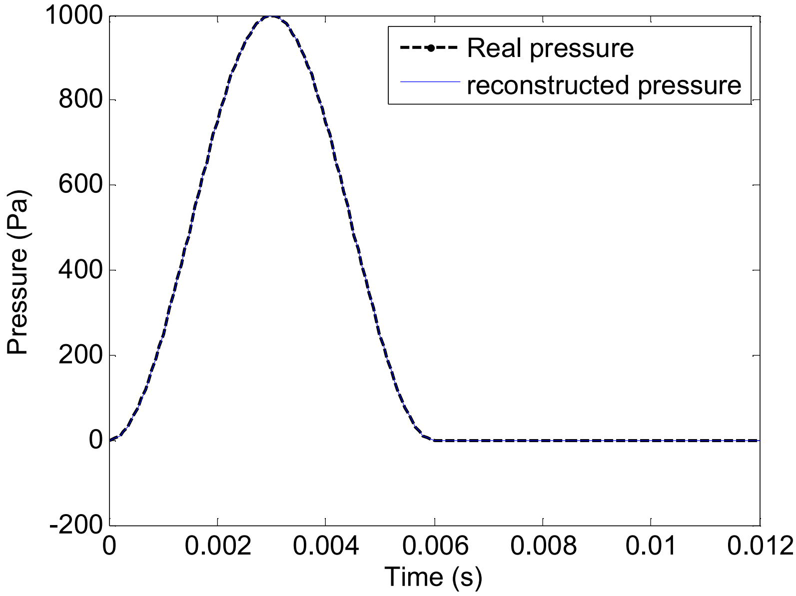

Figure 5: Comparison of the reconstructed pressure profile with the real input pressure for the input signal shown in figure 2 and having period $T = 0.006\,\mathrm{s}$

Figure 6 presents the superposition of the real impact pressure with the pressure profile as obtained by the inverse problem solution: $(x_0 = 0.0683\,\mathrm{m}, y_0 = 0.0683\,\mathrm{m})$ and $(x = 0.041\,\mathrm{m}, y = 0.041\,\mathrm{m})$ and a pulse period $T = 4\mathrm{ms}$. The truncation order is 179.

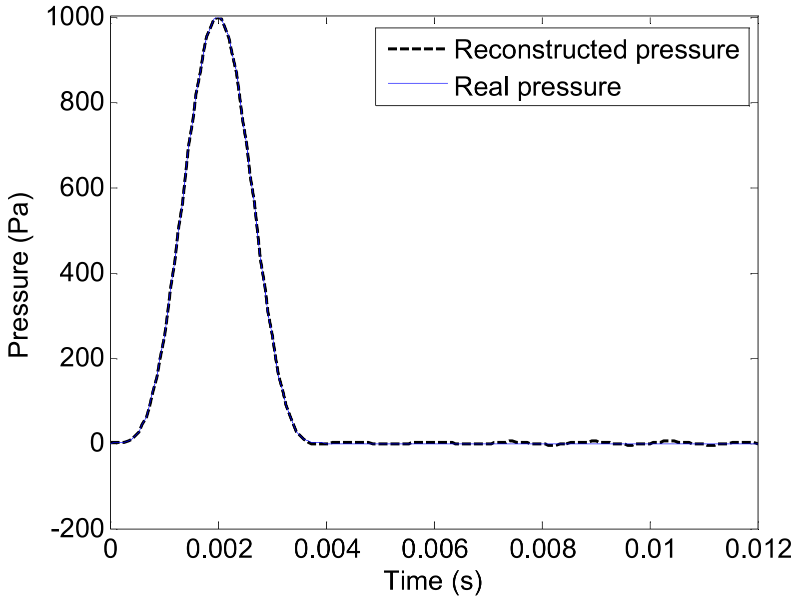

Figure 6: Comparison of the reconstructed pressure profile with the real input pressure having the form of figure 2 but with a period $T = 0.004\,\mathrm{s}$

DISCUSSION

Using the L-curve criterion for the previous problem, the same impact pressure is obtained. The identified truncation order which is determined by the L-curve criterion is 73. The associate CPU time is 146.2. This last does not include extra programming tasks needed by this method.

Considering now a new central impact point with $(x_0 = 0.1025\,\mathrm{m}, y_0 = 0.1025\,\mathrm{m})$ and $(x = 0.041\,\mathrm{m}, y = 0.041\,\mathrm{m})$ and pulse period $T = 6\mathrm{ms}$, solution of the inverse the inverse problem yields same results as those depicted in figure 5. The same truncation order was obtained 160.

It is clear that the new proposed method and the L-curve based method permit the exact reconstruction of the signal, but the first one, in which the order of truncation is, defined as the index of the maximum value of the calculated displacement presents a lower computational cost, gain is about 47%. This methodology was proven to yield results that are independent from the impact location, the measurement point and the pulse period. But, it is necessary that the profile of the impacting force should have a half sine like form. This is not a real limitation since most of the impacting force signals have this general form.

Conclusion

A new method for reconstruction of distributed force in case of non punctual object impacting an elastic rectangular plate was proposed. This method is based on generalized singular value decomposition of Toeplitz like matrix obtained for the discrete convolution problem relating the displacement dynamical response at a given point and the impact pressure signal. This last was assumed to be uniform over a rectangular patch of the plate and to have a half sine profile. To build the filter needed for regularization of the inverse problem, the truncation based method was used. The order of truncation was identified to be the index of time associated to the maximum measured displacement. This methodology was proven to yield results that are independent from the impact location, the measurement point and the pulse period. Even when the signal is not a half sine one, good results are obtained. The computational cost of this method is lower than that of the classical truncation method.

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REFERENCES

Adams, R. and J.F. Doyle, 2000. Multiple force identification for complex structures, Experimental Mechanics 42(1): 25-37.

Djamaa, M.C., N. Ouelaa, C. Pezerat and J.L. Guyader, 2007. Reconstruction of a distributed force applied on a thin cylindrical shell by an inverse method and spatial filtering. Journal of Sound and Vibration 301(3-5): 560-575.

Doyle, J.F., 1984. Further developments in determining the dynamic contact law. Experimental Mechanics 24(4): 265–270.

Doyle, J.F., 1987. Experimentally determining the contact force during the transverse impact of an orthotropic plate. Journal of Sound and Vibration 118 (3): 441–448.

Hansen, P.C., 1998. Rank-Deficient and Discrete Ill-posed Problems, SIAM, Philadelphia, PA.

Hansen, P.C., 1999. The L-curve and its use in the numerical treatment of inverse problems, Tech. Report, IMM-REP 99-15, Dept. of Math. Model. Tech. Univ. of Denmark.

Jacquelin, E., A. Bennani and P. Hamelin, 2003. Force reconstruction analysis and regularization of a deconvolution problem. Journal of Sound and Vibration 265: 81-107.

Jiang, X.Q. and H.Y. Hu, 2009. Reconstruction of distributed dynamic loads on an Euler beam via mode-selection and consistent spatial expression, Journal of Sound and Vibration 316(1-5): 122-136.

Jiang, X.Q. and H.Y.Hu , 2009. Reconstruction of distributed dynamic loads on a thin plate via mode-selection and consistent spatial expression. Journal of Sound and Vibration 323(3-5): 626-644.

Liu, Y. and S. Shepard, 2005. Dynamic force identification based on enhanced least-squares schemes in the frequency domain. Journal of Sound and Vibration 282: 37-60.

Liu, Y., and W.S. Shepard Jr., 2006. An improved method for the reconstruction of a distributed force acting on a vibrating structure. Journal of Sound and Vibration 291(1-2): 369-387.

Mair, B.A., 1994. Tikhonov regularization for finitely and infinitely smoothing operators. SIAM Journal of Mathematical Analysis 25(1): 135–147.

Martin, M.T and J.F. Doyle, 1996. Impact force identification from wave propagation responses. International journal of Impact Engineering 18(1): 65-77.

Meirovitch, L., 1990. Dynamics and Control of Structures. John Wiley, New York.

Tikhonov, A.N. and V. Y. Arsenin, 1977. Solutions of ill-posed problems, Wiley, New York.

Timoshenko S. and Woinovsky-Krieger, 1940, Theory of plates and shells, McGraw-Hill Book Co., Eng. Mech. Series collection, 1940 (reprinted 1959), 580 p.