

## Review of Empirical Equations of Estimating Saturated Hydraulic Conductivity Based on Soil Grain Size Distribution

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### ABSTRACT

Knowledge of hydraulic conductivity is essential for many problems involving water flow and pollution transport in soils. A variety of field and laboratory techniques have been developed to directly measure the unsaturated hydraulic conductivity. In this research, 25 set of soil samples with sand texture have been used. The results showed that among seven empirical formulae (Hazen, Kozeny-Carman, Breyer, Slitcher, Terzaghi, USBR and Alyamani & Sen), the Slitcher formula predicted Ks, saturated hydraulic conductivity, better than other formulae with 0.671 R; 6.08 RMSE; 5.06 MAE; 20.75% RE & 1.393 DT and the Breyer formula estimated Ks, with largest prediction error.

**KEYWORDS**-empirical equations; hydraulic conductivity; soil particle diameter and soil grain size.

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### I. INTRODUCTION

Saturated hydraulic conductivity of soil (Ks) is one of the most important soil physical characteristics that are very important in ground water studies, water infiltration, leaching, design of drainage systems and hydrological studies. Direct measurement of hydraulic conductivity is costly, difficult and time-consuming either field or laboratory. Therefore, indirect methods as a way to partially solve this problem are presented. The methods of predicting hydraulic conductivity by empirical formulae based on particle-size distribution properties have been developed. Various studies indicate that the hydraulic conductivity is dependent on grain size and the particle size distribution in porous media [1].

Empirical formula is one of the methods that used to estimate hydraulic conductivity that empirical formulas have been developed. Curve of particle size distribution is one of the basic physical properties of soils that usually expressed the cumulative probability distribution of the diameter of the soil particles [2]. In recent years, many studies have been to predict soil hydraulic functions based on soil grain size distribution [3-5]. Several studies have been carried out to quantify soil physical properties such as particle size distribution. Many studies have attempted to estimate soil hydraulic properties [6].

Since hydraulic properties have a high spatial variation, Therefore in order to determine the hydraulic characteristics of each region requires a lot of soil samples. Since measurement of hydraulic conductivity is difficult and costly. Therefore, the empirical formulas have been developed [7-9].

Many different relationships have been developed from particle-size distribution. Research findings showed that USBR and Slitcher equation calculated the amounts of hydraulic conductivity less than the other equations [10]. The research results showed that the Breyer equation is very suitable for soils with low uniformity coefficient [11].

Carrier (2003) noted that the Kozeny-Carman equation calculated hydraulic conductivity with high accuracy compared to Hazen equation [8]. Odong (2007) have been evaluated several empirical equations to prediction hydraulic conductivity [12]. Jarvis et al. (2002) presented the functions based on relative frequency of particle and geometric mean of particles diameter [13]. Huang and Zhang Evaluated soil water retention curve [14].

Heuvelmans et al. (2005) Regional sated the parameters of a hydrological model with artificial neural networks and linear regression model[15]. Regression transport functions are suitable method for modeling of soil hydraulic properties [15 & 16].

Odong (2007) evaluated the various empirical equations that predicted soil hydraulic conductivity and the results showed that the Kozeny-Carman equation was the best estimator among the studied equations. However, some of the equations underestimated or overestimated hydraulic conductivity [12].

Vukovic and Soro (1992) determined Hydraulic Conductivity of Porous Media from Grain-Size Composition and after the studding of previous researches provide the following general equation [17]:

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$$K = \frac{g}{v} \cdot C \cdot f(n) \cdot d_e^2 \quad (1)$$

Where K is hydraulic conductivity; g is gravity acceleration; v is kinematic viscosity; C is coefficient of sorting; f (n) is function of porosity, and  $d_e$ , is effective particle diameter.

Hazen (1982) proposed the equation to prediction of saturated hydraulic conductivity by soil particle diameter,  $K_s = c(d_{10})^2$ , Where  $K_s$  is expressed in cm/sec, c is a constant that varies from 1.0 to 1.5, and  $d_{10}$  size gives the diameter for which 10% (by mass) of the particles in a soil sample are finer. This equation is obtained for the washed sand clay and silt causes the hydraulic conductivity calculated by hazel formula does not accurate [18 & 19]. Also, Hazen equation is presented in the following form.

$$K = \frac{g}{v} \times 6 \times 10^{-4} [1 + 10(n - 0.26)] d_{10}^2 \quad (2)$$

Kozeny-Carman formula is one of the most applicable equations used for determine of hydraulic conductivity as a function of the soil properties.

$$K = \frac{g}{v} \times 8.3 \times 10^{-3} \left[ \frac{n^3}{(1-n)^2} \right] d_{10}^2 \quad (3)$$

Ahuja et al. (1989) predicted  $K_s$  by the Kozeny-Carmen equation. This equation is not suitable for clay soils [8]. Breyer presented the following formula [12]:

$$K = \frac{g}{v} \times 6 \times 10^{-4} \log \frac{500}{U} d_{10}^2 \quad (4)$$

Where U is the particle uniformity coefficient:

$$U = \left( \frac{d_{60}}{d_{10}} \right) \quad (5)$$

Where  $d_{60}$  and  $d_{10}$  size gives the diameter for which 60% and 10% (by mass) of the particles in a soil sample are finer.

In Breyer method, the parameter of porosity been considered and consequently, the porosity function is considered one [12].

The Breyer equation is often applicable and useful for heterogeneous and non-uniform distribution material. Slitcher presented the following formula [12]:

$$K = \frac{g}{v} \times 1 \times 10^{-2} n^{3.287} d_{10}^2 \quad (6)$$

Slitcher formula often can be used for particle sizes between 0.01 to 5 mm [12].

Terzaghi presented the following formula:

$$K = \frac{g}{v} \cdot C_i \cdot \left( \frac{n - 0.13}{\sqrt[3]{1-n}} \right)^2 d_{10}^2 \quad (7)$$

Where the  $C_i$  is coefficient of sorting [20].

The following equation presented by U.S. Bureau of Reclamation (USBR) [12]:

$$K = \frac{g}{v} \times 4.8 \times 10^{-4} d_{20}^{0.3} \times d_{20}^2 \quad (8)$$

USBR formula does not depend on porosity and  $d_{20}$  size gives the diameter for which 20% (by mass) of the particles in a soil sample are finer [12].

Alyamani and Sen (1993) offered the following formula [21]:

$$K = 1300 [I_o + 0.025(d_{50} - d_{10})]^2 \quad (9)$$

Where  $K_s$  is hydraulic conductivity,  $I_o$  is the intercept.

Cronican and Gribb (2004) offered multiple linear regressions based on limited soil data [18].

In this study several empirical equations have been evaluated to calculate hydraulic conductivity by particle size.

## II. MATERIALS AND METHODS

In this study in order to prediction hydraulic conductivity by grain size distribution (PSD) used the 25 sets samples of sandy soil texture. Finally parameters of  $d_{10}$ ,  $d_{50}$  and  $d_{60}$  determined where  $d_{10}$ ,  $d_{50}$  and  $d_{60}$  size gives the diameter for which 10%, 50% and 60% (by mass) of the particles in a soil sample are finer.

The values of parameters of  $d_{10}$ ,  $d_{50}$ ,  $d_{60}$  and saturated hydraulic conductivity are showed in Table 1.

TABLE I. VALUES OF STATISTICS OF  $d_{10}$ ,  $d_{50}$ ,  $d_{60}$  AND SATURATED HYDRAULIC CONDUCTIVITY

Statistics	Parameters			
	$d_{10}^a$	$d_{50}$	$d_{60}$	$K_s$
Mean	0.253	0.707	0.936	24.38

Minimum	0.16	0.42	0.61	15.1
Maximum	0.36	1.10	1.38	36.1
Std. Deviation	0.061	0.185	0.248	5.96
Skewness	-0.171	0.179	0.287	0.204

The results were studied by statistics such as Root Mean Square Error (RMSE), Correlation Coefficient (R), Mean Absolute Error (MAE), Relative Error (RE) and Deviation Time (DT) using equation (10), (11), (12) and (13) respectively, where n is the number of the data series and  $O_i$  and  $P_i$  are observed and estimated, and  $O_{ave}$  and  $P_{ave}$  are mean values of observed and estimated respectively.

$$R = \left[ \frac{\sum_{i=1}^n (O_i - O_{ave})(P_i - P_{ave})}{\sqrt{\sum_{i=1}^n (O_i - O_{ave})^2 (P_i - P_{ave})^2}} \right]^{0.5} \quad (10)$$

$$RMSE = \left[ \frac{\sum_{i=1}^n (P_i - O_i)^2}{n} \right]^{0.5} \quad (11)$$

$$MAE = \sum_{i=1}^n [P_i - O_i] / n \quad (12)$$

$$RE = (MAE / O_{ave}) \times 100$$

$$Log_{10}^{DT} = \left[ n^{-1} \sum_{i=1}^n [Log_{10}(P_i / O_i)]^2 \right]^{0.5} \quad (13)$$

Empirical equations including Hazen, Kozeny-Carmen, Breyer, Slitcher, Terzaghi, Alyamani and Sen and USBR Equations have been evaluated.

### III. RESULTS

The values of different statistics of empirical formula were indicated in Table 2.

TABLE II. VALUES OF STATISTICS OF EMPIRICAL EQUATIONS ESTIMATE TO HYDRAULIC CONDUCTIVITY

Equation	Statistics				
	R <sup>a</sup>	RMSE	MAE	RE	DT
Hazen	0.656	53.33	47.65	195.50	2.948
Kozeny-Carmen	0.666	41.74	38.13	156.42	2.597
Breyer	0.637	56.19	48.28	198.04	2.948
<b>Slitcher</b>	<b>0.671</b>	<b>6.08</b>	<b>5.06</b>	<b>20.75</b>	<b>1.393</b>
Terzaghi	0.671	17.17	14.79	60.68	1.667
USBR	0.474	24.77	18.96	77.79	1.889
Alyamani and Sen	0.699	16.72	12.45	51.07	1.625

It is concluded that the slitcher equation was the best model with 0.671 R, 6.08 RMSE, 5.06 MAE, 20.75% RE and 1.393 DT. Also it is concluded that the Breyer equation predicted Ks, saturated hydraulic conductivity with high prediction error with 0.637 R, 56.19 RMSE, 48.28 MAE, 198.04% RE and 2.948 DT. Meanwhile, the high value of DT statistic indicates that the accuracy and efficiency of the model in estimating the saturated hydraulic conductivity is low.

The difference between measured values of saturated hydraulic conductivity with estimated values has been reported by various researchers [12, 21 & 22].

Odong (2007) evaluated the various empirical equations that predicted soil hydraulic conductivity and the results showed that the Kozeny-Carman equation was the best estimator among the studied equations. However, some of the equations underestimated or overestimated hydraulic conductivity [12].

It is concluded that the values of hydraulic conductivity calculated by the Slitcher equation is lower than the other equations, which is match with the results by Cheng and Chen (2007), Vukovic and Soro (1992) and Odong (2007) [10, 12 & 17]. Alyamani and Sen, Terzaghi and USBR equations calculated saturated hydraulic conductivity lower than Breyer, Hazen and Kozeny-Carmen equations.

It is concluded that the accuracy of hydraulic conductivity estimated by the Kozeny-Carmen equation is rather than the Hazen equation, which is consistent with the conclusions by Carrier (2003) [8].

#### IV. CONCLUSION

In this research various empirical formula to calculate hydraulic conductivity based on grain size distribution have been evaluated. The results showed that the slitcher equation was the best formula for predicting saturated hydraulic conductivity among studied equations. The results showed that the Breyer formula predicted saturated hydraulic conductivity with high prediction error. It is concluded that the values of hydraulic conductivity calculated by the Slitcher equation is lower than the other equations. Alyamani and Sen, Terzaghi and USBR equations calculated saturated hydraulic conductivity lower than Breyer, Hazen and Kozeny-Carmen equations.

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