J. Appl. Environ. Biol. Sci., 4(3)333-339, 2014

ISSN: 2090-4274
Journal of Applied Environmental and Biological Sciences
www.textroad.com

A Slack- Based Elasticity Measure in CRS and VRS Production Technologies

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Received: December 5 2013
Accepted: January 27 2014

ABSTRACT

Elasticity is one of the most important concepts in neoclassical economic theory. In this study a new slack-based elasticity measure is proposed and linear programs are provided to calculate it. In fact, we investigate the optimal scale size of efficient units from the mean of outputs points of view. One important property of this measure is applicable not only in variable technology but also in constant technology.

KEYWORDS: DEA, Elasticity, Returns to Scale (RTS), Technology Set**,** Environmental Efficiency.

1. INTRODUCTION

Data Envelopment Analysis (DEA) has been long serving as a methodology to evaluate the performance of various organizations in public and private sectors. Also, Environmental assessment recently becomes a major policy issue all over the world. In order to see the relationship between them, and some applications can be seen [6, 8, 9, 13, 14]. In data envelopment analysis, returns to scale (RTS) were first addressed by Banker [1] and Banker et al. [2]. Elasticity is one of the most important concepts in neoclassical economic theory. The elasticity measure, in the one input and one output case, is defined as the ratio of the marginal productivity to the average productivity at an efficient unit. In the multiple inputs and outputs case, Banker and Therall [3] defined the right-hand and left-hand elasticities as the minimum and maximum slopes of tangential lines to a plane section of the efficient frontier at a specific point on the frontier. According to the RTS definition, all points on the VRS technology can be partitioned into increasing returns to scale (IRS), decreasing returns to scale (DRS) and constant returns to scale (CRS) classes.

Hadjicostas and Soteriou [7] presented a theoretical framework that integrated existing economics and management science literature on RTS, and provided a solid foundation for research work in this area, defining the right-hand and left-hand elasticities as the one- sided derivatives of a convex and linear piecewise function and indicating that both definitions are equivalent.

The output response function is an optimal value function which assigns the maximum proportion of the output vector possible in a production possibility set, T, to a proportion of the input vector. The scale elasticity is defined as the ratio of its marginal productivity of the output response function (where it exists) to its average productivity [12].

The right-hand and left-hand elasticities are used to overcome the non-differentiability problem of the output response function. By replacing the output response function with another appropriate function, various scale elasticity measures have been obtained. As an example, if you consider the output response function as the maximum proportion of a specific single output possible in T, to a proportion of the input vector or a specific single input, you will be able to calculate the marginal rates of the corresponding output to the input vector or the corresponding input, respectively. See [12] for more details. Podinovski et al. [12] and Podinovski and Forsund [11] invoked the theorem of the directional derivative of the optimal value function and showed how this can be used to define and calculate the required elasticities without any simplifying assumptions. In fact, they found a mathematical framework to define and to calculate the different scale elasticity measures. Here, we are going to introduce the mean outputs response function which assigns the maximum of the mean of proportions of outputs possible in T to a proportion of the input vector. The main motivation behind this effort is to evaluate the scale of the boundary points from the mean of outputs point of view. Following Podinovski et al. [11] we also propose a new elasticity measure and subsequently a RTS definition based on this function in different technologies. It will be observed that a major part of DMUs are IRS in each technology according to the proposed RTS definition.

2. BACKGROUND

Let T_C and T_V denote the CRS (Constant Returns to Scale} and VRS (Variable Returns to Scale) production technologies were defined, respectively, by observed units $(X_j, Y_j), j = 1,2,\ldots,$n where $X_j \in R^s, Y_j \in R^m$ and

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both of them are nonzero and nonnegative vectors. Let $X = (x_j^T) \in R^{m\times n}$ and $Y = (y_j^T) \in R^{s\times n}$ are input and output matrixes, respectively.

In general, scale elasticity is studied in input and output orientation separately. In this paper ,we focus only on the output orientation.

Definition 1. $(x, y) \in T$ is called Pareto efficient if there is no point $(\bar{x}, \bar{y}) \in T$ such that $\bar{x} \leq x, \bar{y} \geq$ and $(x, y) \neq (\bar{x}, \bar{y})$.

Definition 2. (Cooper et al. 2006) The output-oriented CCR and BCC efficiency measures are defined as follows, respectively:

$$\Phi_o = Max\,\{\Phi : (x_o, \Phi y_o) \in T_C\}$$
$$\varphi_o = Max\,\{\varphi : (x_o, \varphi y_o) \in T_V\}$$

Where

$$T_C = \{(x, y) : X\lambda \leq x, Y\lambda \geq y, \lambda \geq 0\}$$
$$T_V = \{(x, y) : X\lambda \leq x, Y\lambda \geq y, e\lambda = 1, \lambda \geq 0\}$$

Consider the following optimal value function:

$$\begin{aligned} f(b) &= Max\ cx \\ s.t.\ Ax &= b \\ x &\geq 0 \end{aligned} \tag{2.1}$$

Where $A \in R^{m\times s}$, $b \in R^m$. We have from linear programming that $f: dom(f) \to R$ is a piecewise linear convex function such that $dom(f) = \{b \in R^m : the\ model\ is\ feasible\}$ (Theorem 8.9 of [10]).

Definition 3. ([4]) Let $E \subseteq R^m$ and $f: E \to R$ is a convex function. We say that a vector $\xi \in R^n$ is a subgradient of f at point $x \in E$ if

$f(z) \geq f(x) + (z - x)^t d, \forall z \in T.$

The set of all subgradient vector of f at $x \in R^m$ is called the sub-differential set of f at x and is denoted by $\partial f(x)$.

Theorem 4. For $b \in dom(f)$, $\partial f(x) = \{\pi : \pi\ is\ an\ dual\ optimal\ solution\ of\ (2.1)\}$.

Recall from convex analysis that f is differentiable at b if and only if $\partial f(x)$ is singleton.

It said to vector $d \in R^m$ is a feasible direction at point $x \in dom(f)$ if there is a $\delta_0 > 0\ such\ that\ x + \gamma\, d \in dom(f)\ \forall\, 0 \leq \gamma \leq \delta_0$.

Definition 5. Let vector $d \in R^m$ is a feasible direction at point $x_0 \in dom(f)$. The *directional derivative* of f at x_0 in the direction d is defined by

$$f'(x_0, d) = lim_{\{\delta \to 0\}} \frac{f(x_0 + \delta\, d) - f(x\_0)}{\delta}$$

Proposition 6. ([4]) For any $b \in dom(f)$ and feasible direction d,we have:

$$f'(b, d) = max_{\{\xi \in \partial f(b)\}}\ \xi^t . d \tag{2.2}$$

Assume $(x_o, y_o) \in T$ and $\beta \geq 0$. Define $\Delta = \{\alpha : \exists\, \beta \in R^+ : (\alpha x_o, \beta y_o) \in T\}$. It is easy to see that $\Delta = [\bar{\alpha}, +\infty)$. For $\alpha \in \Delta$, the *output response function* is defined as follows:

$$\beta(\alpha) = max\,\{\beta : (\alpha x_o, \beta x_o) \in T_v\} \tag{2.3}$$

Now suppose that

$$h(b) = max\,\{\beta : X\lambda \leq b_1^t;\ Y\lambda - \beta y_o \geq 0^t\ ; e\lambda = 1; \lambda \geq 0\} \tag{2.4}$$

Where $b_1 \in R^m, 0 \in R^s$ and $b = (b_1, 0,1)$.

Let $b_0 = (x_o, 0,1)$ and $\alpha = 1 + \delta$ where $\delta \geq 0$. With respect to the definition of the *output response function*, it follows that

$\beta(1) = h(b_0)$ and $\beta(\alpha) = h(b_0 + \delta\, d);\ d = (x_o, 0,0)$.

Note that $\beta : \Delta \to R$ is a piecewise linear convex function of the parameter α (Theorem 8.4 of Murty 1983).

By substituting the above equalities in the right-hand derivative rule of β at 1, we obtain

$$\begin{cases} \beta'_+(1) = h'(b_0; d),\ d = (x_o, 0,1) \\ \beta'_-(1) = -h'(b_0; d), d = -(x_o, 0,1) \end{cases} \tag{2.5}$$

From the Theorem (4), we have $\partial h(b)$ is a nonempty, convex and compact set. So, $h'(b, d)$ always exists and is finite.

Corollary 7. The $\beta'_+(1)$ always exists and finite however $\beta'_-(1)$ exists and finite only if that $d = -(x_o, 0,1)$ is a feasible direction.

Definition 8. ([12]) The scale elasticity $\varepsilon(x, y)$ at any point $(x, y) = (\alpha x_o, \beta(\alpha) y_o)$ is defined as follows:

$$\varepsilon(x, y) = \acute{\beta}(\alpha) \frac{\alpha}{\beta(\alpha)}$$

provided that $\acute{\beta}(\alpha)$ exists and be finite. In particular, at the efficient unit (x_o, y_o), we have $\varepsilon(x_o, y_o) = \acute{\beta}(1)$.

Definition 9. (Banker and Thrall 1992) Suppose that $\beta(1) = 1$ and $1 \in int(\Delta)$. We define output-oriented RTS as follows:

- IRS prevail at (x_o, y_o) if $\varepsilon_-(x_o, y_o) \geq \varepsilon_+(x_o, y_o) > 1$.
- DRS prevail at (x_o, y_o) if $1 > \varepsilon_-(x_o, y_o) \geq \varepsilon_+(x_o, y_o)$.

- CRS prevail at (x_o, y_o) if $\varepsilon_-\ (x_o, y_o) \geq\ 1 \geq \varepsilon_+\ (x_o, y_o)$.

3. A slack- based RTS

The main goal is to estimate the response of each outputs to the proportion change of the input vector. We attempt to calculate the response of the mean of outputs. Suppose that $(x_o, y_o) \in T$. Define $\tilde{\Delta} = \{\alpha: \exists\ , \tilde{\beta} \in R_+^s, and, \tilde{\beta} \geq \mathbf{1},\ s.t.\left(\alpha x_o, \tilde{\beta} \otimes y_o\right) \in T\}$, where β_r is the response of the rth output, $\tilde{\beta} = (\beta_1, \beta_2, \dots, \beta_s)$, $1 = (1,1, \dots ,1) \in R_+^s$ and $\left(\tilde{\beta} \otimes y_o\right)^t = (\beta_1\ y_{1o}, \beta_2 y_{2o}, \dots, \beta_s y_{so})$.
It is easy to see that $\tilde{\Delta} = [\tilde{\alpha}, +\infty)$ that contains 1 too. Let $\beta_r = 1 + s_{ro}^+$ where $s_{ro}^+ \geq\ 0$ for $r \in\ O$. Then

$$\beta_r\ y_{ro} = y_{ro} + s_{ro}^+ y_{ro} = 1 + s_{ro} \quad , for\ \ r \in\ O$$

Where $s_{ro} = s_{ro}^+ y_{ro}$ and $\frac{1}{s}\sum_{r=1}^{s}\beta_r\ =\ 1 + \frac{1}{s}\sum_{r=1}^{s}\frac{s_{ro}}{y_{ro}}$.
Following that, we investigate the elasticity of this slack- based measure in CRS and VRS technologies.

3.1 CRS Technology

Suppose that $(x_o, y_o) \in T_c$.We define $\Delta_c = \{\alpha: \exists \in R_+^s\ s.t.(\alpha x_o, y_o - s) \in T_c\}$, the output response function can be determined as the optimum value of the following linear program for $\alpha \in \Delta_c$:

$$\gamma_c(\alpha) = 1 + max\frac{1}{s}\sum_{r=1}^{s} s_r / y_{ro}$$

$$s.t. \sum_{j=1}^{n}\lambda_j x_{ij} \leq \alpha x_{io},\ \ i \in I$$

$$\sum_{j=1}^{n}\lambda_j y_{rj} - s_{ro} = y_{ro},\ \ r \in O$$

$$\lambda_j \geq 0, s_r \geq 0,\ \forall j, r \tag{3.1}$$

In the above program ,vector s is the variable used for optimization, and α is a parameter kept constant while the optimization is performed. The corresponding dual problem for $\alpha = 1$ is as follows:

$$\gamma_c(1) = 1 + Min\ (\sum_{i=1}^{m} v_i x_{io}\ - \sum_{r=1}^{s} u_r y_{ro})$$

$$s.t.\ \ \sum_{i=1}^{m} v_i x_{ij}\ - \sum_{r=1}^{s} u_r y_{rj}\ \geq\ 0, \qquad j \in\ J$$

$$u_r \geq \frac{1}{s y_{ro}}, \qquad r \in\ O$$

$$v_i \geq\ 0, \quad i \in\ I \tag{3.2}$$

Let Ω_c^o to be the optimal solution set of the model for $\alpha = 1$, corresponding to the unit (x_o, y_o).
By applying Theorem (4) and Proposition (6), one-sided derivatives $\gamma_c'^{+}(1)$ and $\gamma_c'^{-}(1)$ can be determined as follows, provided that $\gamma_c\ (1) = 1$ and $1 \in \Delta_c$:

$$\gamma_c'^{+}(1) = \ Min\ VX_o$$
$$s.t.\ \ (U, V) \in\ \ \Omega_c^o \tag{3.3}$$

And

$$\gamma_c'^{-}(1) = \ Max\ VX_o$$
$$s.t.\ \ (U, V) \in\ \ \Omega_c^o \tag{3.4}$$

Obviously, $\gamma_c'\ (1)$ exists and is finite if and only if $\gamma_c'^{+}(1) = \ \gamma_c'^{-}(1)$.
Definition 10. Let $(x_o, y_o) \in T_c$ and $\gamma_c(1) = 1$. If $\gamma_c{}'\ (1)$ exists and be finite, the mean scale elasticity $\varepsilon_c\ (x_o, y_o)$ is defined as follows:

$$\varepsilon_c(x_o, y_o) = \gamma_c{}'\ (1) \tag{3.5}$$

Definition 11. Let $(x_o, y_o) \in T_c$ and $\gamma_c(1) = 1$. If $\gamma'\ (1)$ is not exist, the one sided mean scale elasticities $\varepsilon_c^+\ (x_o, y_o)$ and $\varepsilon_c^-\ (x_o, y_o)$ are defined as follows:

$$\varepsilon_c^+\ (x_o, y_o) = \ \gamma_c'^{+}(1)$$
$$\varepsilon_c^-\ (x_o, y_o) = \ \gamma_c'^{-}(1),\ \ if\ 1 \in int(\Delta_c)\ . \tag{3.6}$$

Definition 11. (Output-Oriented Mean RTS (MRTS)) Suppose that $\gamma(1) = 1$ and $1 \in\ int\Delta_c)$. We define Output-Oriented Mean RTS (MRTS) as follows:

- Increasing MRTS prevail at (x_o, y_o) if $\varepsilon_c^-\ (x_o, y_o) \geq \varepsilon_c^+\ (x_o, y_o) >\ 1$.
- Decreasing MRTS prevail at (x_o, y_o) if $1 > \varepsilon_c^-\ (x_o, y_o) \geq \varepsilon_c^+\ (x_o, y_o)$.
- Constant MRTS prevail at (x_o, y_o) if $\varepsilon_c^-\ (x_o, y_o) \geq 1 \geq\ \varepsilon_c^+\ (x_o, y_o)$.

Notice that if 1 is a boundary point of Δ_c, we define MRTS only based on $\varepsilon_c^+ \ (x_o, y_o)$ similarly.
Proposition 12. Let $(x_o, y_o) \in T_c$ and $\gamma_c(1) = 1$. We always have $\varepsilon_c^+ \ (x_o, y_o) \geq 1$.
Proof: Let $(u, v) \in \Omega_c^o$. The constraint set $u_r \geq \frac{1}{sy_{ro}}, r \in O$ in Ω_o^c would guarantee that $\sum_{r=1}^{s} u_r y_{ro} \geq 1$. By the hypotheses$\gamma_c(1) = 1$, we have $vx_o \geq 1$. Hence $\gamma_c'^{+} = Min_{\Omega_c^o} vx_o \geq 1$ and this complete the proof.
Corollary 12. Let (x_o, y_o) T_c and $\gamma_c(1) = 1$. It is not possible that decreasing MRTS prevail at (x_o, y_o).
Consider the following model which has obtained by deleting the non-negativity constraints of s_r, $r \in O$ in the model(3.2).

$$\overline{\gamma_c}(1) = 1 + Min\left(\sum_{i=1}^{m} v_i x_{io} - \sum_{r=1}^{s} u_r y_{ro}\right)$$
$$s.t. \quad \sum_{i=1}^{m} v_i x_{ij} - \sum_{r=1}^{s} u_r y_{rj} \geq 0, \qquad j \in J$$
$$u_r = \frac{1}{sy_{ro}}, \qquad r \in O$$
$$v_i \geq 0, \quad i \in I \tag{3.7}$$

Proposition 13. If $\bar{\gamma} = 1 \ for \ (x_o, y_o)$, then constant MRTS prevail at (x_o, y_o).
Proof: It is easy to see that $\bar{\gamma} \geq \gamma(1)$ and so $\gamma(1) = 1$. On the other hand, if $(\bar{u}, \bar{v})$ is an optimal solution of the model (3.7), then $(\bar{u}, \bar{v}) \in \Omega_c^o$ where $\bar{V}X_o = \bar{U}Y_o = 1$. Due to $vx_o \geq 1$ over the Ω_c^o, then $\varepsilon_+(x_o, y_o) = 1$.
According to the corollary (7), we are able to identify the MRTS nature of each boundary point of T_c only by solving the model (3.7).
Example 1: In this example, we consider 5 units with one input and two outputs in CRS technology. The data set are displayed in Table 1.

Table 1. Input and Output date for DMUs in the Example1

DMUs	Input	Output1	Outpu2	CCR efficiency
A	1	1	4	0.80
B	1	1.5	4	1
C	1	3	3.5	1
D	1	4	2	1
E	1	4	1	0.66

Table 2 shows the obtained results of applying the models (SBM efficiency in output-oriente [5]), (3.3), (3.4) (right-hand and left-hand elasticities, respectively) and (3.7).

Table 2. The results obtained by the proposed models for units in the Example 1

DMUs	$\gamma(1)$	$\bar{\gamma}(1)$	$\gamma_c'^{+}(1)$	$\gamma_c'^{-}(1)$	MR TS	
A	1.25	-	-	-		
B	1.00	1.58	1.58	1.58		IMRTS
C	1.00	1.00	1.00	1.00		CMRTS
D	1.00	1.25	1.25	1.68		IMRTS
E	1.50	-	-	-		

As you can see, the units B, C and D are efficient. Regarding Proposition (6) and Definition MRTS, Unit B is CMRTS and C and D are IMRTS. With regard to the output values and Figure 1, this result was expected.

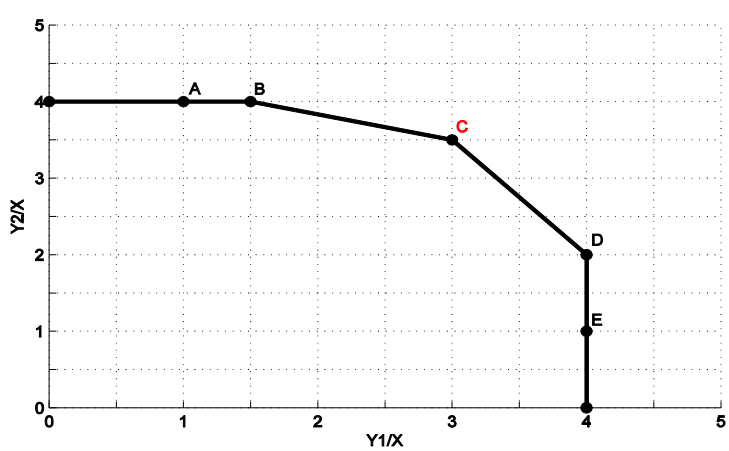

Figure 1. Farrell frontier for data set of DMUs in the Example 1

3.2 VRS Technology

In this section, we investigate the slack- based elasticity measure over the T_c. Suppose that $(x_o, y_o) \in T_c$. Define $\Delta_v = \{\alpha : \exists\, s \in R_+^s\ s.t.\ (\alpha x_o, y_o - s) \in T_v\}$ for $\alpha \in \Delta_v\}$, consider the following *optimal value function*:

$$\gamma_v(\alpha) = 1 + max \frac{1}{s} \sum_{r=1}^{s} s_r / y_{ro}$$
$$s.t. \sum_{j=1}^{n} \lambda_j x_{ij} \le \alpha x_{io}, \quad i \in I$$
$$\sum_{j=1}^{n} \lambda_j y_{rj} - s_{ro} = y_{ro}, \quad r \in O$$
$$\lambda_j \ge 0, s_r \ge 0, \ \forall j, r \tag{3.8}$$

The corresponding dual problem is as follows:

$$\gamma_v(1) = 1 + Min\ (\sum_{i=1}^{m} v_i x_{io} - \sum_{r=1}^{s} u_r y_{ro} + \sigma)$$
$$s.t. \quad \sum_{i=1}^{m} v_i x_{ij} - \sum_{r=1}^{s} u_r y_{rj} + \sigma \ \ge\ 0, \qquad j \in J$$
$$u_r \ge \frac{1}{s y_{ro}}, \qquad r \in O$$
$$v_i \ge 0, \quad i \in I \tag{3.9}$$

Let Ω_v^o to be the optimal solution set of the model for $\alpha = 1$, corresponding to the unit (x_o, y_o).
Similarly, one-sided derivatives $\gamma_v'^{+}(1)$ and $\gamma_v'^{-}(1)$ can be determined as follows, provided that $\gamma_v\ (1) = 1$ and $1 \in \Delta_v$:

$$\gamma_v'^{+}(1) = Min\ VX_o$$
$$s.t.\ (U, V) \in \ \Omega_v^o \tag{3.10}$$

And

$$\gamma_v'^{-}(1) = Max\ VX_o$$
$$s.t.\ (U, V) \in \ \Omega_v^o \tag{3.11}$$

Obviously, $\gamma_c'\ (1)$ exists and is finite if and only if $\gamma_v'^{+}(1) = \ \gamma_v'^{-}(1)$.

Definition 14. Let $(x_o, y_o) \in T_v$ and $\gamma_v(1) = 1$. If $\gamma_v{}'\ (1)$ exists and be finite, the mean scale elasticity $\varepsilon_c\ (x_o, y_o)$ is defined as follows:

$$\varepsilon_v(x_o, y_o) = \gamma_v{}'\ (1) \tag{3.12}$$

Definition 15. Let $(x_o, y_o) \in T_v$ and $\gamma_v(1) = 1$. If $\gamma'\ (1)$ is not exist, the one sided mean scale elasticities $\varepsilon_v^+\ (x_o, y_o)$ and $\varepsilon_v^-\ (x_o, y_o)$ are defined as follows:

$$\varepsilon_v^+\ (x_o, y_o) = \ \gamma_v'^{+}(1)$$
$$\varepsilon_v^-\ (x_o, y_o) = \ \gamma_v'^{-}(1), \ if\ 1 \in int(\Delta_v)\ . \tag{3.13}$$

Definition 11. (Output-Oriented Mean RTS (MRTS)) Suppose that $\gamma(1) = 1$ and $1 \in int\Delta_v)$. We define Output-Oriented Mean RTS (MRTS) as follows:

- Increasing MRTS prevail at (x_o, y_o) if $\varepsilon_v^-\ (x_o, y_o) \ge \varepsilon_v^+\ (x_o, y_o) > \ 1$.
- Decreasing MRTS prevail at (x_o, y_o) if $1 > \varepsilon_v^-\ (x_o, y_o) \ge \varepsilon_v^+\ (x_o, y_o)$.
- Constant MRTS prevail at (x_o, y_o) if $\varepsilon_v^-\ (x_o, y_o) \ge 1 \ge \ \varepsilon_v^+\ (x_o, y_o)$.

Notice that if 1 is a boundary point of Δ_v, we define MRTS only based on $\varepsilon_v^+\ (x_o, y_o)$ similarly.

Proposition 12. Let $(x_o, y_o) \in\ T_c$ and $\gamma_c(1) = 1$. We always have $\varepsilon_c^+\ (x_o, y_o) \ge 1$.

Proof: Let $(\bar{u}, \bar{v}, \bar{\sigma}) \in \Omega_o^v$. It is easy to verify that $(\frac{\bar{u}}{\bar{u}y_o}, \frac{\bar{v}}{\bar{u}y_o}, \frac{\sigma}{\bar{u}y_o})$ is an optimal solution for the multiplier form of the output-oriented BBC model. Now if $\varepsilon_+\ (x_o, y_o) \ge\ 1$, then $\bar{\sigma} < 0$ by the Theorem 5.2 of [5]. Hence,

$$\bar{v}x_o = \bar{u}y_o - \sigma$$
$$> \bar{u}y_o$$
$$\ge 1$$

and finally, $\gamma_v'^{+}(1) = \ Min_{\Omega_o^v} vx_o \ge 1$ and this complete the proof.

Example 2: In this example, we consider 4 units A,B,C and D with one input and two outputs in VRS technology. The data set are displayed in Table 3. Also, Table 4 shows the obtained results of applying the proposed models and the resulting technology is represented in Figure2.

Table 3. Input and Output date for DMUs in the Example 2

DMUs	Input	Output1	Outpu2	BBC efficiency	RTS
A	1.00	3.00	2.00	1	CRS
B	1.25	3.00	4.00	1	CRS
C	1.50	4.50	3.00	1	CRS
D	1.75	1.87	4.50	1	DRS

Table 4. The results obtained by the proposed model for units in the Example 2

DMUs	$\gamma(1)$	$\gamma_c'^{+}(1)$	$\gamma_c'^{-}(1)$	MRTS
A	1.00	2.00	2.00	IMRTS
B	1.00	0.00	1.55	CMRTS
C	1.00	0.00	2.00	CMRTS
D	1.00	0.00	0.00	DMRTS

The CCR efficient frontier is the triangle ABC, which includes the three original units, and the unit D is BCC efficient. Using the results in Table 2, we can improve unites A and D by increasing and deceasing their scale size, respectively. Figure 2 confirms this as well.
The CCR efficient frontier is the triangle ABC, which includes the three original units, and the unit D is BCC efficient. Using the results in Table 2, we can improve unites A and D by increasing and deceasing their scale size, respectively. Figure 2 confirms this as well.

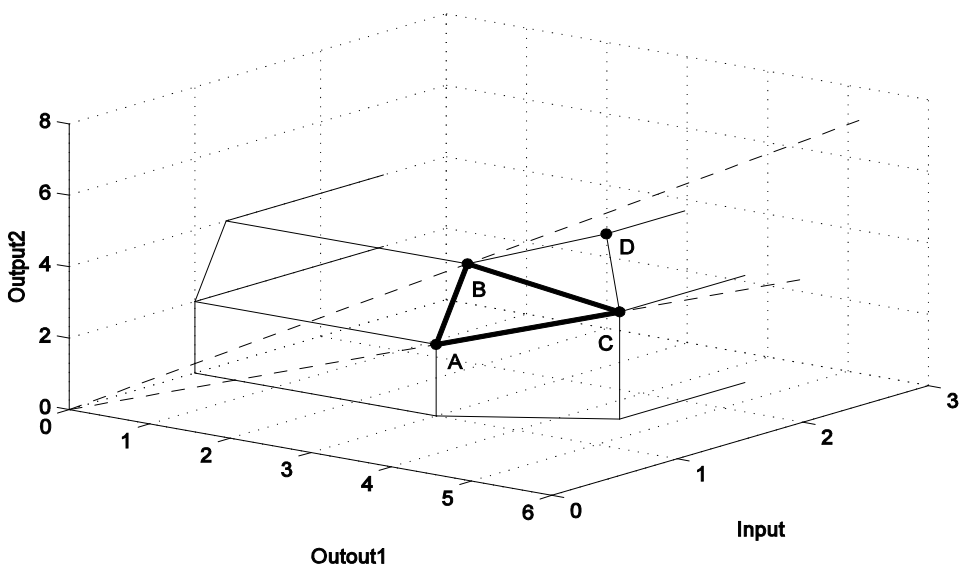

Figure 2. Production Possibility Set (T_v) for data set of Example 2

Results obtained using the propose definition to unit D is quite consistent with the results of classic definition of RTS. But, as for unit A, our approach reveals some shortage in unit A to achieve proper size (CMRTS).

4. CONCLUSION

In this study, we proposed a new elasticity measure in CRS and VRS production technologies and provided linear program models for the calculation of elasticity measures. In addition, a new RTS definition has also been proposed. It is observed that the CRS points themselves are classified as DRS, CRS or IRS according to the proposed RTS definition. However, we implied the elasticity measure in CRS and VRS production technologies, it can be applied to NIRS and NDRS production technologies as well.

Acknowledgment

The authors declare that they have no conflicts of interest in the research.

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