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Improving the Performance of Hu Moments for Shape Recognition

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ABSTRACT

In this paper, we show that the performance of invariant moments in shape recognition can be significantly improved. This is achieved without any additional computational cost by taking the moments of 3 to 4 annuli around the center of gravity of a given picture. Our idea is applied to different datasets of leaves and achieves in some cases an improvement of the recognition rate from 57% to 84%.

KEYWORDS: Invariant moments, Hu moments, Chen improved moments, shape recognition, neural networks

1 INTRODUCTION

Invariant moments are by now more than 50 years old. In 1962, Hu [1] introduced 7 invariants that he derived using the theory of algebraic invariants and that later became known as Hu's moments. Six of these moments are given by the expressions:

$$
\begin{aligned}
&\phi_1 = \mu_{20} + \mu_{02}, \\
&\phi_2 = (\mu_{20} - \mu_{02})^2 + 4\mu_{11}^2 \\
&\phi_3 = (\mu_{30} - 3\mu_{12})^2 + (3\mu_{21} - \mu_{03})^2 \qquad (1) \\
&\phi_4 = (\mu_{30} + \mu_{12})^2 + (\mu_{21} + \mu_{03})^2 \\
&\phi_5 = (\mu_{30} - 3\mu_{12})(\mu_{30} + \mu_{12})\,[(\mu_{30} + \mu_{12})^2 - 3(\mu_{21} + \mu_{03})^2] - (\mu_{03} - 3\mu_{21})(\mu_{03} + \mu_{21})\,[3(\mu_{30} + \mu_{12})^2 - (\mu_{21} + \mu_{03})^2] \\
&\phi_6 = (\mu_{20} - \mu_{02})\,[(\mu_{30} + \mu_{12})^2 - (\mu_{21} + \mu_{03})^2] + 4\mu_{11}(\mu_{03} + \mu_{21})(\mu_{30} + \mu_{12})
\end{aligned}
$$

where

$$\mu_{pq} = \mu_{pq}(\Omega) = \frac{\iint_\Omega (x - x_c)^p (y - y_c)^q \, dxdy}{\left(\iint_\Omega dxdy\right)^{\frac{(p+q+2)}{2}}} \qquad (2)$$

is the central moment of the domain Ω and (x_c, y_c) are the coordinates of the center of gravity of Ω. Hu [1], showed that these computationally simple to obtain moments are invariant to rotation, scaling and translation and have the discriminative power to be a reliable tool for shape recognition. According to [3] and [4], Hu's moments are considered as a numerical, global and non-preserving technique for shape recognition as it uses all of the pixels of a picture (rather than just the boundary) and from which one cannot recover the original picture. This is why many researchers sought to generalize and improve Hu's moments into a complete theory of invariant moments that is now fully and beautifully summarized in the recent book by Flusser et. al [2]. In fact, in order to remedy the redundancy of Hu's moments and their inability to recover the initial shape of the domain, many turned to orthogonal moments that replace the expression of $(x-x_c)^p(y-y_c)^q$ in (2) by $P_m(x)P_n(y)$ where the family $\{P_m(.)\}_{m=0,\,1,\,2,\ldots}$ is a familyof orthogonal polynomials thus leading to the birth of Zernike moments [5-6], Pseudo-Zernicke moments [7], Legendre moments [8-9], Chebychev moments [10-11], Fourier-Mellin moments [13], Chebychev-Fourier [14] and radial harmonic Fourier moments. Besides removing the redundancy in the original Hu's moments, these invariant moments based on orthogonal polynomials are relatively fast and stable and allow the user to

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recover the shape of the domain in the same way a periodic function is recovered from its Fourier coefficients. One should caution here that the ability of reconstructing a shape from its orthogonal moments is rather limited compared to the standard techniques such as wavelets [2].

Since the original work of Hu did not leave much of an insight on how to obtain higher order moments many authors turned to the complex moments of the form

$$c_{pq} = \iint_{\Omega} (x+iy)^p (x-iy)^q \, dxdy \qquad (3)$$

for domains centered at the origin. Thus many authors were able to obtain higher order moments using different approaches [2], [16-21]. However, except for the work of Flusser [2, 20, 21] many of the obtained higher moments were either incomplete or interdependent. In fact, as it turns out, the original Hu moments are dependent and one can express ϕ_3 in terms of ϕ_4, ϕ_5, and ϕ_7 (see [2] for example) and this is the reason we only listed 6 out of the 7 original moments in equation (1).

Hu's moments and complex moments carry most of the information about a shape in their first few terms, in the same way the first few terms of a series expansion yield a good approximation about a given function. Moreover, one can easily see from equations (2) and (3) that both moments give more weight to pixels that are farthest from the center of gravity of the picture which is mostly a good thing because most of the information about a shape is located on its boundary. In fact, Chen [22] generated moments that are computed only on the boundary of the shape with reasonable success. However, the unequal weight given to pixels tends to give an unfair emphasis of extracted features on portions of the domain and yield a negative impact on the discrimination ability of the moments [23]. In addition, when the resolution of a picture is below a certain threshold moment invariants tend to fluctuate wildly when scaled and rotated (from 47% to 124%) especially for high order moments [24]. The purpose of this paper is to show that this problem can be remedied by simply computing the moments of different annuli whose union constitutes the totality of the shape.

The rest of the paper is organized as follows: In Section II, we explain our method and show that the moments on the annuli are indeed scale and rotation invariant. Section III describes the different data sets that we used and the results of our experiments with a comparison of the recognition rates obtained using our idea versus those of standard methods. Finally Section IV concludes the paper.

2. ANNULI MOMENT INVARIANTS

Suppose that the domain $\Omega$is the disjoint union of *n* subdomains $\Omega_1,\Omega_2, \ldots,\Omega_n$ then it is obvious that the quantities $\mu_{pq}(\Omega_i)$ are rotation and scale invariant. In fact, the domain specific $\phi_j(\Omega_i)$ specific to each subdomain are also rotation invariant. The only remaining question is how to choose to divide the domain Ω into subdomains Ω_i's in a systematic way that is also translation, scaling and rotation invariant. A possible way to achieve this goal is to

a) Define *R* the radius of Ω as the largest distance from the center of gravity of $\Omega$to the boundary of Ω.
b) Consider a finite sequence of positive numbers $1=\alpha_n>\alpha_{n-1}>\alpha_{n-2}>\ldots>\alpha_1>\alpha_0=0$.
c) For *i=1 to n*,
    - Let A_i be the annuli centered at (x_c,y_c) the center of gravity of Ω of inner radius $(\alpha_{i-1}R)$ and outer radius $(\alpha_i R)$.
    - Take $\Omega_i=\Omega \bigcap A_i$

Clearly, if *n=1*, then we are dealing with the regular Hu moments. If on the other hand *n* is too large, then each of the subdomains $\Omega_i$would only contain a few pixels and we would run the risk of representing a shape with a high dimensional feature vector with all the problems that can come with it. Figure 1.1 shows an example of a domain divided into 3 subdomains according to the above algorithm.

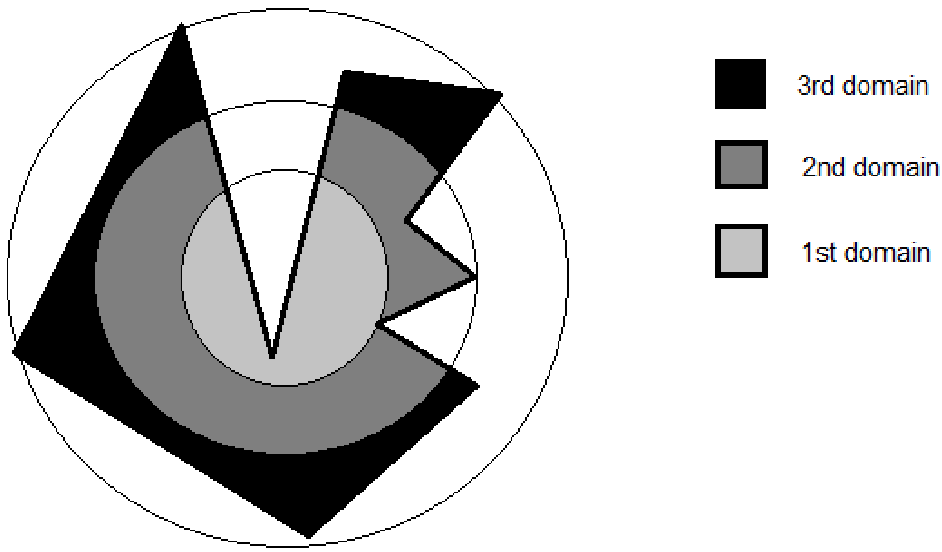

Figure1.1 Dividing a domain into 3 subdomains.

To show the invariance of the moments with respect to scaling and rotation, Table 1.1 presents the moments for each of the subdomains of the leaf in Figure 1.2 in comparison with those computed when the picture is scaled down and up by 20% and 40% respectively and when the picture was rotated by 45 degrees. Clearly, almost all moments of each subdomain are consistent except for the 7^{th} moment.

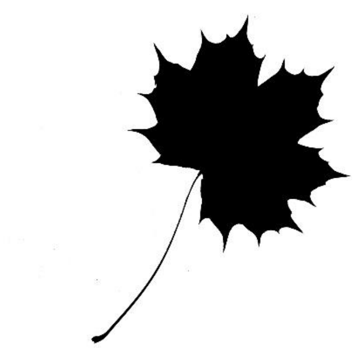

Figure 1.2 (from the Swedish leaves dataset [25])

Moment	1			2			3			4			6			7		
Annulus	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
Original	0.173	0.0044	0.0145	0.00117	1.9E-05	0.0002	6E-05	3E-05	0.0005	6E-06	3E-05	0.0005	-8E-08	1E-07	7E-06	-1.2E-10	1.8E-13	2E-10
80% Size	0.173	0.0044	0.0146	0.00117	1.9E-05	0.0002	6E-05	3E-05	0.0005	6E-06	3E-05	0.0005	-8E-08	1E-07	7E-06	-1.2E-10	-3.2E-13	2E-10
140% Size	0.173	0.0044	0.0145	0.00117	1.9E-05	0.0002	6E-05	3E-05	0.0004	6E-06	3E-05	0.0005	-8E-08	1E-07	7E-06	-1.2E-10	-9.4E-14	2E-10
45 Degree	0.1729	0.0044	0.0145	0.00117	1.9E-05	0.0002	6E-05	3E-05	0.0005	6E-06	3E-05	0.0005	-8E-08	1E-07	7E-06	-1.2E-10	9.5E-14	1E-10

Table1.1: Annuli Moments for Figure 1.2 scaled and rotated

In order to show the relative stability of the annuli invariant moments, we considered two classes of leaves from the Swedish Leaf Dataset. Each class contains 75 different pictures each for which we computed 6 moments of the annuli for these leaves. The averages and standard deviations of the 6 moments for each of the annuli are given in Table 1.2 for the leaf class 1 and Table 1.3 for leaf class 2. Both tables clearly show that the first moment of the first inside annulus is very stable for both classes of pictures. As the degree of the moment increases, the relative size of the standard deviation compared to the mean also increases. This is quite expected as this is a common problem with moment invariants. Moreover, as one moves farther from the center of gravity (to the outer annuli) the ratio σ/μ also increases reflecting the fact that the effect of fluctuations and noise are over magnified as we move away from the center.

		Moment Number for Leaf Class 1					
		1	2	3	4	6	7
Annulus1	Average	0.139122427	0.002184826	9.1615E-06	5.2682E-06	2.3690E-07	-6.1848E-12
	STDev	0.02005192	0.001703412	1.5412E-05	5.3647E-06	2.9642E-07	7.46178E-11
Annulus2	Average	0.034379534	0.00103974	1.6885E-05	1.8025E-05	5.1872E-07	-2.5324E-12
	STDev	0.012023417	0.000529779	1.6498E-05	1.9208E-05	6.0793E-07	4.64824E-11
Annulus3	Average	0.006411139	8.20494E-05	4.5733E-06	3.9907E-06	3.9530E-08	6.59851E-13
	STDev	0.007154877	0.00017011	7.5496E-06	7.9759E-06	1.3564E-07	3.49737E-12

Table1.2: Annuli Moments for Figure 1.2 scaled and rotated

		Moment Number for Leaf Class 2					
		1	2	3	4	6	7
Annulus1	Average	0.175781662	0.005901582	0.0087415	0.000775685	3.05546E-05	0.000122693
	STDev	0.005025624	0.003174222	0.006866421	0.000474111	3.38665E-05	0.000185802
Annulus2	Average	0.005901582	0.0087415	0.000775685	3.05546E-05	0.000122693	0.000102115
	STDev	0.003174222	0.006866421	0.000474111	3.38665E-05	0.000185802	9.08174E-05
Annulus3	Average	0.0087415	0.000775685	3.05546E-05	0.000122693	0.000102115	5.27962E-05
	STDev	0.006866421	0.000474111	3.38665E-05	0.000185802	9.08174E-05	7.31053E-05

Table1.3: Annuli Moments for Figure 1.2 scaled and rotated

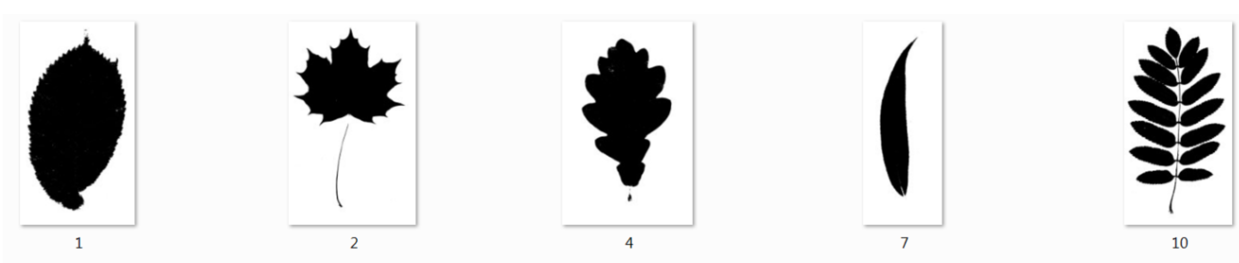

Figure 3.1 Easy dataset

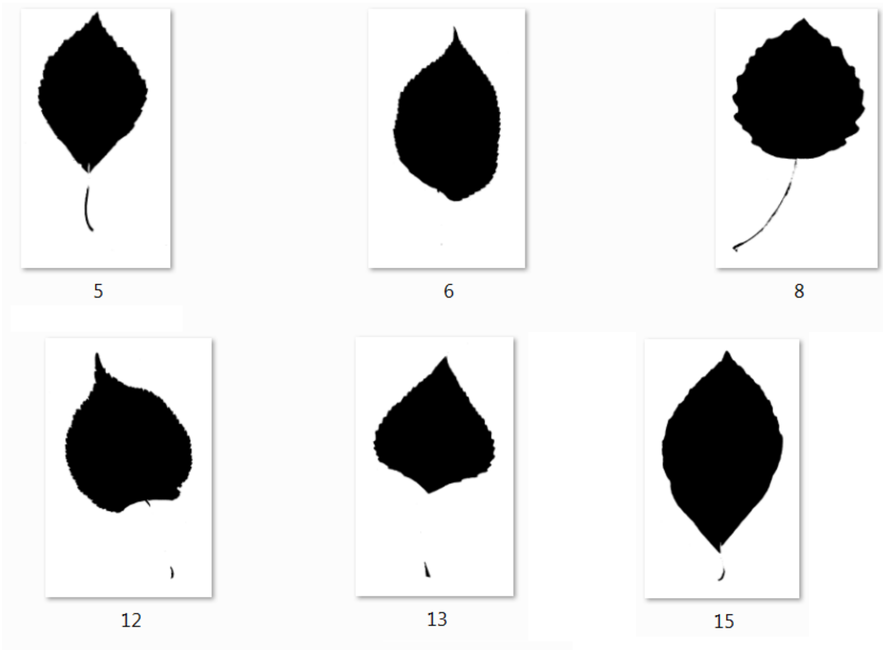

Figure 3.2 Medium dataset

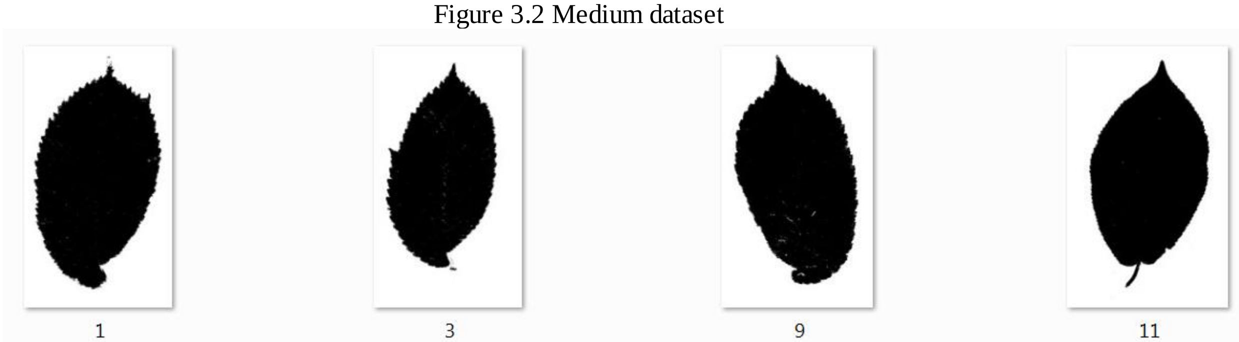

Figure 3.3 Hard dataset

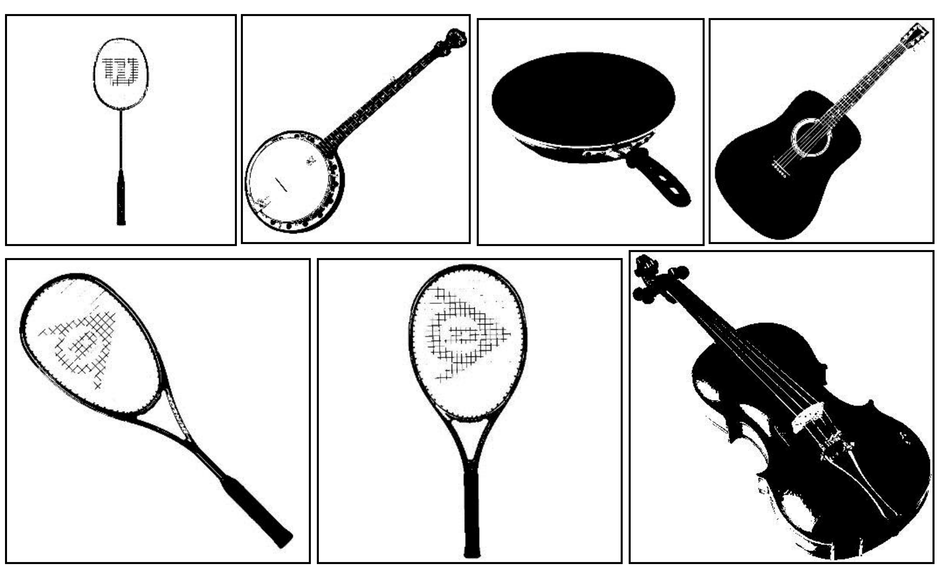

Figure 3.4 Objects with handles

1. DATASETS & RESULTS

For our experiments we use four different sets. The first three sets (fig 3.1-3) are datasets of leaves taken from the Swedish leaf dataset. The first set Fig 3.1 consists of 5 different classes each containing 75 different pictures of leaves. The original pictures are color pictures that we transformed into black and white pictures. We called the first set 'Easy" because the classes are easily distinguishable by humans. The second set that we worked with, is the "medium" set given in Fig 3.2. It also contains 5 classes of leaves with 75 pictures each. The third set consists of 4 classes that are hard to distinguish especially when the pictures were transformed into black and white. The last dataset is a collection of objects with handles and contains 7 classes of pictures of tennis, badminton and squash rackets, banjos, guitars, violins and frying pans. Each class of these objects with handles contains 35-40 pictures each all downloaded from the internet. Figure 3.5 gives an idea about the variability of pictures in the squash class.

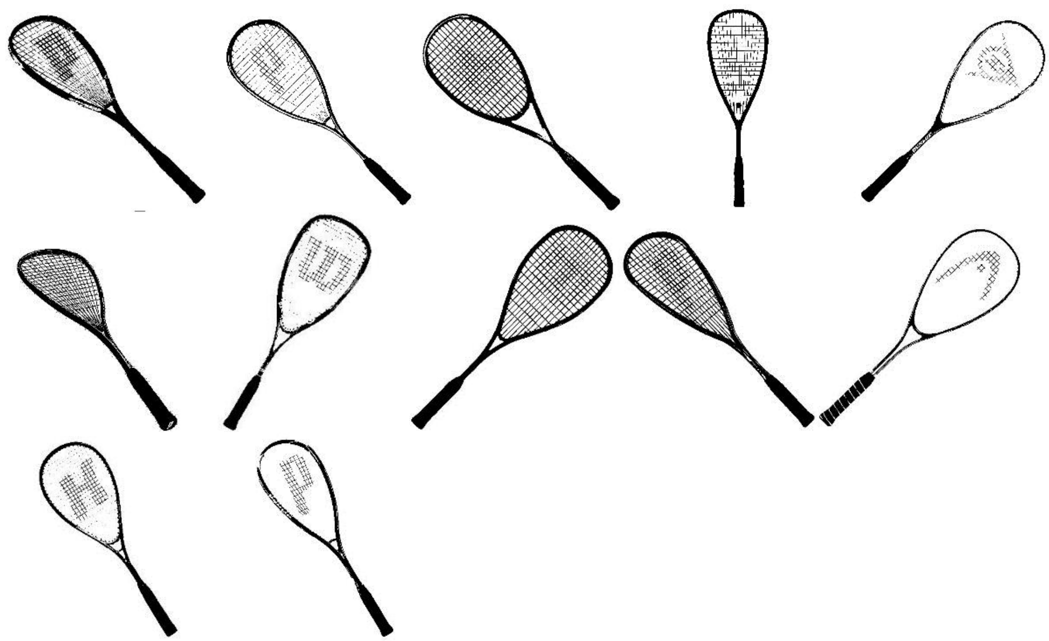

Figure 3.5

For each of the datasets we trained a simple feedforward neural network with one hidden layer using the invariant moments of the 3-4 annuli as inputs. As can be seen from table 3.1.a. when 3 annuli are used, the recognition rate using just the 3 first moments of each annulus varies from 64% for the hard set to 93% for the easy dataset. For the hard dataset, the recognition rate steadily increases as we increase the number of moments used to reach 82.4%. For the easy and medium datasets, the best recognition rates are respectively 97.4% and 93.8% both reached when all 18 moments (6 for each annuli) are used. But we should notice that using only 9 moments in the easy and medium datasets the recognition rates are very comparable with the maximum recognition rates. Using 4 annuli instead of three did not improve the best recognition rate by much in all cases. However, when using only the first moment of each annuli, the recognition rate is definitely better. Again as the number of moments in increased the recognition rate gets better especially for the hard dataset.

	Number of invariants used in the neural network					
	3	6	9	12	15	18
Easy	93.3	97.1	97.2	96.8	96.9	97.4
Medium	77.3	89.1	93.1	92.4	93.3	93.8
Hard	64.1	70.2	75	79.4	80.7	82.4

(a)

	Number of invariants used in the neural network					
	4	8	12	16	20	24
Easy	93.7	96.6	96.4	96.9	96.2	97.2
Medium	81.2	89.6	93.5	93.2	92.9	93.3
Hard	67.7	72.3	81	83.4	80.4	83.6

(b)

Table 3.1 Classification results for the Easy, Medium and Hard datasets (a) with 3 annuli and (b) with 4 annuli.

Table 3.2 gives a comparison between the recognition rates obtained using the moments of the annuli versus those of the regular Hu moments [1] and Chen's improved moments [22]. These improved moments are the same as Hu moments except that they are computed only on the edge of the image by the replacing the double integrals in equation (2) by simple integrals on the boundary of the domain and replacing the exponent of the denominator by $(p+q+1)/2$. For full comparison we also computed improved moments for each of the annuli. We labeled those in table 3.2 as Chen 18 and Chen 24 (6 moments for each annulus). The table clearly shows that the recognition rates for the annuli moments are much better than those of the regular Hu moments and the improved moments for both the Hard leaf dataset and the Objects with handles dataset. The improved moments were not computed for the objects with handles because of the low quality of the pictures in this data set resulting in wild edges (that we obtained for the rackets, and the strings of the musical instruments) that resulted in very poor recognition rates. For the Easy and Medium datasets, the recognition rates of the improved edge annuli moments had the best performance slightly better than the regular Hu moments and the Chen moments. But the annuli moments recognition rates were very comparable.

	6 Moments		**3 Annuli**		**Annuli**	
	Hu's [1]	Chen[22]	18 Hu	18 Chen	24 Hu	24 Chen
Easy	98.7	97.3	97.4	98.9	97.3	99.2
Medium	91.3	86.4	93.8	86.9	93.3	86.0
Hard	56.7	78.0	82.4	81.7	83.6	77.0
Objects with handles	70.2	-	75.9	-	80.8	-

3. CONCLUSION

In this paper, we showed that dividing a domain into 3 or 4 subdomains, and computing their invariant moments improves the recognition rates in a variety of datasets. This division does not result in any extra computations. In fact, each integral or sum computed in the regular Hu moments is simply divided into 3 or 4 partial sums. We believe that this improvement is due to the simple fact that the annuli moments seem to remedy the fact that regular Hu moments tend to give way more weight to pixels that are farthest from the center of gravity. We plan to run the same idea on orthogonal moments to see if the annuli moments would improve the recognition rates and the reconstruction procedures of the original shapes.

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