

# Investigation of Symmetric, Asymmetric and Multi-Fractal Returns of Investment Companies

Fatemeh Samadi<sup>1</sup>, Hashem Nikoomaram<sup>2</sup>, Fereydun Rahnamay Roudposhti<sup>3</sup>

<sup>1</sup>Faculty Member of Shargh Tehran Branch, Islamic Azad University, Tehran, Iran

<sup>2,3</sup>Professor and Faculty Member of Science and Research Branch, Islamic Azad University, Tehran, Iran

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## ABSTRACT

As globalization and international communications increase in transactions especially in capital markets, use of new and appropriate tools is important for managers and decision-makers in finance in order to acquire appropriate return with rational risk. The new paradigm which has been formed based on chaos theory and fractal science in finance and investment areas can change neo-classic ideas which are unable and inefficient in dealing with complexities. In the present research, we try to introduce a new paradigm in finance using physics and math, which is the very fractal space and analyze variations of investment companies returns in Tehran Stock Exchange considering the hypotheses of this new approach.

**KEYWORDS:** symmetric return, asymmetric return, multi-fractal, fractal market

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## 1. INTRODUCTION

Modern financial theory has been the only and main approach to capital market research for more than several decades. However, past decades crises in most countries and the studies conducted show that many failures in financial markets are arisen from modern theories and traditional assumptions in markets. Traditional models are balance models and cannot manage systems in chaos. Further, one of the main assumptions which govern most financial theories is normal distribution of returns. Considering different studies conducted on financial markets, empirical data have the following conditions: most returns have non-Gaussian distribution and have high skewness in the short term. This, in part, can be the main reason for modern financial theories failure. Today, there are many methods for investigation of risk and returns in all affairs especially in portfolio management and many of these methods are based upon modern financial assumptions which rarely match the real world or do not have appropriate tool for investigation of information in market. In the real world, investors do not have similar investment horizons and information structure is mainly dependent on market sensitivity and prices are reflections of short-term transactions combination and long-term evaluation and values of securities are related to economic conditions. In general, purchase and sale, liquidity and short-term period's information are all governing markets and fractal markets assumptions. The aim of the author is to investigate and explore a new and multidisciplinary method for analyzing portfolio return via concentration on fractal assumptions.

## 2. Scientific fundamentals and review of literature

Modern financial theory which actually came to existence in 1900 is considered as the main viewpoint in capital market and has been the base of many studies in spite of its controversial assumptions. Since it is still the dominant paradigm in many studies, calculation of returns and risk are also conducted considering rationality of investors, market efficiency and so on. However, these assumptions are very weak in describing financial markets. After introduction of new financial paradigm and exiting Newtonian restricting assumptions which had covered all financial theories like a halo, we can now introduce a new paradigm and space into thinking substructures and assumptions. The following graph shows that evolutionary trend of classic theories and shocks which are considered as reasons for inefficiency of the theory. This verifies the scientific and theoretical fundamental of the present research (Tania and Lasgossi, 2009).

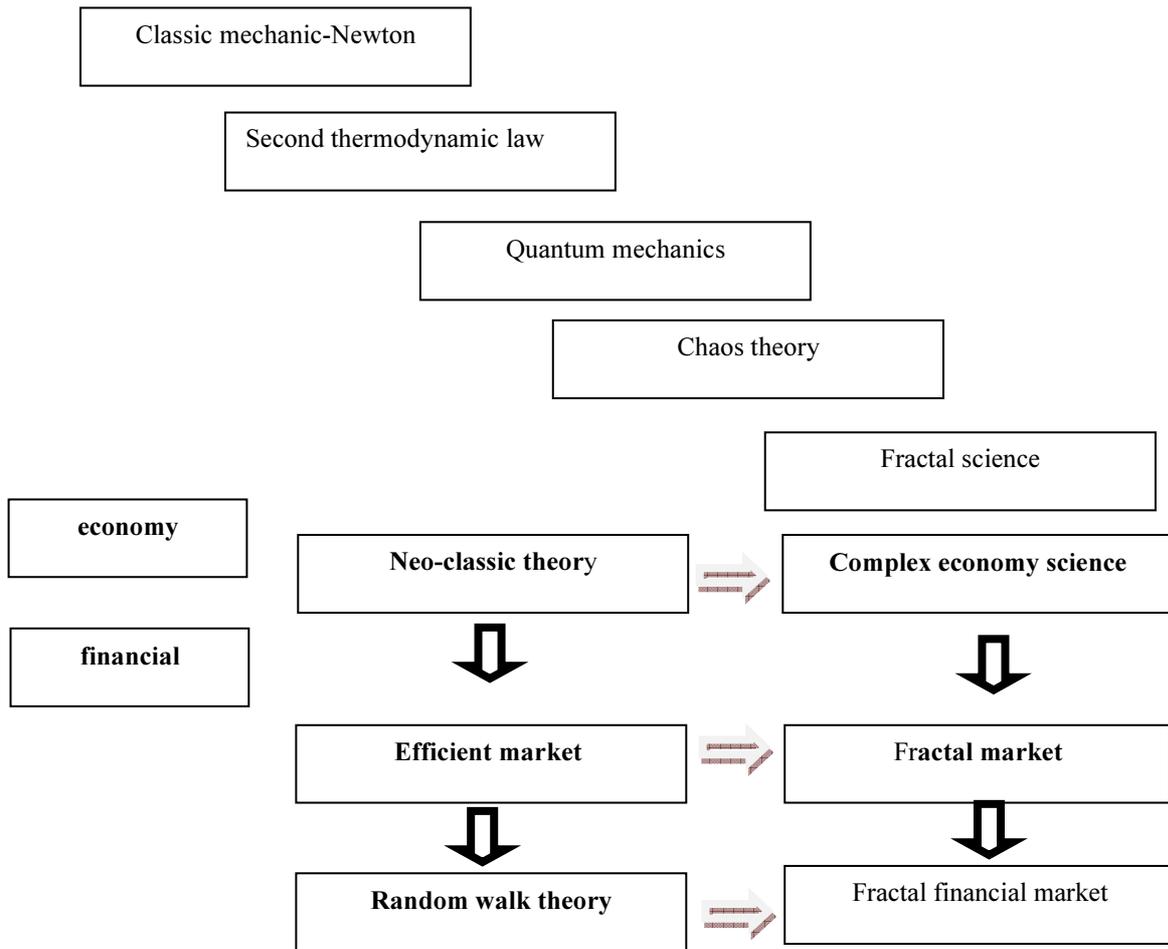


Figure1. Evolutionary trend of different sciences in relation to finance science

As it was said in the past sentences, as approach changed from neoclassic to complex economy, some changes were made to finance. This means that market efficiency theory and random walk theory no longer were useful because they were based on neoclassic fundamentals. Therefore, new approach and viewpoint in finance took the place of the former approach. This new approach to market was the very fractal market and fractal financial market. In Physics and finance, chaos theory and fractals science aim to teach appearance of non-linear periodical behavior resulted from sensitivity of systems to primary conditions which tend to follow paths specified by strong attractors. The main viewpoint which introduces this new paradigm into finance is that markets are like fractals structures in the nature. The new financial paradigm is not based upon neoclassic theories which are related to Newtonian thinking. Therefore, principles of chaos theory and fractals science can be expanded on financial components. Therefore, the key factor is understanding of fractal characteristics in financial markets and this leads to subsequent perceptions and mathematical tools for market analysis. Therefore, models will be more realistic with respect to financial behavior from now on. Fractal theory is a mathematical branch of chaos theory which was discovered by the French mathematician Mandelbrot. He studied fractal theory in 1987-2005 and was an American-French mathematician. He studied the lengths of beaches and found that the beaches were longer when they measured in larger scales than when they were measured in short scales. This irregularity caused emergence of a new branch of math called fractal theory. A fractal is a shape which has been made up of similar shapes in the same path. The two main ideas in fractals is self-steadiness. When a fractal is made up of similar values to the top and bottom, this is called self-steadiness. Self-steadiness idea is a structure which was called primary point by Mandelbrot. There are two important impacts in fractals science which were depicted by Mandelbrot in 1963, 1967 and 1997 and Fama in 1963 and 1965. He stated that distribution of return on assets has a broad sequence and non-Gaussian behavior in relation to normal state which is called Noah impact. Joseph impact means that the trend of great changes is followed by great changes and small changes also follow this trend. In other words, variability has a long-term memory. (Robert Clarson, 2008). These two impacts will be discussed in the following sentences.

Chaos theory and fractals science are new theories. It seems that these theories explain complex components which include non-linearity. Furthermore, it seems that complex non-linear components are more effective and help us with analysis of time series. These theories can help us with analyzing time series which are made up of complex environment, psychological issues and financial components. The two main components in these theories include Hurst and Lyapunov components (Robert Clarson, 2008).

Since emergence, fractals have been classified into two groups: Mono fractals: (one-dimensional fractal) single-fractal process (single-metric), is when behavior metric is described by a constant  $H$ . In fact, it is a case of self-affine linear processes and  $H$  is self-affine or metric component index of process. (multiple fractal) multi-fractal process (multi-metrics) is when different components specify a metric of different distribution moments. In other words,  $H$  component is time-dependent or selection from among many different values is possible. This was introduced by Mandelbrot, Calort and Fiscer in 1997 and was a summary of fractal systems. Since one single view of fractal dimension cannot state system dynamism, a continuous spectrum of views is necessary for this means and this view is called singularity dimension. These views indicate degree of local singularity or order around one single point. Three necessary components in multi-fractal model are: distributions have broad sequences, memory is long-term and transaction time concept is important.

Therefore, as it was mentioned, efficient market is replaced by fractal market from now on. Two important and main points in this new approach include liquidity and investment horizon. In stock market, millions of securities are transacted every day. Pricing method of securities is the result of different variables analysis and each influence prices in a particular manner. Therefore, one of the main challenges ahead of investors, analysts, securities evaluators, brokers, financial and credit institutes is determination of securities expected return. There are many methods for estimation of expected return but expected (predicted) return is not always the same as real return on asset. The probability of presence of difference between real return and predicted return is called risk of that asset. In fact, it can be said that risk of an asset is the probable change of future return resulted from that asset (Raee and Talangi, 2010, 113). The main question in the present research involves exact investigation of prediction of symmetric methods (GARCH), asymmetric methods (EGARCH) and multi-fractal in prediction of stocks return for 5 selected companies in Tehran Stock Market.

The first basic concept is analysis of return. After introduction of chaos theory and fractals geometry, new financial components are defined and this changes neoclassic perception in rationality, complete markets and balanced models and has made mathematical assumptions more continuous, symmetric and smooth. According to chaos theory and fractals science, financial markets characteristics are systems sensitive to primary conditions and irrational and non-linear behaviors are based upon predetermined mechanisms. These models are very important to investigation of variability of return and investment risk because this approach defines markets to be inefficient and constant and assumes that it has variability and is chaotic. These models are based upon this assumption that distribution of returns in financial markets follows a category of distributions called Levy distribution. In comparison with normal distribution, this distribution has broader tails so that second moment has not been defined for this distribution. Considering the existing experiences in the market, it will be closer to reality (Mandelbrot, 2004).

### 3. RESEARCH METHODOLOGY

The present research is an applied research in terms of target and it is a quantitative correlation of time series. The operation in this research involved daily calculation of logarithmic cash return since the beginning of 2001 till the end of December 2012 (including 2971 data).

Then, symmetric, asymmetric and multi-fractal returns on investment of companies registered with Tehran Stock Exchange were calculated. In order to investigate general characteristics and basic features of return series, and in order to estimate models and exactly analyze them, first descriptive statistics concerning time series should be conducted. Furthermore, it must be noticed that conduction of reliability test of the research variable is necessary. The reason for doing such tests is observation of the precondition of avoiding false regression and investigation of possibility of achieving co-integration vectors. Further, it refers to presence of important differences between steady and unsteady time series. This is while impact of shocks on time series is necessarily temporary and its impacts are eliminated as time goes by and the series return to their balanced level. However, the impacts of shocks will be permanent for steady time series. Generalized Dickey Fuller statistic and Philips Proun statistic were used for investigation of steadiness of the series. The sub-tests used in this research include:

1. Investigation of normality: goodness of fit test was used for investigation of normality of distribution of return. This means we investigated agreement between EPS of investment companies and specified theoretical distribution. Usually, common methods for goodness of fit test include Kai-squared test, Kolmogorov-Smirnov test and Jarque-bera test are used.
2. investigation of auto-correlation of time series: in order to investigate auto-correlation of time series of share price, Ljung-Box test was used.

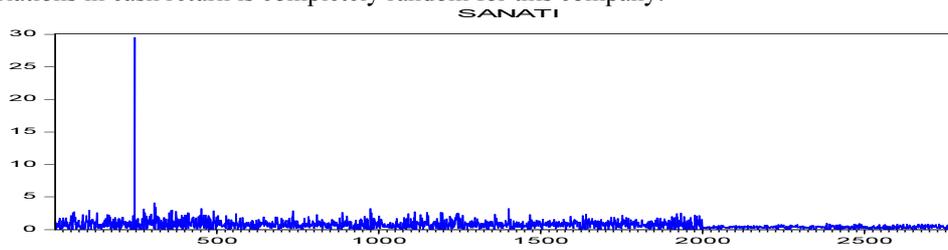
3. investigation of stability of variance over time: this question is answered by investigation of stability of variance over time: whether variance is stable over time or not. If the variance of stability is stable, companies which calculate endangered value do not need to calculate variance for every day.

**4. Research hypothesis**

Calculation of portfolio return in investment companies in Tehran Stock Exchange with fractal approach is more efficient than the traditional method of Marquitz which is based upon asymmetric and symmetric methods.

**5. Research model**

In the present research, multi-fractal method was used for prediction of return. Further, the methods used were based upon GARCH, EGARCH and Multi-fractal methodologies. First, we investigate the trend of cash return of Industrial Development Investment Company. Considering the following graph, it can be seen that the trend of variations in cash return is completely random for this company.



In the following sentences, descriptive statistics of time series of cash return of Industrial Development Investment Company have been presented.

SANATI	
Mean	0.696189
Median	0.568152
Maximum	29.48859
Minimum	0.000000
Std. Dev.	0.761946
Skewness	19.75578
Kurtosis	732.3653
Jarque-Bera	62045661
Probability	0.000000
Sum	1943.063
Sum Sq. Dev.	1619.766
Observations	2791

According to the results summarized in the above table, kurtosis of cash return of the company is more than the kurtosis of normal distribution and its skewness is to the right side. However, considering the Jarque-bera statistic and significance level, it can be observed that probability distribution of cash return series of the company is not normal and skewed distributions will probably be more able to predict these series. In the next parts, steadiness status of the time series of cash return of the company will be investigated.

Table1: results of steadiness test on the level of model variables

Test type and significnace	Dickey-Fouler		Generalized Dickey-Fouler		Philips-Proun	
	statistic	Critical value (%5)	statistic	Critical value(%5)	statistic	Critical value (%5)
SANATI	-15/78	-2/86	-4/06	-1/94	-57/99	-2/86

Considering the results of the above table, cash return variable of the company is steady in (1%) level and therefore it can be guaranteed that the results of the regression will not be led to false regression. In the following sentences, we estimate ARMA model of time series of cash return of Industrial Development Investment Company. MA(1) and MA(2) were verified in the model. considering the akaike index of optimum lag for ARMA model of the time series of cash return of the company was selected to be (18, 1). In order to make sure that the lags are optimum in the model, the structure of unit root for this series can be observed.

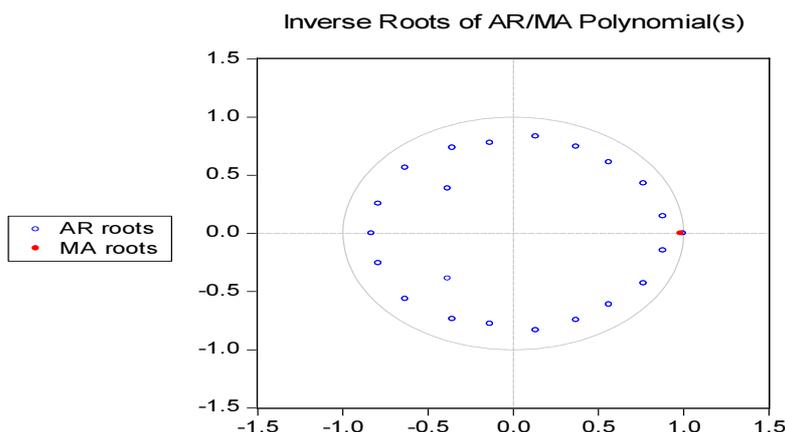


Figure 2 : investigation of unit root in ARMA model

In the next sentences, we investigate Arch effect in the estimated model. Considering the results, it can be observed that Arch effect can be observed in data of time series of cash return of the company. Therefore, it is necessary to use GARCH model because of optimum lag of ARMA model. According to the above estimation model, we predicted 191 data from among 2791 data in three states: normal distribution, t distribution and skewed t distribution. Finally, we estimated the model based on multi-fractal method in three states: normal distribution, t distribution and skewed t distribution. A comparison of the above three methods in terms of precision of prediction of MSE evaluation criterion has been mentioned in three states: normal distribution, t distribution and skewed t distribution.

Table2: out-of-sample performance

		MSE	Rank
EGARCH	NORMAL	0.94647	8
EGARCH	t_student	0.92575	6
EGARCH	Skw_t	0.92645	7
EGARCH	Pareto distribution	0.90434	5
GARCH	NORMAL	0.979758	12
GARCH	t_student	0.95666	11
GARCH	Skw_t	0.95569	10
GARCH	Pareto distribution	0.95688	9
Multi-fractal	NORMAL	0.89745	4
Multi-fractal	t_student	0.88650	3
Multi-fractal	Skw_t	0.87144	1
Multi-fractal	Pareto distribution	0.87232	2

Considering the above table, it can be observed that multi fractal method based on MSE index predicts the model in Pareto and skewed t states better than normal distribution state. Further, skewed t estimation model has been more efficient than other methods. All calculations have been conducted on 10 investment companies in Tehran Stock Exchange. General results are summarized in the following tables. In this research, we investigated the possibility of presence of fractal structure in EPS of investment companies in Tehran Stock Exchange. First, descriptive statistics of the investment companies are as follows:

	BEHSHAHR	BOALI	DAMAVAND	MADAN	MLI
Mean	0.329037	0.713322	0.294731	0.308035	0.647225
Median	0.309435	0.652985	0.282748	0.309403	0.581544
Maximum	1.187424	2.913394	1.226954	1.211165	6.250751
Minimum	0.000000	0.000000	0.000000	0.000000	0.000000
Std. Dev.	0.189920	0.444364	0.195963	0.211771	0.491948
Skewness	0.701380	0.956760	0.662165	0.420168	1.485071
Kurtosis	3.781914	4.507467	3.664710	3.067708	10.77331
Jarque-Bera	299.9308	690.0763	255.3403	82.65429	8052.740
Probability	0.000000	0.000000	0.000000	0.000000	0.000000
Sum	918.3413	1990.883	822.5931	859.7263	1806.406
Sum Sq. Dev.	100.6337	550.9118	107.1406	125.1232	675.2151
Observations	2791	2791	2791	2791	2791

	NERO	SANATI	SEPAH	SRBEMAH	TOSEHMLI
Mean	0.261434	0.696189	0.410674	0.300957	0.316471
Median	0.238277	0.568152	0.318845	0.262447	0.262565
Maximum	1.450795	29.48859	26.29853	1.589207	5.025353
Minimum	0.000000	0.000000	0.000000	0.000000	0.000000
Std. Dev.	0.212117	0.761946	0.609872	0.250397	0.287394
Skewness	0.916728	19.75578	27.70932	1.105286	2.769333
Kurtosis	4.127077	732.3653	1164.852	4.780318	30.93612
Jarque-Bera	538.6471	62045661	1.57E+08	936.8645	94324.57
Probability	0.000000	0.000000	0.000000	0.000000	0.000000
Sum	729.6618	1943.063	1146.190	839.9708	883.2717
Sum Sq. Dev.	125.5319	1619.766	1037.724	174.9286	230.4406
Observations	2791	2791	2791	2791	2791

Considering the significance level of Jarque-bera statistic, it can be observed that distribution of time series is not normal in all cases. Considering the investigation of descriptive statistics of companies, skewness level in all of them is above 3 and this shows that their distribution functions are not normal. Further, Jarque-bera statistic also verifies this non-Gaussian state in all 10 companies. According to unit root test, steadiness of EPS for 2002-2012 time periods was investigated. Results show that the time series under investigation in the above time period for investment companies selected in this study are as follows. Therefore, Dickey-Fouler, generalized Dickey-Fouler and Philips-Proun test verify that EPS of investment companies in Tehran Stock Exchange is unsteady. In general, according to the results, it can be said that presence a fractal structure in 2001-2012 time period is verified.

Table3: results of steadiness test on model variables level

Philips-Proun		generalized Dickey-Fouler		Dickey-Fouler		Test type and significance level
Critical value (5%)	statistic	Critical value (5%)	Critical value (5%)	statistic	Critical value (5%)	
-2.86	-59.01	-1.94	-11.26	-2.26	-12.24	TOSEHMLI
Philips-Proun		generalized Dickey-Fouler		Dickey-Fouler		Philips-Proun
Critical value (5%)	statistic	Critical value (5%)	Critical value (5%)	statistic	Critical value (5%)	
-2.86	-53.90	-1.94	-5.18	-2.86	-13.22	SRBEMAH
Philips-Proun		generalized Dickey-Fouler		Dickey-Fouler		Philips-Proun
Critical value (5%)	statistic	Critical value (5%)	Critical value (5%)	statistic	Critical value (5%)	
-2.86	-57.48	-1.94	-3.74	-2.86	-10.78	SEPAH
Philips-Proun		generalized Dickey-Fouler		Dickey-Fouler		Philips-Proun
Critical value (5%)	statistic	Critical value (5%)	Critical value (5%)	statistic	Critical value (5%)	
-2.86	-51.72	-1.94	-1.25	-2.86	-23.58	MLI
Philips-Proun		generalized Dickey-Fouler		Dickey-Fouler		Philips-Proun
Critical value (5%)	statistic	Critical value (5%)	Critical value (5%)	statistic	Critical value (5%)	
-2.86	-53.92	-1.94	-2.4	-2.86	-16.37	BOALI
Philips-Proun		generalized Dickey-Fouler		Dickey-Fouler		Philips-Proun
Critical value (5%)	statistic	Critical value (5%)		Critical value (5%)	statistic	Critical value (5%)
-2.86	-57.99	-1.94		-4.06	-2.86	-15.78
Philips-Proun		generalized Dickey-Fouler		Dickey-Fouler		Philips-Proun
Critical value (5%)	statistic	Critical value (5%)	Critical value (5%)	statistic	Critical value (5%)	
-2.86	-49.48	-1.94	-3.38	-2.86	-24.29	DAMAVAND
Philips-Proun		generalized Dickey-Fouler		Dickey-Fouler		Philips-Proun
Critical value (5%)	statistic	Critical value (5%)	Critical value (5%)	statistic	Critical value (5%)	
-2.86	-59.08	-1.94	-2.12	-2.86	-10.89	BEHSHAHR
Philips-Proun		generalized Dickey-Fouler		Dickey-Fouler		Philips-Proun
Critical value (5%)	statistic	Critical value (5%)		Critical value (5%)	statistic	Critical value (5%)
-2.86	-34.58	-1.94		-2.06	-2.86	-10.8
Philips-Proun		generalized Dickey-Fouler		Dickey-Fouler		Philips-Proun
Critical value (5%)	statistic	Critical value (5%)	Critical value (5%)	statistic	Critical value (5%)	
-2.86	-52.31	-1.94	-7.34	-2.86	-18.07	NERO

Considering the results of the above table, it can be observed that all data are significant and estimation of the models of the above series does not lead to false regression. akaike index was used for investigation of presence of optimum lag in different models in all 10 companies. Considering the akaike index level, optimum lag for ARMA model of the time series of the return has been selected. In the following sentences, we optimum lag has been investigated considering akaike index in different models.

Table4: Determinations of optimum lag level in ARMA model

Company name	BEHSHAHR	BOALI	DAMAVAND	MADAN	MLI
Akaike statistic value	0.0323	1.1379	1.1455	0.0017	0.1491
ARMA optimum lag	(17,1)	(19,2)	(16,2)	(18,0)	(18,2)
Company name	NERO	SANATI	SEPAH	SRBEMAH	TOSEHMLI
Akaike statistic value	2.7642	0.3467	1.8145	0.0014	0.2442
ARMA optimum lag	(16,0)	(15,1)	(18,0)	(16,2)	(16,2)

In the following sentences, considering the different methods and different distributions, multi-fractal and Marco Switching models and the two symmetric and asymmetric states in three distributions normal, skewed t and Pareto for 10 investment companies are specified. Further, the most effective method in prediction of return has been specified in investment companies.

Table 5: comparison of precision of different methods in different distributions in investigated companies

Distribution type			Estimation model		company name
Pareto distribution	Skewed t distribution	T distribution	Normal distribution	Multi-fractal	asymmetric symmetric
					MLI
					MADAN
					DAMAVAND
				✓	BOALI
				✓	BEHSHAHR
					TOSEHMLI
					SRBEMAH
					SEPAH
					SANATI
					NERO

Considering the above results, it can be observed that multi-fractal method is the most precise method for estimation in all 10 companies. Further, investigation of different models reveals that skewed t distribution is more precise than other statistical distributions in prediction of cash return of investigated companies in 7 states. Only in three companies Pareto distribution was better in prediction. In other words, multi-fractal methods model and skewed t distribution are more precise than other methods of prediction of cash return in stock companies. In fact, research main hypothesis is supported.

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