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Oscillatory Flow of Unsteady Oldroyd-B fluid with magnetic field Between Two Vertical Plates

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ABSTRACT

In this article the Non-Newtonian fluid flow between two oscillating vertical plates is examined. The constitutive equations of Non-Newtonian differential type unsteady Oldroyed-B fluid have been used for modelling of the problem. The fluid is electrically conducted and a constant magnetic field acting in the transverse direction to the flow field. Two different varieties of flow problems have been modelled in terms of nonlinear *PDE, s* with certain physical conditions. The modelled *PDE, s* have been solved analytically by using the (*OHAM*) Method. This method is frequently used for the solution of such type problem arise in various applied and engineering sciences and are found quite useful. The effect of several modelled parameters of velocity profile has been studied graphically.

KEYWORDS: Unsteady Second Grade Fluid, Verticals plates, (OHAM) and (ADM).

INTRODUCTION

From last twenty years non-Newtonian fluids have been getting's particular importance in areas of research, especially in applied mathematics, industry and in the field of engineering. Examples of such types of fluids are plastic manufacturing, food process, biological fluids, wire and fibres coating, papers manufacturing, drilling mud. Numerous complex fluids like as polymer melts, paint, shampoo, mud, ketchup, blood, certain oils and greases, and other some emulsions are involved in the class of non-Newtonian fluids. These fluids are described by a non-linear relation between stress and the rate of deformation tensors. Therefore, several models have been proposed .Hence, a number of fluid models have been suggested to predict the non-Newtonian performance of different kinds of materials. Due to industrial and technological usages non-Newtonian fluids have become its important part therefore researchers also take a great interest in it. Fetecau at all [2-5] have done a great work on Oldroyed –B fluids. They discussed Stock First problem for Oldroyed –B fluid. They also investigated the flow effect of an Oldroyd-B fluid in unsteady case generated by a uniformly accelerating plate between two side walls perpendicular to the plate. All the physical and numerical explanation has been given in their work .The oscillating effect of the same fluid were also examined in their work.Hayat [6-7] examined some simple's flows of Oldroyed –B fluid. Further they worked on Coueete and Poiseuille flows of oldroyed 6-Constant fluid with MHD. The Coueete flow of second order fluid in porous medium is investigated by Hayat and et al. [9]. From last three years Gul et al. [9-13] examined different type fluids in varieties of article. They investigated different effects of several physical variables on flow profiles. Volume flux, average velocity, and the heat distribution of the flow field were shown in their work. In most of their work they used two analytical techniques to obtain better results. For solution they used *ADM*, *OHAM, HPM* techniques. Exact solutions of for Oldroyed–B fluid in porous medium in unsteady state have been investigated by Khan et al. [14]. Burdujan [15] examined Oldroyed –B fluid for some particular class and discuss it for various cases. The oscillating flow of an Oldroyd-B fluid through a plane wall was discussed by Anjum et al [16]. Shahid et al [17] investigated Oldroyd-B fluid over an infinite flat plate; they examined the oscillating effect and found the exact solution for the modelled problem. Ghosh et al [19-21] have a research work on Oldroyd-B fluid with hydro magnetic effect on a pulsating plate and studied in unsteady state with induced and half rectified sine pulses. Shah et al [22] used *OHAM* for the solution of third grad fluid modelled problem. Marinca et al. [25, 26] obtained the approximate solution of non-linear steady flow problem modelled with fourth order fluid model by using *OHAM*. They observed from the obtained a result that *OHAM* is more effective than other techniques. Kashkari [27] examined *OHAM* solution of nonlinear Kawahara equation. He used *HPM* and *VIM*technique for comparison and found that OHAM is more effective method. Mabood et al. [31] examined the solution of non-linear Riccati differential equation by using *OHAM*.

Basic Equation

The magneto hydrodynamic equations (momentum, mass equations and Ohms' law) governing the problem are written as

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$$\rho\left[\frac{DU}{Dt}+(U.\nabla)U\right]=\Delta.\mathrm{T}+\mathbf{J}\times\mathbf{B}+\rho\mathbf{g}. \tag{1}$$

$$\nabla.U=0 \tag{2}$$

Where symbol ρ is used for the fluid density. $\mathbf{J}\times\mathbf{B}$ is the magnetic body force in where $\mathbf{J}$ show electric current density and $\mathbf{B}$ is magnetic induction.The generalized Ohm's law is

$$\mathbf{J}=\sigma\left(\boldsymbol{E}+\boldsymbol{U}\times\mathrm{B}\right)\ . \tag{3}$$

$\mathbf{E}$ sthows, electric field which we take constant and. σ is the electrical conductivity. $\mathbf{J}\times\mathbf{B}$ is also called the Lorentz force per unit volume and here we define it as

$$\mathbf{J}\times\mathbf{B}=\left[0,-\sigma B_0^2\mathrm{u},0\right]. \tag{4}$$

Here $\mathbf{B}=\left(0,B_0,0\right)$ is magnatic filed which is taken uniform in which B_0 is the acting magnetic force . $\mathbf{J}$ is the current density,in term of magnatic field, we also express as

$$\nabla\times\mathbf{B}=\mu_0\mathbf{J} \tag{5}$$

Where μ_0 is the magnetic permeability. The Cauchy stress tensor, T is

$$\mathrm{T}=-p\mathbf{I}+\mathbf{S}, \tag{6}$$

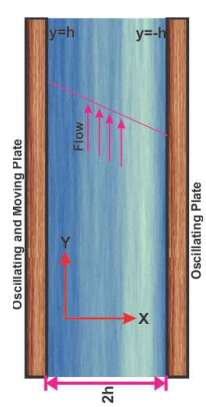
$$\mathbf{S}+\lambda_r\frac{\mathbf{D}\,\mathbf{S}}{Dt}=\mu\left[1+\lambda_t\frac{D}{Dt}\right]\mathbf{A}_1, \tag{7}$$

$$\mathbf{A}_0=\mathbf{I}\ ,\ \mathbf{A}_1=\mathbf{L}+\mathbf{L}^T,\mathbf{L}=gradu\ , \tag{8}$$

$\mathbf{S}$ is a symbol for the tress tensor, $\mathrm{P}\mathbf{I}$ show isotropic stress, A is used for the Rivlin Ericksen tensor and μ means the coefficient of viscosity. λ_r and λ_t are material constant.

Statement of the Couette flow Problem:

We suppose incompressible non-Newtonian Oldroyd-B fluid between two Vertical and parallel plates $y=h$ and $y=-h$ .Considered that both plates are oscillating and moving with constant velocity U .The total thickness of the fluid between the plates supposed to be $y=2h$ moving and oscillating plate caries with it a liquid of width $y=\delta$. The configuration of fluid flow is along the Y-axis and perpendicular to x-axis. A constant magnetic field is acting on the upward vertical direction where the plate is non-conducting. The pressure P is kept constant in this section that is pressure gradient is zero in x- direction. We have supposed the unsteady flow, which is laminar and incompressible. Gravitational force and magnetics force causes the fluid motion. Velocity field for state problem is defined as



"**Fig. 1**", Geometry of the problem

$$\mathbf{u} = \left[0, u\left[x,t\right], 0\right], \text{and } \tau = \left[x,t\right], \tag{9}$$

The boundary conditions are

$$u_0\left(h,t\right) = U\left(1+\cos\omega t\right),\ \ u_0\left(-h,t\right) = U\cos\omega t. \tag{10}$$

The frequency of oscillating plates is ω. Inserting all the assumption in momentum equation (1) so it reduced to the form of

$$\rho\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \frac{\partial \tau_{xy}}{\partial x} - \sigma B_0{}^2 u + \rho g. \tag{11}$$

$$\frac{\partial p}{\partial y} = \frac{\partial \tau_{yy}}{\partial y}, \tag{12}$$

$$\frac{\partial p}{\partial z} = 0. \tag{13}$$

With the use of (9), Equation (7) reduced in the form of

$$\tau_{xx} + \lambda_r\left[\frac{\partial \tau_{xx}}{\partial t} - 2\tau_{xy}\frac{\partial u}{\partial x}\right] = -2\mu\lambda_t\left[\frac{\partial u}{\partial x}\right]^2, \tag{14}$$

$$\tau_{xy} + \lambda_r\left[\frac{\partial \tau_{xy}}{\partial t} - \tau_{yy}\frac{\partial u}{\partial x}\right] = \mu\left(\frac{\partial u}{\partial x}\right) + \lambda_t\mu\left(\frac{\partial^2 u}{\partial t\partial x}\right), \tag{15}$$

$$\tau_{yy} + \lambda_r\frac{\partial \tau_{yy}}{\partial t} = 0, \tag{16}$$

Then equation (16) reduces to

$$\tau_{yy} = \Pi\left(y\right)e^{-\frac{t}{\lambda_1}}. \tag{17}$$

Here $\Pi\left(y\right)$ is an arbitrary function. If $t<0$, then $\left(\tau_{yy}=0\right)$, which proves that $\left(\Pi\left(y\right)=0\right)$. Hence, from the (11) and (15), we obtained

$$\left(1+\lambda_r\frac{\partial}{\partial t}\right)\frac{\partial u}{\partial t} = v\left(1+\lambda_t\frac{\partial}{\partial t}\right)\frac{\partial^2 u}{\partial x^2} - \frac{\sigma B_0{}^2}{\rho}\left(1+\lambda_r\frac{\partial}{\partial t}\right)u + \rho g. \tag{18}$$

Introducing non-dimensional variables

$$u = \frac{\tilde{u}}{U}, x = \frac{\tilde{x}}{\delta}, t = \frac{\mu\tilde{t}}{\rho\delta^2}, k_1 = \frac{\lambda_r\mu}{\rho\delta^2}, k_2 = \frac{\lambda_t\mu}{\rho\delta^2}, \omega = \frac{\tilde{\omega}\delta^2\rho}{\mu}, m = \frac{\delta^2\rho g}{\mu U}, M = \frac{\sigma B_0^2\delta^2}{\mu}. \tag{19}$$

Where $\tilde{\omega}$, k_1, k_2, m, M is oscillating, relaxation, retardation, gravitational, and magnetic parameters respectively.

Using these non-dimensional parameters from Eqn (19) into Eqn (18), boundary conditions from Eqn (10) and inserting bars we get

$$\left(1+k_1\frac{\partial}{\partial t}\right)\frac{\partial u}{\partial t} = \left(1+k_2\frac{\partial}{\partial t}\right)\frac{\partial^2 u}{\partial x^2} - M\left(1+k_1\frac{\partial}{\partial t}\right)u + m. \tag{20}$$

$$u_0\left(1,t\right) = 1 + c\cos\omega t, u_0\left(-1,t\right) = \cos\omega t. \tag{21}$$

Basic Theory of OHAM

To obtain the solution of the muddled problem we use *OHAM* method .First we explain the preliminary idea of *OHAM*. Consider a general *PDE* of the type

$$\Im(\mathrm{u}(\mathrm{x},t)) + \Re(\mathrm{u}(\mathrm{x},t)) + \aleph(\mathrm{x},t) = 0,\ \ \mathrm{B}\left(\mathrm{u}(\mathrm{x},t), \frac{\partial u(\mathrm{x},t)}{\partial x}\right) = 0,\ \ \mathrm{x}\in\Gamma, \tag{22}$$

Here $\Im$ is linear and $\Re$ is non-linear operator, $\aleph$ is determine, $\mathrm{u}(\mathrm{x},t)$ is the undetermined function, x independent and t is dependent variable where B is the boundary operator, $\aleph$ show the domain of the function.

From basic theory of *OHAM* we define the optimal homotopy $\Pi(\mathrm{x},t,p):\Pi\times[0,1]\to R$, and it must satisfy the following equation

$$[1-\mathrm{P}]\left[\Im\Pi(\mathrm{x},t,\mathrm{P})+\aleph(\mathrm{x},t)\right]=\begin{bmatrix} H(\mathrm{P})\left[\Im\Pi(\mathrm{x},t,\mathrm{P})+\aleph(\mathrm{x},t)+\Re\Pi(\mathrm{x},t,\mathrm{P})\right], \\ B\left(\Pi(\mathrm{x},t,\mathrm{P}),\dfrac{\partial\Pi(\mathrm{x},t,\mathrm{P})}{\partial x}\right)=0 \end{bmatrix}. \tag{23}$$

P shows the embedding parameter and $\mathrm{P}\in[0,1]$, $\mathrm{H(P)}$ is the auxiliary function $(\mathrm{H(P)}\neq 0)$ and for $\mathrm{P}\neq 0$, $(H(0)=0)$. From equation (23) we can clearly write

$$\mathrm{P}=0\Rightarrow H(\Pi(\mathrm{x},t,0),0)=H(0)\left[\Im\Pi(\mathrm{x},t)+\aleph\Pi(\mathrm{x},t,p)\right]=0, \tag{24}$$

$$\mathrm{P}=1\Rightarrow H(\Pi(\mathrm{x},t,1),1)=H(1)\left[\Im\Pi(\mathrm{x},t)+\aleph\Pi(\mathrm{x},t,p)+\Re\Pi(\mathrm{x},t,p)\right]=0, \tag{25}$$

Clearly, it satisfied that when $\mathrm{P}=0,\Pi(\mathrm{x},t,0)=u_0(\mathrm{x},t)$ and when $\mathrm{P}=1$ then $\Pi(\mathrm{x},t,1)=u(\mathrm{x},t)$ we obtain $u_0(\mathrm{x},t)$ by inserting $\mathrm{P}=0$ in equation (23)

$$\Im\Pi(\mathrm{x},t)+\aleph\Pi(\mathrm{x},t,\mathrm{P})=0. \tag{26}$$

Here we take the auxiliary function as

$$H(\mathrm{P})=\mathrm{P}C_1+\mathrm{P}^2C_2+\mathrm{P}^3C_3......, \tag{27}$$

Where $C_1,C_2,C,....$ are called auxiliary constants.

For the approximate solution, the unknown function $\Pi(\mathrm{x},t,\mathrm{P})$ is expanding as

$$\Pi(\mathrm{x},t,\mathrm{P},c_i)=u_0(\mathrm{x},t)+\sum_{n\geq 1}u(\mathrm{x},t,\mathrm{P},c_i)\mathrm{P}^n, \tag{28}$$

By inserting equation (28) into equation (23) and comparing the equal power of p, we obtained the 0th, 1st and 2nd order problem are $u_0(\mathrm{x},t)$, $u_1(\mathrm{x},t)$ and $u_2(\mathrm{x},t)$. So the equation is

$$\Im(u_1(\mathrm{x},t))+\Re(\mathrm{x},t)=c_1\Re_0(u_0(\mathrm{x},t)),\quad B\left(\frac{\partial u_1(\mathrm{x},t)}{\partial x}\right)=0, \tag{29}$$

$$\Im(u_2(\mathrm{x},t))-\Im(u_1(\mathrm{x},t))=\begin{bmatrix} c_2\Re_0(v_0(\mathrm{x},t))+c_1\left[\Im(v_1(\mathrm{x},t))+\aleph_1(u_0(\mathrm{x},t),u_1(\mathrm{x},t))\right], \\ B\left(u_2(x),\dfrac{\partial u_2(\mathrm{x},t)}{\partial x}\right) \end{bmatrix}=0, \tag{30}$$

The general governing equations for $u_n(\mathrm{x},t)$ are given by

$$\Im(u_n(\mathrm{x},t))-\Im(u_{n-1}(\mathrm{x},t))=\begin{bmatrix} c_n\Re_0(u_0(\mathrm{x},t))+\sum_{i=1}^{n-1}c_i\begin{bmatrix}\Im(u_{n-i}(\mathrm{x},t))+ \\ \Re_{k-1}(u_0(\mathrm{x},t),\mathrm{u}_1(\mathrm{x},t).....\mathrm{u}_{n-i}(\mathrm{x},t))\end{bmatrix}, \\ k=2,3,...,B\left(v_k(\mathrm{x},t),\dfrac{\partial u_n(\mathrm{x},t)}{\partial x}\right)=0, \end{bmatrix} \tag{31}$$

Here $\Im_m(u_0(\mathrm{x},t),\mathrm{u}_1(\mathrm{x},t)....\mathrm{u}_{m-1}(\mathrm{x},t))$ is the coefficient of P^m, in the expansion of $\Re\Pi(\mathrm{x},t,\mathrm{P})$.

$$\Re(\Pi(\mathrm{x},t,\mathrm{P},c_i))=\Re_0(u_0(\mathrm{x},t))+\sum_{m=1}^{\infty}\Re_m(u_0(\mathrm{x},t),\mathrm{u}_1(\mathrm{x},t)....\mathrm{u}_m(\mathrm{x},t))\mathrm{P}^m. \tag{32}$$

The convergence of the Series in equation (31) depend upon the auxiliary constants $c_1,c_2,...$ If it converges at $\mathrm{P}=1$, then the mth order approximation u is

$$u\left(\mathrm{x},\mathrm{C}_1,\mathrm{C}_2\ldots.C_m\right)=u_0\left(\mathrm{x},t\right)+\sum_{i=1}^{m}u_i\left(\mathrm{x},\mathrm{C}_1,\mathrm{C}_2\ldots.C_i\right). \tag{33}$$

Using Eq. (33) in Eq.(32), the residual is obtained as:

$$\wp\left(\mathrm{x},t,c_i\right)=\Im\left(u\left(\mathrm{x},t,c_i\right)\right)+\aleph\left(\mathrm{x},t\right)+\Re\left(u\left(\mathrm{x},t,c_i\right)\right),\ i=1,2..m \tag{34}$$

Several methods, e.g. Ritz Method, Method of Least Squares, Galerkin's Method and Collocation Method is used to obtain the optimal values of $C_i, i=1,2,3,4\ldots$ We apply the Method of Least Squares in our problem as given below:

$$j\left(C_1,\mathrm{C}_2,\ldots\mathrm{C}_n\right)=\int_a^b\wp^2\left(\mathrm{x},t,\mathrm{C}_1,\mathrm{C}_2,\ldots\mathrm{c}_m\right)dx. \tag{35}$$

Where a and b are the constant values taking from domain of the problem.

Auxiliary constants $C_1,\mathrm{C}_2,\ldots\mathrm{C}_n$ can be identified from:

$$\frac{\partial j}{\partial C_1}=\frac{\partial j}{\partial C_1}=\ldots=0. \tag{36}$$

At last, from these constants, the approximate solution is obtained.

The OHAM Solution for the Problem

We apply *OHAM* method to obtain the analytical solution on equation (20) with boundary condition in equation (21). The component problems are given on orders that 1st, 2nd and 3rd order problem of velocity profile are

$$p^0:\frac{\partial^2u_0\left(\mathrm{x},t\right)}{\partial x^2}+m=0, \tag{37}$$

$$p^1:\frac{\partial^2u_1\left(\mathrm{x},t\right)}{\partial x^2}=\left(1+c_1\right)\frac{\partial^2u_0}{\partial x^2}-k_1c_1\frac{\partial^2u_0}{\partial t^2}+k_2c_1\frac{\partial}{\partial t}\left(\frac{\partial^2u_0}{\partial x^2}\right)-c_1\left(1+Mk_1\right)\frac{\partial u_0}{\partial t}+m\left(1+c_1\right)-Mc_1u_0. \tag{38}$$

$$p^2:\frac{\partial^2u_1\left(\mathrm{x},t\right)}{\partial x^2}=c_2\frac{\partial^2u_0}{\partial x^2}+k_2c_2\frac{\partial}{\partial t}\left(\frac{\partial^2u_0}{\partial x^2}\right)+\left(1+c_1\right)\frac{\partial^2u_1}{\partial x^2}+k_2c_1\frac{\partial}{\partial t}\left(\frac{\partial^2u_1}{\partial x^2}\right)-k_1c_1\frac{\partial^2u_1}{\partial t^2}-$$
$$c_2\left(1+Mk_1\right)\frac{\partial u_0}{\partial t}-c_1\left(1+Mk_1\right)\frac{\partial u_1}{\partial t}+c_2\left(m-Mu_0\right)-Mc_1u_1. \tag{39}$$

Solutions to equations (37-39) using boundary condition in equation (21) are

$$u_0\left(\mathrm{x},t\right)=\cos\left[t\omega\right]+\frac{1}{2}\left(1+m+x-mx^2\right). \tag{40}$$

$$u_1\left(\mathrm{x},t\right)=\frac{1}{24}\begin{pmatrix}Mc_1\left(6+5m+2x-6x^2-6mx^2-2x^3+x^4\right)+12M\mathrm{Cos}\left[t\omega\right]c_1\left(1-x^2\right)-12\omega\mathrm{Sin}\left[t\omega\right]c_1\\ \left(1+x^2\right)-12\omega^2\mathrm{Cos}\left[t\omega\right]c_1k_1\left(1-x^2\right)+12M\omega\mathrm{Sin}\left[t\omega\right]c_1k_1\left(1+x^2\right)\end{pmatrix}. \tag{41}$$

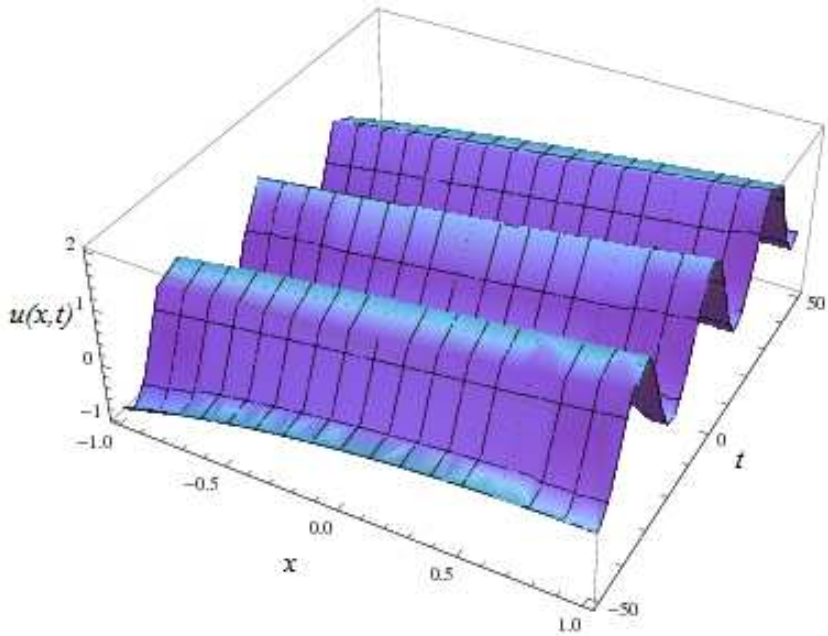
$$u_2(\mathrm{x},t)=\frac{1}{720}\begin{pmatrix} Mm(50+150m+60x-180x^2-180mx^2-60x^3+30mx^4)+360M\cos[t\omega]c_1(1-x^2) \\ +360\omega\sin[t\omega]c_1(x^2-1)+Mc_1^2(180+150m+75M+61mM+60x+7Mx-180x^2- \\ 180mx^2-90Mx^2-75mx^2-60x^3-10Mx^3+30mx^4+15Mx^4+15mMx^4+3Mx^5- \\ mMx^6)+M\cos[t\omega]c_1^2(360+150M-360x^2-180Mx^2+30Mx^4)+\omega^2\cos[t\omega]c_1^2 \\ (-150+180x^2-30x^4)+\omega\sin[t\omega]c_1^2(-360-300M+360x^2+360Mx^2-60Mx^4)+ \\ Mc_2(180+150m+60x-180x^2-180mx^2-60x^3+30mx^4)+360M\cos[t\omega]c_2(1-x^2) \\ 360\omega\sin[t\omega]c_2(x^2-1)+360\omega^2\cos[t\omega]c_1k_1(x^2-1)+360M\omega\sin[t\omega]c_1k_1(x^2-1)+ \\ \omega^2\cos[t\omega]c_1^2k_1(-360-600M+360x^2+720Mx^2-120Mx^4)+M\omega\sin[t\omega]c_1^2k_1 \\ (-360-300M+360x^2+360Mx^2-60M^2x^4)+\omega^3\sin[t\omega]c_1^2k_1(300-360x^2+60x^4) \\ +360\omega^2\cos[t\omega]c_2k_1(x^2-1)+360M\omega\sin[t\omega]c_2k_1(x^2-1)+M^2\omega^2\cos[t\omega]c_1^2k_1^2 \\ (-150+180x^2-30x^4)+\omega^4\cos[t\omega]c_1^2k_1^2(150-180x^2+30x^4)+M\omega^3\sin[t\omega]c_1^2k_1^2 \\ (300-360x^2+60x^4)+360\omega^2\cos[t\omega]c_1^2k_2(x^2-1)+360M\omega\sin[t\omega]c_1^2k_2(x^2-1) \\ 360M\omega^2\cos[t\omega]c_1^2k_1k_2(x^2-1)+360\omega^3\sin[t\omega]c_1^2k_1k_2(1-x^2) \end{pmatrix}. \tag{42}$$

The series solutions of velocity profile are obtained as

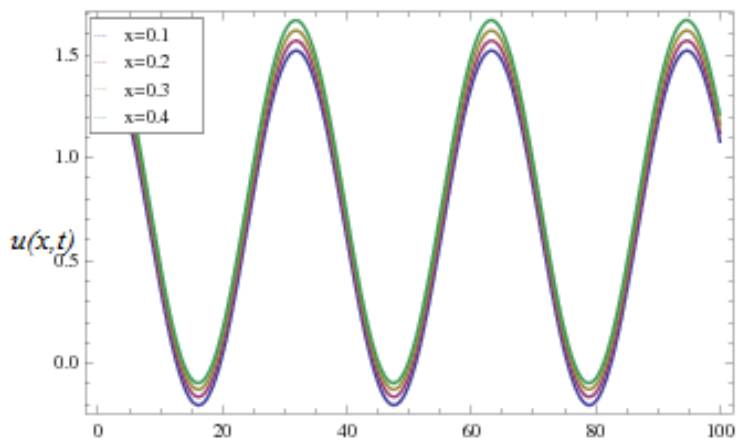
$$u(\mathrm{x},t)=u_0(\mathrm{x},t)+u_1(\mathrm{x},t)+u_2(\mathrm{x},t), \tag{43}$$

$$u(\mathrm{x},t)=\cos[t\omega]+\frac{1}{2}\left(1+m+x-mx^2\right)+\frac{1}{24}(Mc_1\left(6+5m+2x-6x^2-6mx^2-2x^3+x^4\right)+12M$$
$$\mathrm{Cos}[t\omega]c_1\left(1-x^2\right)-12\omega\mathrm{Sin}[t\omega]c_1\left(1+x^2\right)-12\omega^2\mathrm{Cos}[t\omega]c_1k_1\left(1-x^2\right)+12M\omega\mathrm{Sin}[t\omega]c_1k_1\left(1+x^2\right))$$
$$+\frac{1}{720}Mc_1(180+150m+60x-180x^2-180mx^2-60x^3+30mx^4)+360M\cos[t\omega]c_1(1-x^2)+360\omega$$
$$\sin[t\omega]c_1(x^2-1)+Mc_1^2(180+150m+75M+61mM+60x+7Mx-180x^2-180mx^2-90Mx^2-$$
$$75mx^2-60x^3-10Mx^3+30mx^4+15Mx^4+15mMx^4+3Mx^5-mMx^6)+M\cos[t\omega]c_1^2(360+150M$$
$$-360x^2-180Mx^2+30Mx^4)+\omega^2\cos[t\omega]c_1^2(-150+180x^2-30x^4)+\omega\sin[t\omega]c_1^2(-360-300M+$$
$$360x^2+360Mx^2-60Mx^4)+Mc_2(180+150m+60x-180x^2-180mx^2-60x^3+30mx^4)+360M$$
$$\cos[t\omega]c_2(1-x^2)+360\omega\sin[t\omega]c_2(x^2-1)+360\omega^2\cos[t\omega]c_1k_1(x^2-1)+360M\omega\sin[t\omega]c_1k_1$$
$$(x^2-1)+\omega^2\cos[t\omega]c_1^2k_1(-360-600M+360x^2+720Mx^2-120Mx^4)+M\omega\sin[t\omega]c_1^2k_1+(-360$$
$$-300M+360x^2+360Mx^2-60M^2x^4)+\omega^3\sin[t\omega]c_1^2k_1(300-360x^2+60x^4)+360\omega^2\cos[t\omega]$$
$$c_2k_1(x^2-1)+360M\omega\sin[t\omega]c_2k_1(x^2-1)+M^2\omega^2\cos[t\omega]c_1^2k_1^2(-150+180x^2-30x^4)+\omega^4\cos$$
$$[t\omega]c_1^2k_1^2(150-180x^2+30x^4)+M\omega^3\sin[t\omega]c_1^2k_1^2(300-360x^2+60x^4)+360\omega^2\cos[t\omega]c_1^2k_2$$
$$(x^2-1)+360M\omega\sin[t\omega]c_1^2k_2(x^2-1)360M\omega^2\cos[t\omega]c_1^2k_1k_2(x^2-1)+360\omega^3\sin[t\omega]c_1^2k_1k_2(1-x^2). \tag{44}$$

The values of C_i for the velocity components are $C_1=-0.9979302338072563$ and $C_2=0.0015788902084095113$



"**Fig. 2**", The effect of different time level, on velocity profile when
$\omega = 0.2, m = 0.4, M = 0.3, k_1 = 0.4, k_2 = 0.3, c_2 = 0.00157, c_1 = 0.99793.$



"**Fig. 3**", The velocity distribution graphs when we take the velocity profile when
$\omega = 0.2, m = 0.4, M = 0.3, k_1 = 0.3, k_2 = 0.4, c_2 = 0.00157, c_1 = 0.99793.$

Poiseuille flow Problem:

In this section, both plates are at rest. The flows between the plates are totally due to the existence of a constant pressure gradient, the modelled equations with boundary area

$$\left(1+\lambda_r \frac{\partial}{\partial t}\right)\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \nu\left(1+\lambda_t \frac{\partial}{\partial t}\right)\frac{\partial^2 u}{\partial x^2} - \frac{\sigma B_0^{\,2}}{\rho}\left(1+\lambda_r \frac{\partial}{\partial t}\right)u + \rho g. \tag{45}$$

$$u_0(1,t) = 1 + c\cos\omega t, \mathrm{u}_0(-1,t) = 1 + \cos\omega t. \tag{46}$$

The non-dimensional form of [45] and [46]

$$\left(1+\lambda_r \frac{\partial}{\partial t}\right)\frac{\partial u}{\partial t} = -\Omega + v\left(1+\lambda_t \frac{\partial}{\partial t}\right)\frac{\partial^2 u}{\partial x^2} - \frac{\sigma B_0{}^2}{\rho}\left(1+\lambda_r \frac{\partial}{\partial t}\right)u + \rho g. \tag{47}$$

$$u_0(1,t) = c\cos\omega \mathrm{t}, \mathrm{u}_0(-1,t) = \cos\omega \mathrm{t}. \tag{48}$$

$$u_0(\mathrm{x},t) = \frac{1}{2}(1+x)(\mathrm{m}+\mathrm{x}) + \Omega(1-\mathrm{x}) + \cos\omega \mathrm{t}. \tag{49}$$

$$u_1(\mathrm{x},t) = \frac{1}{24}\left[\begin{array}{l} -24\left(1-x^2\right) - 24\Omega\left(1-x^2\right) + Mc_1\left(5m-6x^2+mx^4\right) + 24\Omega c_1(1+M-x^2-6\mathrm{M}x^2+Mx^4) \\ +12M\mathrm{Cos}\omega \mathrm{t}c_1\left(1-x^2\right) - 12\omega\mathrm{Sin}\omega \mathrm{t}c_1\left(1+x^2\right) - 12\omega^2\mathrm{Cos}\omega \mathrm{t}c_1k_1\left(1-x^2\right) + 12M\omega\mathrm{Sin}\omega \mathrm{t}c_1 \\ k_1\left(x^2-1\right). \end{array}\right] \tag{50}$$

Here u_2 is extremely large, so we give just its graphical solution

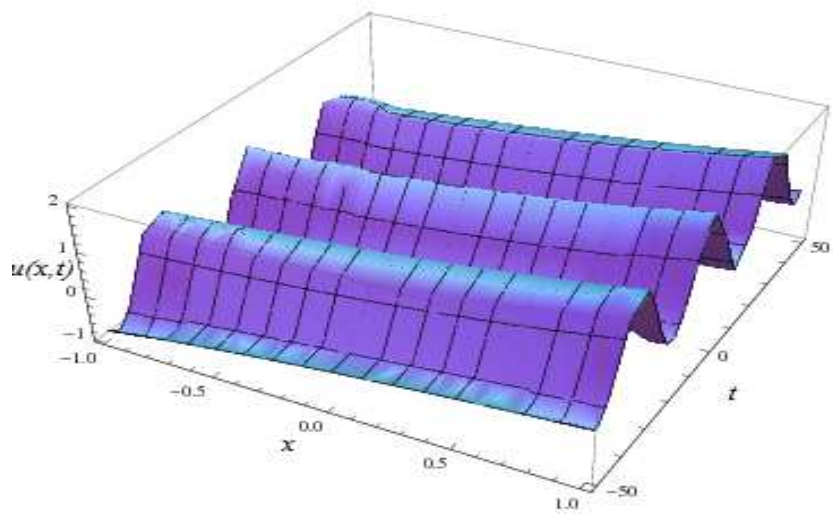
The series solutions up to two terms of velocity profile is obtained as

$$u(\mathrm{x},t) = u_0(\mathrm{x},t) + u_1(\mathrm{x},t) \tag{51}$$

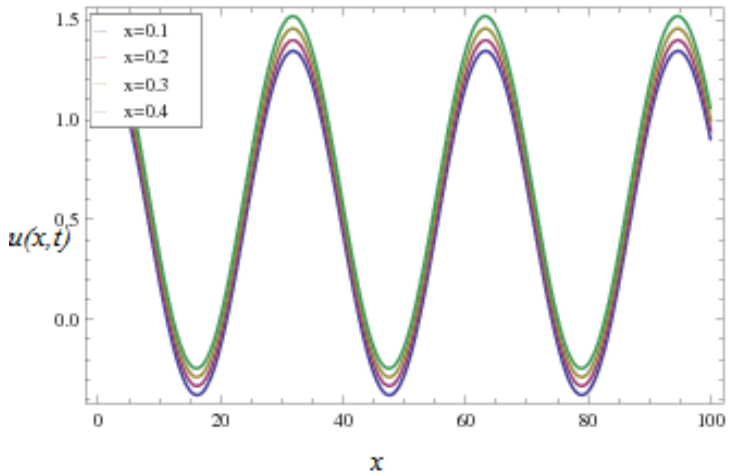
$$\begin{array}{l} u(\mathrm{x},t) = \frac{1}{2}(1+x)(\mathrm{m}+\mathrm{x}) + \Omega(1-\mathrm{x}) + \cos\omega \mathrm{t} + \frac{1}{24}(-24\left(1-x^2\right) - 24\Omega\left(1-x^2\right) + Mc_1\left(5m-6x^2+mx^4\right) \\ +24\Omega c_1(1+M-x^2-6\mathrm{M}x^2+Mx^4) + 12M\mathrm{Cos}\omega \mathrm{t}c_1\left(1-x^2\right) - 12\omega\mathrm{Sin}\omega \mathrm{t}c_1\left(1+x^2\right) - 12\omega^2\mathrm{Cos}\omega \mathrm{t}c_1k_1 \\ \left(1-x^2\right) + 12M\omega\mathrm{Sin}\omega \mathrm{t}c_1k_1\left(x^2-1\right)). \end{array} \tag{52}$$

The values of C_i for the velocity components are

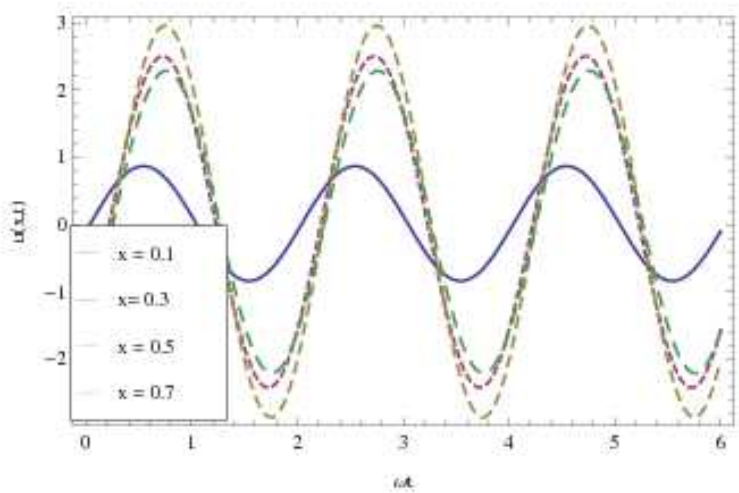
$$C_1 = -1.0243141149119446; C_2 = -0.0006983026317248786;$$



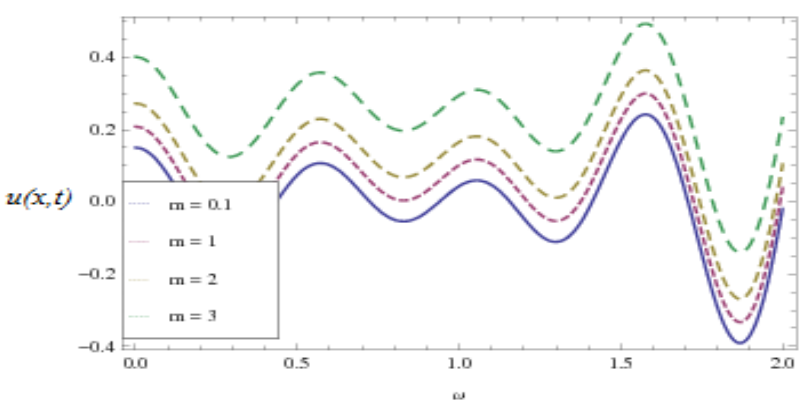
"**Fig. 5**"The effect of various time levels, on the velocity profile in case of Poiseuille flow wing $\omega = 0.2, m = 0.4, M = 0.3, k_1 = 0.3, k_2 = 0.3, c_2 0.00157, c_1 = 0.99793.$



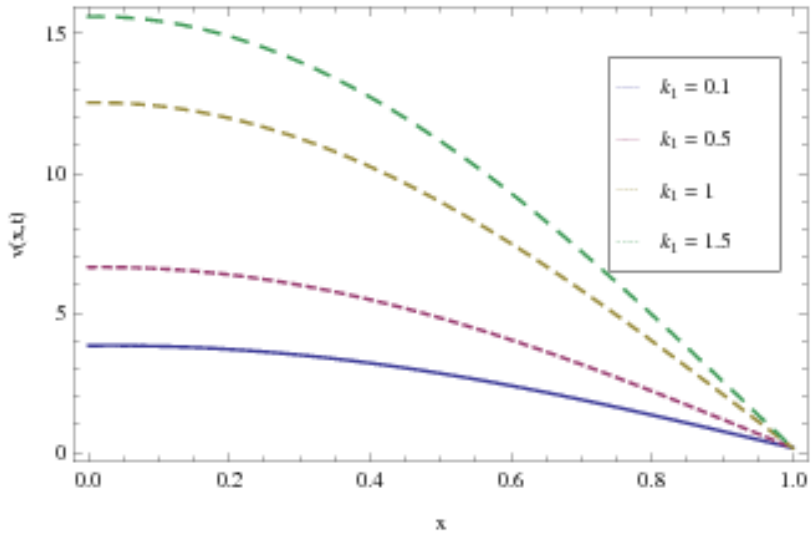
"**Fig. 6**"The effect of various time levels, on the velocity profile in case of Poiseuille when $\omega = 0.2, m = 0.4, M = 0.3, k_1 = 0.3, k_2 = 0.4, c_2 0.00157, c_1 = 0.99793.$



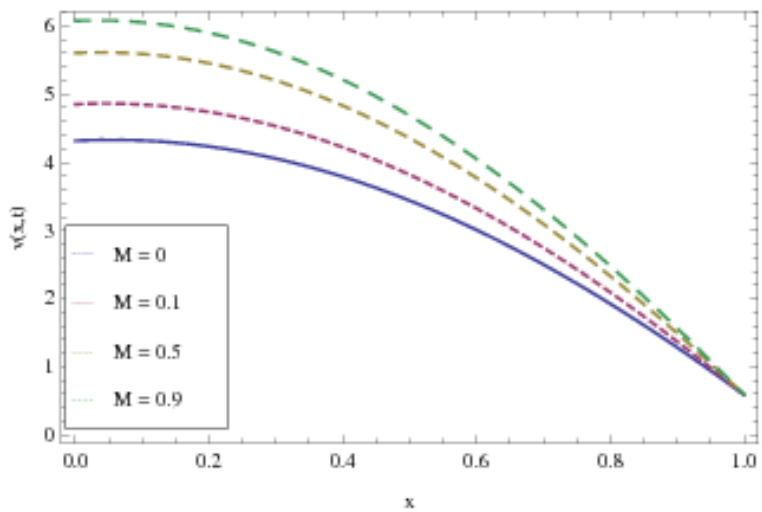
"**Fig. 7**"The effect oscillating frequency. When $k_1 = 0.8, k_2 = 0.5, \omega = 03.14, m = 0.5, M = 4.$



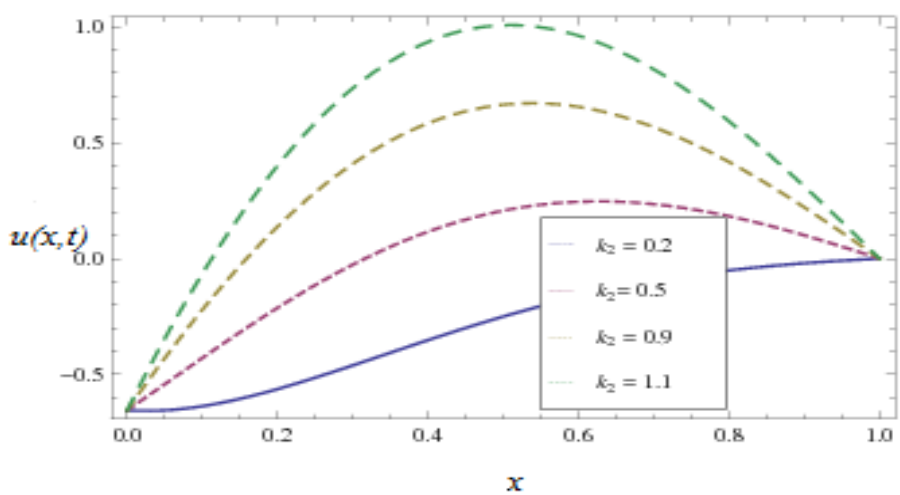
"**Fig. 7**"The effect gravitational parameter. When $k_1 = 0.8, k_2 = 0.5, \omega = 0.3, M = 4, t = 1.$



"**Fig. 8**"The effect of non-Newtonian parameter k_1. When $\omega = 2, m = 0.4, M = 4, k_2 = 0.2, t = 1$.



"**Fig. 9**"The effect of Magnetic parameter. When $\omega = 2; m = 0.4, k_1 = 0.5, k_2 = 0.8, t = 1$.



"**Fig. 10**"The effect of non-Newtonian parameter k_1 .When $\omega = 2, m = 0.4, M = 4, k_1 = 0.2, t = 1$.

Results and Discussion

An unsteady MHD Oldroyd-B fluid over an oscillating flow in vertical plat has been studied in this article. The governing nonlinear partial differential equations is muddled and solved analytically for velocity profile by using OHAM methods. Fig. 1 Illustrate the physical geometry of the problem [1]. Fig [2-3] shows the effect of different time level and velocity distribution of velocity field in case of couette flow problem. Fig [5-6] shows the same result for Poiseuille flow problem. The influence of first three periods, $\omega t\ \epsilon[0,\ 3\pi]$ are used to study the osallating effect near the plate as shown in Fig. 7. The fluid close the plate oscillates mutually with the plate in the similar period, this is due to the no-slip condition. The amplitude fluid motion rises slowly to the direction of the surface of the fluid flow. The influence of magnetic field on velocity field is studied in Fig. [9].The magnetic field show increase in the flow motion. Fig. 10 shows an increase in the fluid motion when m(gravitational parameter) is increases. This is actually occurred due to friction force which seems greater near the plates and less on the fluid surface. The effects of k_1 and k_2 (relaxation and retardationtime parameter),are shown Figs. 8 and 11. When we Increased k_1 and k_2 it increased fluid motion.

Conclusion:

We have considered for unsteady and MHD flow of an Oldoyed-B fluid flow. The non-linear problem arises from the modelling are solved by using Optimal Homotopy Asympotic Method. The result obtained is an anyllitical.The problem is discussed in two cases.In first case couette flow of the fluid is studied in vertical plates .The graphical solution have been discussed and physical interruption of different parameter is given with detailed.In second case poisulle flow problem is discussed for Oldroyed –B fluid in vertical plates. For solution of the problems OHAM methed is used. k_1 and k_2 are (relaxation time parameter) and (retardation time parameter) its effect is shown graphically.

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