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Flow of Oldroyd-B Fluid between Two Inclined and Oscillating Plates

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ABSTRACT

The studies of this paper related to the unsteady magnetohydrodynamics (MHD) incompressible Oldroyd-B fluid between two periodically vibrated inclined plates. The basic governing equations of the above mentioned fluid are modeled and simplified in the form of nonlinear PDEs. The solution of the above non-linear PDEs has been obtained by using two analytical methods, namely HPM and OHAM. The comparison of these two methods are also analyzed and found in very good agreement. Finally, the physical effects of modelled parameters have been studied graphically.

KEYWORDS: Unsteady motion, thin film, MHD, Oldroyd-B fluid, inclined belt, OHAM and HPM.

I. INTRODUCTION

In everyday life and in engineering flow of non-Newtonion fluid are frequently occurred. It is ubiquitous in nature and technologies. Therefore, to understand there mechanics is essential in most of applications. Most of it problems appeared in several areas varying from blood to the engineering and technologies problem. In non-Newtonian fluids Thin film flows have a large variety of practical applications in nonlinear sciences and engineering industries. For modelling of non-Newtonion fluid flow problem we used several model lik (second grade, third grad, fourth grad oldoryd four, oldoyed-B) etc fluids model by which we got linear or nonlinear ode,s or pde,s which may be solved exactly, analyticaly or numerically. For solution of these problems different varieties of methods are used, in which (ADM,VAM,HAM,HPM,OHAM) are frequently used. In this work we have modelled a non linear pde,s by using oldroyd-B fluid model. Solution of the problem is obtained by using OHAM and HPM. Anakira et all [1] have used OHAM for the solution of Delay differential equations. Akselet all [2] have been examined the solutions of for some unsteady of oldroyd-B fluids in unidirectional flows. Ayubet all [3] discussed the oscillating motions in plane wall for said fluid. Burdujan [4] take it in particular class and give a brief discussion on it. Fetecau et al. [5-6] studied oldroyd –B fluid in form of constantly accelerating flow over a flat plate and in second paper he discussed transient oscillating motion using same model in cylindrical domains. In both case he obtained the exact solutions.

The related work can be seen in [3-6].Ghosh et all [7-9] examined Oldroyd-B fluid under the effect of *MHD* in different case that is pulsating flow, Chanel flow and induced by rectified sine pulses. All of details is given briefly in their works. Gul et al. [10, 12] investigated the MHD thin film flow of non-Newtonian fluid on a vertical and in inclined oscillating belts. They obtained analytical solution of using the Asymptotic perturbation methods (ADM , OHAM and HPM). The obtained results of the lift and drainage problems for velocity and as well as for temperature field are compared and given graphically. The properties of several physical parameters are also explained. Hameedet all [13] discussed thin film fluids in vertical belt. Haitao [14] et all studied unidirectional flow of an oldoyd-B fluid. He JH [15-20] has been given a brief disruption on HPM methods. Khan et al [21-23]studied Oldroyed –b fluid in rotating system with effect of MHD and also studied it in unsteady stat and find some exact solutions for it. They examined it in porous space. Kashkari [24] examined the OHAM application and approximate solution of nonlinear Kawahara equation. Lie et all [25-26] studied Axial MHD flow of generalized Oldroyd-B fluid due to two oscillating cylinder. They gives basic introduction of HPM with detail examples. Marinca et al. [26-29] applied OHAM on non-linear steady flow problem modelled from fourth order fluid and find approximate solution. They used OHAM for thin film flow problem and for nonlinear oscillators with discontinuities. Mabood et al [30-31] gives application of Optimal homotopy asymptotic method for the approximate solution and they also examined the heat transfer on MHD stagnation point flow in porous medium. Nofel [32] examined temperature conduction and fractional vanderpol damped nonlinear oscillator.

Shahid et al [33] studied unsteady flow of Oldroyd-B fluid in steady stats. Transient solution have been obtained with used of Laplace and Fourier series. Shah et al. [34] examined thin film flow of third order fluid on moving inclined plane and found it solution by using OHAM. Siddiqui et al. [35-37] examined the thin film

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flow of a third order fluid over an inclined plane as well they studied it in moving vertical belt. For solution of the non-linear problems they used HPM in first case and OHAM in other case.

II. Basic Equation

Momentum and continuity equation are written as

$$div\mathbf{v} = 0\,, \tag{1}$$

$$\rho\frac{D\mathbf{v}}{Dt} = div\mathrm{T} + \mathbf{J}\times\mathbf{B} + \rho g\sin\theta. \tag{2}$$

Where

$$\mathbf{J}\times\mathbf{B} = \left[0, -\sigma B_0^2 \mathrm{u}, 0\right], \tag{3}$$

$$\mathbf{J} = \sigma\left(\mathbf{E} + \mathbf{U}\times\mathbf{B}\right), \quad \nabla\times\mathbf{B} = \mu_0\mathbf{J}.$$

$\frac{D}{Dt}$ shows the material time derivative ρ is density, and g is body force and v is the velocity.Where B is the uniform induction filed , σ is the electrical conductivity and $\mathbf{J}$ is current density.

$$\frac{D\mathbf{\Psi}_1}{Dt} = \frac{\partial\mathbf{\Psi}_{11}}{\partial t} + \left(\mathbf{v}.\nabla\right)\mathbf{\Psi}_{11} - \left(\nabla\mathbf{v}\right)\mathbf{\Psi}_{11} - \mathbf{\Psi}_{11}\left(\nabla\mathbf{v}\right)^T. \tag{5}$$

The above model can be reduced to different types of fluid depend on λ_r (relaxation time) and λ_t (retardation time).In equation (5), if $\lambda_r = \lambda_t$ the fluid becomes viscous. When $\lambda_t = 0$, it becomes a Maxwell fluid and reduced to Oldroyd-B fluid when $0 < \lambda_t < \lambda_t < 1$.

The cauchy stress tensor, T is

$$\mathbf{T} = -\mathrm{P}\mathbf{I} + \boldsymbol{\tau}, \tag{6}$$

$p\mathbf{I}$ is isotropic stress.

$$\mathbf{T} + \lambda_1\frac{\mathbf{D}\,\mathbf{T}}{Dt} = \mu\left[1 + \lambda_2\frac{D}{Dt}\right]\mathbf{A}_1, \tag{7}$$

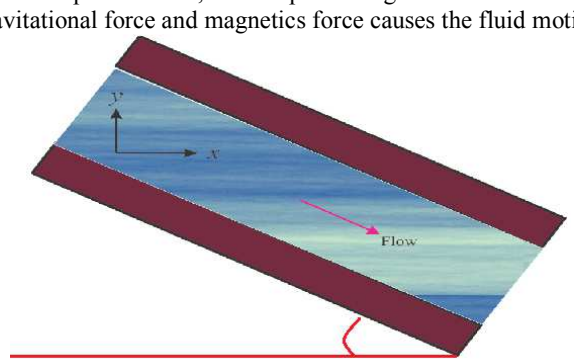
$\mathbf{T}$ is used for extra tress tensorand μ is used for cofficient of viscosity.

$$\mathbf{A}_0 = \mathbf{I}\ ,\ \mathbf{A}_1 = \nabla v + (\nabla v)^T, = \nabla v\,, \tag{8}$$

$\mathbf{A}_1$ is the Rivlin Ericksen stress tensor`.

III. Formulation of the Problem

Consider incompressible non-Newtonian Oldroyd-B fluid between two inclined and parallel plates $y = h$ and $y = -h$ .Both plates are oscillating.The configuration of fluid flow is along the y-axis and perpendicular to x-axis. A transvers uniformmagnetic field is applied on inclined directionl where the plate is non conductiong.The pressure P is kept constant , that is pressure gradient is zero .The flow is supposed to be unsteady flow, laminar .Gravitational force and magnetics force causes the fluid motion.



"**Fig. 1**",Geometry of the problem

The velocity field is of the form

$$\mathbf{v}=\left(u\left(\mathrm{y},t\right),0,0\right),\text{ and }\boldsymbol{\tau}=\left(\mathrm{y},t\right) \tag{9}$$

subject to the boundary conditions

$$u\left(-\mathrm{h},t\right)=U\cos\omega t\ \ ,u\left(\mathrm{h},t\right)=U\cos\omega t\ , \tag{10}$$

Where ω is the frequency of the oscillating belt.
The momentum equation (2) is reduced to

$$\rho\frac{\partial v}{\partial t}=-\frac{\partial p}{\partial x}+\frac{\partial\tau_{xy}}{\partial y}+\rho\mathrm{g}\sin\theta-\sigma B_0{}^2\,\mathrm{v}, \tag{11}$$

$$\frac{\partial p}{\partial y}=\frac{\partial\tau_{yy}}{\partial y}, \tag{12}$$

$$\frac{\partial p}{\partial z}=0. \tag{13}$$

It follows from (7) and (9) that

$$\tau_{xx}+\lambda_1\left[\frac{\partial\tau_{xx}}{\partial t}-2\tau_{xy}\frac{\partial\mathrm{v}}{\partial y}\right]=-2\mu\lambda_2\left[\frac{\partial\mathrm{v}}{\partial y}\right]^2, \tag{14}$$

$$\tau_{xy}+\lambda_1\left[\frac{\partial\tau_{xy}}{\partial t}-\tau_{yy}\frac{\partial\mathrm{v}}{\partial y}\right]=\mu\left(\frac{\partial\mathrm{v}}{\partial y}\right)+\lambda_2\mu\left(\frac{\partial^2\mathrm{v}}{\partial t\partial y}\right), \tag{15}$$

$$\tau_{yy}+\lambda_1\frac{\partial\tau_{yy}}{\partial t}=0, \tag{16}$$

Equation (16) reduces to

$$\tau_{yy}=\Psi\left(y\right)e^{-\frac{t}{\lambda_1}}. \tag{17}$$

Where $\Psi\left(y\right)$ any arbitrary function. When $t<<0$, then $\tau_{yy}=0$. Equations (11) and (15) and in the presence of zero pressure gradient, we obtain

$$\left[1+\lambda_1\frac{\partial}{\partial t}\right]\frac{\partial v}{\partial t}=v\left[1+\lambda_2\frac{\partial}{\partial t}\right]\frac{\partial^2 v}{\partial y^2}-\frac{\sigma B_0{}^2}{\rho}\left[1+\lambda_1\frac{\partial}{\partial t}\right]v+\rho g\sin\theta. \tag{18}$$

Introducing non-dimensional variables

$$v=\frac{\breve{v}}{V},y=\frac{\breve{y}}{\delta},t=\frac{v\breve{t}}{\rho\delta^2},\mathrm{K}_1=\frac{\lambda_1 v}{\rho\delta^2},\mathrm{K}_2=\frac{\lambda_2 v}{\rho\delta^2},\omega=\frac{\breve{\omega}\delta^2\rho}{\mu},m=\frac{\delta^2\rho g\sin\theta}{\mu V},M=\frac{\sigma B_0^2\delta^2}{\mu}. \tag{19}$$

where, M is the magnetic parameter, ω is the oscillating parameter, K_1 is the relaxation paramter, K_2 is the retardation paramete and m is the gravitational parameter.
Using (19) in (18) and (10) and dropping bars we obtain

$$\left[1+\mathrm{K}_1\frac{\partial}{\partial t}\right]\frac{\partial u}{\partial t}=\left[1+\mathrm{K}_2\frac{\partial}{\partial t}\right]\frac{\partial^2 u}{\partial y^2}-M\left[1+\lambda_1\frac{\partial}{\partial t}\right]u+m. \tag{20}$$

$$v\left(-1,t\right)=\cos\omega t\ ,\ v\left(1,t\right)=\cos\omega t\ , \tag{21}$$

IV. The HPM Solution for the Problem

By applying HPM to (20) and (21) component form is asT

$$\mathrm{P}^0:\frac{\partial^2 v_0\left(y,t\right)}{\partial y^2}=-m\ , \tag{45}$$

$$\mathrm{P}^1:\ \frac{\partial^2 v_1\left(y,t\right)}{\partial y^2}=2\frac{\partial^2 v_0}{\partial y^2}-k_1\frac{\partial^2 v_0}{\partial t^2}+k_2\frac{\partial}{\partial t}\left(\frac{\partial^2 v_0}{\partial y^2}\right)-Mk_1\frac{\partial v_0}{\partial t}+2m-Mv_0, \tag{46}$$

$$\mathrm{P}^2:\ \frac{\partial^2 v_2(y,t)}{\partial y^2}=2\frac{\partial^2 v_1}{\partial y^2}-\frac{\partial v_1}{\partial t}-\mathrm{K}_1 M\frac{\partial^2 v_1}{\partial t^2}+\mathrm{K}_2\frac{\partial}{\partial t}\left(\frac{\partial^2 v_1}{\partial y^2}\right)-Mv_1\ , \tag{47}$$

Solutions of equations (22-24) using boundary condition in equation (20) is

$$v_0(y,t)=\left\{\frac{1}{2}\mathrm{m}(y^2-1)+\cos[t\omega]\right\}. \tag{48}$$

$$v_1(y,t)=\frac{1}{24}\left\{\begin{array}{l}\mathrm{m\ M}\left(5\text{ - }6\ \ \mathrm{y}^2+\ \mathrm{y}^4\right)+12M\mathrm{Cos}[t\omega]\left(1-y^2\right)-12\omega\mathrm{Sin}[t\omega]\left(y^2-1\right)-12\omega^2\mathrm{Cos}[t\omega]\mathrm{K}_1\\ \left(1-y^2\right)-12M\omega\mathrm{Sin}[t\omega]\mathrm{K}_1\left(1-y^2\right)\end{array}\right\} \tag{49}$$

$$v_2(y,t)=\left\{\begin{array}{l}Mm(300+61M-360y^2-75mMy^2+60y^4+15My^4-My^6)+M\cos[t\omega](7200+150\\ M-720y^2-180My^2+30Mx^4)+\omega^2\cos[t\omega](-150+180y^2-30y^4)+\omega\sin[t\omega]c_1^2(-720\\ -300M+720y^2+360My^2-60My^4)+M(180+150m+60x-180x^2-180mx^2-60x^3+\\ 30mx^4)+\omega^2\cos[t\omega]\mathrm{K}_1(-700-600M+720y^2+720My^2-120My^4)+M\omega\sin[t\omega]\mathrm{K}_1(-720\\ -300M+720y^2-60My^4)+\omega^3\sin[t\omega]\mathrm{K}_1(-360y^2+60y^4)+M^2\omega^2\cos[t\omega]\mathrm{K}_1^2(-150+180y^2\\ -30y^4)+\omega^4\cos[t\omega]\mathrm{K}_1^2(150-180y^2+30y^4)+M\omega^3\sin[t\omega]\mathrm{K}_1^2(300-360y^2+60y^4)+360\omega^2\\ \cos[t\omega]\mathrm{K}_2(\mathrm{y}^2-1)+360M\omega\sin[t\omega]\mathrm{K}_2(\mathrm{y}^2-1)360M\omega^2\cos[t\omega]\mathrm{K}_1\mathrm{K}_2(\mathrm{y}^2-1)\\ +360\omega^3\sin[t\omega]\mathrm{K}_1\mathrm{K}_2(1-y^2).\end{array}\right\} \tag{50}$$

The series solutions of velocity profile up to second component is

$$v(y,t)=v_0(y,t)+v_1(y,t)+v_2(y,t). \tag{51}$$

$$\begin{array}{l}\mathrm{v(y,t)}=\frac{1}{2}\mathrm{m}(y^2-1)+\cos[t\omega]+\frac{1}{24}[\mathrm{m\ M}\left(5\text{ - }6\ \ \mathrm{y}^2+\ \mathrm{y}^4\right)+12M\mathrm{Cos}[t\omega]c\left(1-y^2\right)-12\omega\mathrm{Sin}[t\omega]\\ \left(y^2-1\right)-12\omega^2\mathrm{Cos}[t\omega]\mathrm{K}_1\left(1-y^2\right)-12M\omega\mathrm{Sin}[t\omega]\mathrm{K}_1\left(1-y^2\right)]+(Mm(300+61M-360y^2-75mMy^2\\ +60y^4+15My^4-My^6)+M\cos[t\omega](7200+150M-720y^2-180My^2+30Mx^4)+\omega^2\cos[t\omega](-150\\ +180y^2-30y^4)+\omega\sin[t\omega](-720-300M+720y^2+360My^2-60My^4)+Mc_2(180+150m+60x\\ -180x^2-180mx^2-60x^3+30mx^4)+\omega^2\cos[t\omega]\mathrm{K}_1(-700-600M+720y^2+720My^2-120My^4)+M\omega\\ \sin[t\omega]\mathrm{K}_1(-720-300M+720y^2-60My^4)+\omega^3\sin[t\omega]k_1(-360y^2+60y^4)+M^2\omega^2\cos[t\omega]\mathrm{K}_1^2(-150\\ +180y^2-30y^4)+\omega^4\cos[t\omega]\mathrm{K}_1^2(150-180y^2+30y^4)+M\omega^3\sin[t\omega]\mathrm{K}_1^2(300-360y^2+60y^4)+360\omega^2\\ \cos[t\omega]\mathrm{K}_2(\mathrm{y}^2-1)+360M\omega\sin[t\omega]\mathrm{K}_2(\mathrm{y}^2-1)360M\omega^2\cos[t\omega]\mathrm{K}_1\mathrm{K}_2(\mathrm{y}^2-1)+360\omega^3\sin[t\omega]\mathrm{K}_1\mathrm{K}_2(1-y^2).\end{array} \tag{52}$$

V. The OHAM Solution for the Problem

The component problems of velocity profile are when OHAM method is applied

$$\mathrm{P}^0:\frac{\partial^2 v_0(y,t)}{\partial y^2}=-m. \tag{53}$$

$$\mathrm{P}^1:\frac{\partial^2 v_1(y,t)}{\partial y^2}=-\mathrm{K}_1c_1\frac{\partial^2 v_0}{\partial t^2}+\left(1+c_1\right)\frac{\partial^2 v_0}{\partial y^2}+\mathrm{K}_2c_1\frac{\partial}{\partial t}\left(\frac{\partial^2 v_0}{\partial y^2}\right)-c_1\left(1+M\mathrm{K}_1\right)\frac{\partial v_0}{\partial t}+m\left(1+c_1\right)-Mc_1v_0 \tag{54}$$

$$p^2 : \frac{\partial^2 v_1(y,t)}{\partial y^2} = -\mathrm{K}_1 c_1 \frac{\partial^2 v_1}{\partial t^2} + c_2 \frac{\partial^2 v_0}{\partial y^2} + k_2 c_2 \frac{\partial}{\partial t}\left(\frac{\partial^2 v_0}{\partial y^2}\right) + (1+c_1)\frac{\partial^2 v_1}{\partial y^2} + k_2 c_1 \frac{\partial}{\partial t}\left(\frac{\partial^2 v_1}{\partial y^2}\right)$$
$$-c_1(1+M\mathrm{K}_1)\frac{\partial v_1}{\partial t} + c_2(m - Mv_0) - Mc_1 v_1 - c_2(1+M\mathrm{K}_1)\frac{\partial v_0}{\partial t}. \tag{55}$$

Solutions to equations (30-32) using boundary condition in equation (21) are

$$v_0(y,t) = \left\{\frac{1}{2}\mathrm{m}(y^2-1) + \cos[t\omega]\right\}. \tag{56}$$

$$v_1(y,t) = \frac{1}{24}\left\{\begin{matrix} \mathrm{m\,Mc_1}(5 - 6\ \mathrm{y}^2 + \mathrm{y}^4) + 12\mathrm{c}_1 M\mathrm{Cos}[t\omega](1-y^2) - 12\mathrm{c}_1\omega\mathrm{Sin}[t\omega](y^2-1) \\ -12\mathrm{c}_1\omega^2\mathrm{Cos}[t\omega]\mathrm{K}_1(1-y^2) - 12\mathrm{c}_1 M\omega\mathrm{Sin}[t\omega]\mathrm{K}_1(1-y^2) \end{matrix}\right\} \tag{57}$$

The second component solution for velocity (in OHAM) is very larg, therefore, only graphical representations up to second order are given.

The series solutions of velocity profile is obtained as

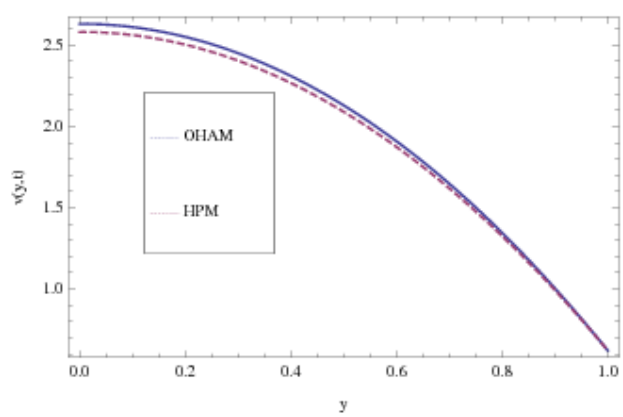
$$v(y,t) = v_0(y,t) + v_1(y,t) + v_2(y,t) \tag{58}$$

The values of C_i for the velocity components are

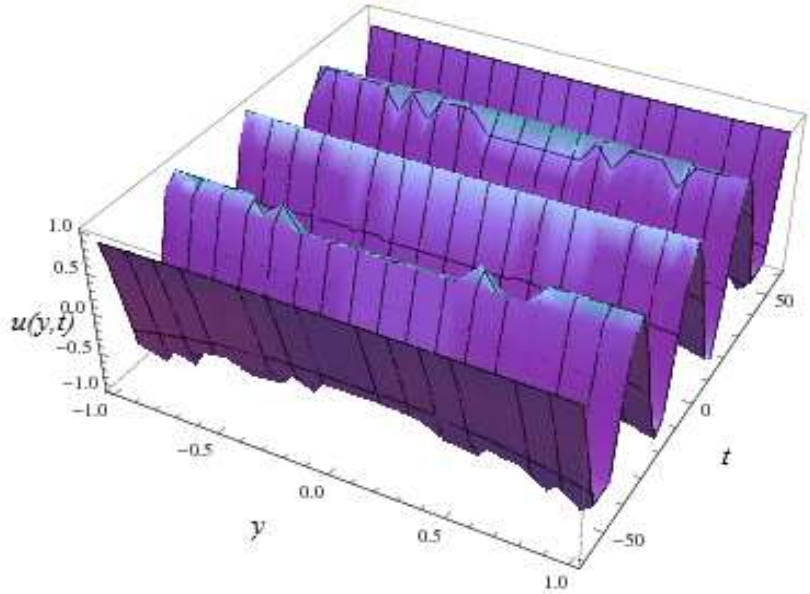
C_1 = -1.0575680997844723, C_2=0.008909892406518072.

"Table 1"Numarical Comparison of *OHAM* and *HPM* for the velocity profile, when $M = 0.02, t = 5, k_1 = 0.02, k_2 = 0.09, t = 10, \omega = 0.09, m = 4.$

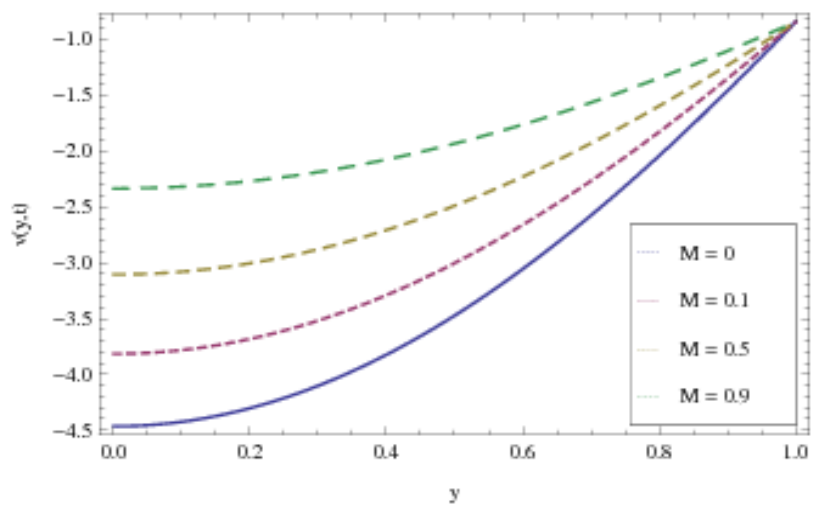
x	OHAM	ADM	Absolute Error
0.0	2.63195	2.58258	0.049368
0.1	2.61188	2.56287	0.0490063
0.2	2.55167	2.50377	0.047905
0.3	2.45131	2.4053	0.0460163
0.4	2.31078	2.26752	0.0432602
0.5	2.13004	2.09052	0.0395247
0.6	1.90906	1.87439	0.0346659
0.7	1.64779	1.61928	0.028508
0.8	1.34616	1.32532	0.0208429
0.9	1.00413	0.992699	0.0114309
1.0	0.62161	0.62161	-1.8311×10^{-18}



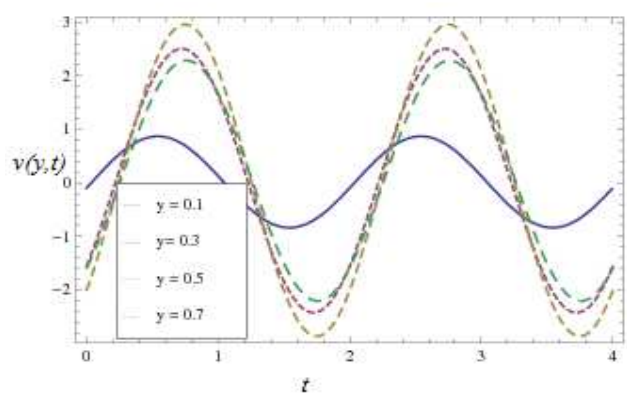
"**Fig.2**", Comparison of OHAM and HPMsolutions for velocity profileby when $\omega = 0.2, m = 0.4, M = 0.5, t = 5, k_1 = 0.6, k_2 = 0.3,$



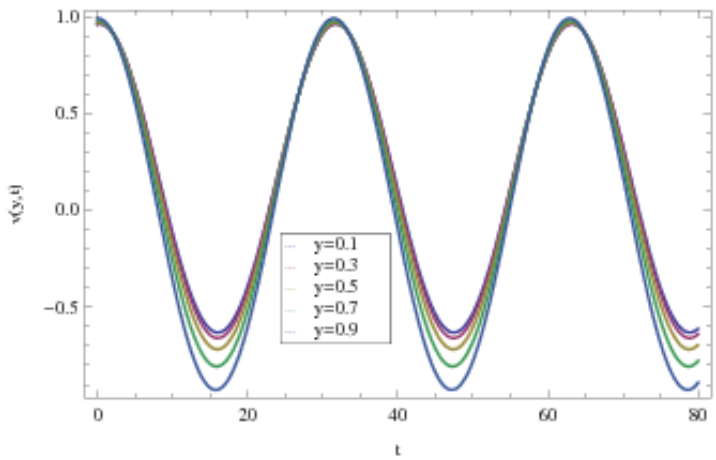
"**Fig. 3**"Influence of different time level when $\omega = 0.2, m = 0.4, M = 0.5, t = 5, k_1 = 0.6, k_2 = 0.3,$



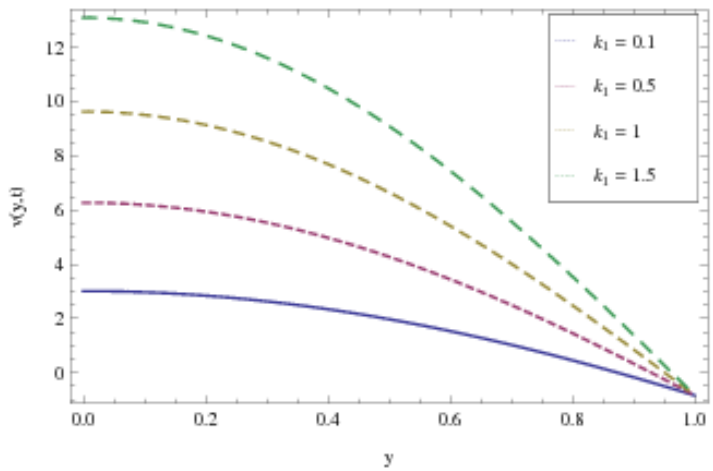
"**Fig. 4**",Influence of magnatic parameter on velocity profile when $\omega = 0.2, m = 0.5, t = 5, k_1 = 0.5, k_2 = 0.7.$



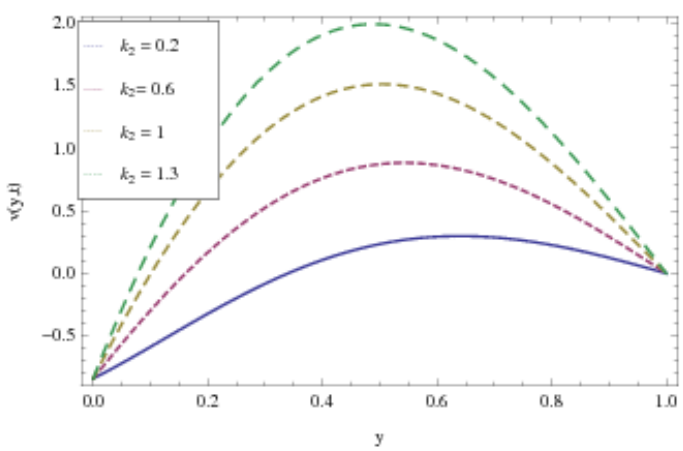
"**Fig. 5**",velocity distrubation at diffrient time level
$\omega = 0.2, m = 0.4, t = 1, k_1 = 0.5, k_2 = 0.7.$



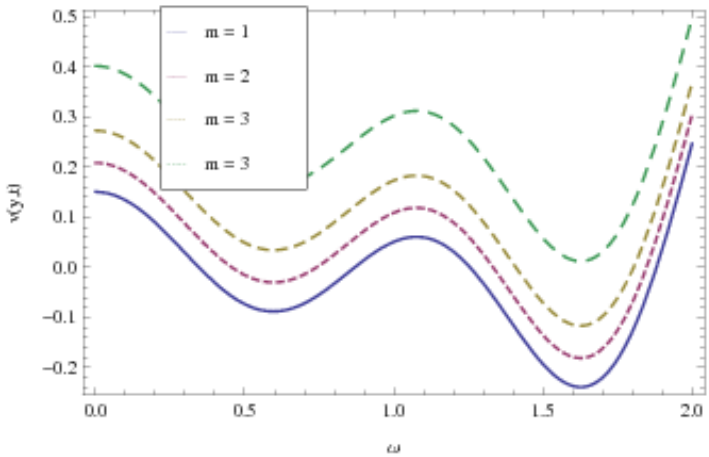
"**Fig. 6**",Velocity distribution graphs when $\omega = 0.2, m = 0.4, M = 0.5, t = 5, k_1 = 0.6, k_2 = 0.3,$



"**Fig.7**", Effect of non-Newtonian parameter k_1 on velocity profiles when $m = 0.4, M = 0.3, t = 1, k_2 = 0.7.$



"**Fig.8**",Effect of non-Newtonian parameter k_2 on velocity profiles when $\omega = 0.2, y = 0.4, M = 0.5, t = 5, k_1 = 0.6.$



"**Fig.9**",Effect of gravity parameter m on the velocity profile when $\omega = 0.2, y = 0.5, \mathrm{M} = 0.5, t = 5, k_2 = 0.7.$

VI. RESULTS AND DISCUSSION

In this work we have studied Oldroyd-B fluid in two inclined oscillating parallel plates.The fluid is considered in unsteady form under the effect of MHD.The obtained results from both OHAM and HPMmethods are compared.We have found that these results are in excellent agreement.Tables [1] show the numerical comparisons of OHAM and HPM.The absolute errors between both methods are calculated and shown.Fig.[1] shows the geometry of the problem. Fig. 2 analyise the comparison of OHAM and HPM solutions by using various values of parameters that is $(\omega, \mathrm{M}, t, k_2, k_1)$ .Fig[3,5,6]show the influence of velocity distribution using parameters $(\omega, \mathrm{M}, t, k_2, k_1)$ and show that velocity field is increases by increasing these parameters.All results for the Oldroyd-B fluid near the plates are showed in the *y*-coordinate only for a specific domain $y \in [0,1]$ .The influence M on velocity field is shownin Fig [4] which give a result that if we increase it,themotionof fluidflowis also increased. The effects of K_1 (relaxation time parameter) and K_2 (retardation time parameter) are shown Figs. [7, 8]. Increase in these parameters increases the velocity profile.Fig [9] show the gravity effect m . Increasing it fluid motion is clearly increased.

VII. CONCLUSION

We have concluded the excellent agreement of HPM and OHAM for the mentioned modelled problem.

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