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Zero Divisor Graph on Modules

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ABSTRACT

Suppose R is a displacement and unit ring and M is a $-\mathrm{R}$ module. In this article, the graph depends on M, we show with $\Gamma(\mathrm{M})$, so that M=R, then $\Gamma(\mathrm{M})$ is classic zero divisor graph. We show that the $\Gamma(\mathrm{M})$ graph with $diam(\Gamma(M)) \leq 3$ is a connected graph and in M Lesser module with condition $Z(M)^* \neq M/\{O\}$, have $gr(\Gamma(M)) = \infty$, if and only if $\Gamma(M)$ is a star graph.

KEYWORDS: Module, Zero divisor graph of modulus, Round, Diameter, Complete bipartite graph

1. INTRODUCTION AND PRELIMINARIES

The first time in 1988, Beck [10] stated the concept of zero divisor graph for a commutative ring. Beck was considered all members of the displacement and unit ring as vertex of the graph and his main task was to find the necessary and sufficient conditions for the finiteness of chromatic number of a graph. Also, according to definition of0 and 1,two vertex of x and y Were adjacent, if and only if xy=0. Reviews related to graph coloring continued by Anderson and Nasir [4]. But from this graph is not obtained interesting results. And in addition to were obvious properties. For example, all its vertices were adjacent to zero. Recently, the zero divisor graph from displacement ring has been extended to the graph maker absurd ideal from displacement ring. (Two ideals I and J are adjacent, whenever $IJ = (O)$).

In [8], the classic zero divisor graph has been extended to modules on displacement rings. According to [11], $m, n \in M$ are adjacent, if and only if $(mR :_R M)(nmR :_R M)M = o$ that is a direct extension of classical zero divisor graph. In [4] and [13],the authors presented two different graphs to a M module$-\mathrm{R}$ according to the first duality $M^* = Hom(M, R)$. Although they necessarily is not generalizations of the classical divisor graph, but there are some deep mutual relations between these two graphs and type of its classic. We first analyzed the expression of several basic definitions.

Definition (1-1): Suppose M is a Abelian collective group and R is displacement ring, in this case the M calls a right $-\mathrm{R}$ module, whenever a scalar multiplication of M elements defined in the following way.

$: M \times R \to M$

$(m, r) = m.r$

So that for each $r, r_1, r_2 \in R$ and $m, m_1, m_2 \in M$ have:

1. $(m_1 + m_2) = m_1 r + m_2 r$
2. $m(r_1 + r_2) = mr_1 + mr_2$
3. $m(r_1 r_2) = (mr_1) r_2$

And if R is unit and $m.1_R = m$, then call M as a unitary module$-\mathrm{R}$.

Definition (1-2): In the $G = (V, E)$ graph (V is represents the vertex, and E is represents the graph edge)a cycle with $n-1$ length is a series of $x_i \in V$ distinct vertices, as $x_1 - x_2 - x_3 - ... - x_n$ such $x_1 = x_n$.

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Definition (1-3): In the $G=(V,E)$ graph the shortest path length between two u and v vertex show with $\mathrm{d(u,v)}$ and the diameter of the graph are defined as follows:

$diam(G)=SUP\{d(u,v)\mid u,v\in V\}$

Definition (1-4): The shortest distance in a graph called back graph that show with $\mathrm{gr(G)}$ symbol. For example, the back cube to length 4.

Definition (1-5): The $G=(V,E)$ graph called bipartite whenever can the set of v vertices partitioned into two sets, so that between the vertices of any set, there is no edge. In addition, the G bipartite graph call complete if any two vertices in a set are not partitioning be adjacent to each other. Complete graph with n of vertex showed with $\mathrm{K_n}$

Definition (1-6): Suppose R is a unit displacement ring and M is a module $-\mathrm{R}$. In the dependent graph on M means $\Gamma(\mathrm{M})$, we say $\mathrm{m,n}\in\mathrm{M}$ are adjacent if and only if $(mR:_R M)(nR:_R M)M=o$.**2. Main Results**

In this chapter, we look to articulate definitions, theorems, concepts and the main results in conjunction with the complete and bipartite graph from zero divisor graph. In the following figure, we show zero divisor graph the some of the $-\mathrm{Z}$ modules. (Example 2-1)

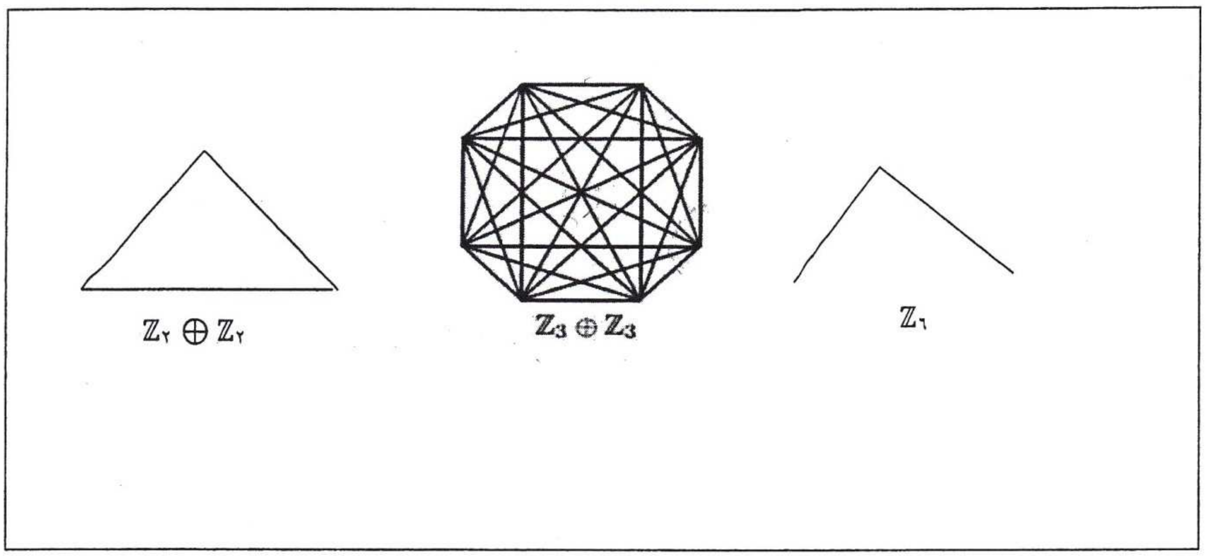

Figure1.

According to the above example, we know that zero divisor graph from $\mathrm{Z_3}\oplus\mathrm{Z_3}$ is a complete graph. Therefore, it is possible for the P first number can be result that the Z module $\mathrm{M}=\mathrm{Z_p}\oplus\mathrm{Z_p}$ is a complete graph with $\left|Z(M)^*\right|=P^2-1$.

Theorem (2-1): Suppose S and S' are the two $-R$ module of simple identical and $M=S\oplus S'$ then $Z(M)^*=M\setminus\{o\}$ and $\Gamma(\mathrm{M})$ is a complete graph.

Proof: Suppose that M_1 and M_2 are two identical modules. Then $\Gamma(M_1)\simeq\Gamma(M_2)$ .With this fact is enough to prove the theorem. Web show that $\mathrm{M}\simeq\mathrm{S}\oplus\mathrm{S}$. For each $o\neq x\in S$ have $((x,o)R:ann(S)$. Also for each $(a,b)\in M$ have $(a,b)((x,o)R:M)=o$ and also (x,o) is adjacent to each element of $\mathrm{(a,b)}\in\mathrm{M}$ nonzero. This result is achieved to $\mathrm{(x,o)}$. Now suppose that x and y is two nonzero element from S. It is easy to show that $\mathrm{a_{nn}(x)=a_{nn}(y)=a_{nn}(S)}$ is a ideal maximal from R. It is obvious that $\mathrm{((x,y)R:M)}$ is including $a_{nn}(S)$. On the other hand if $1\in((x,y)R:M)$ then we have $(x,o)|\in(x,y)R$ and also for each $r\in R$, we have $(x,o)=(x,y)r$. Then $yr=0$ be result that $r\in a_{nn}(y)=a_{nn}(x)$ and also $x=xr=0$, which is a inconsistency. Therefore, $((x,y)R:M)=a_{nn}(S)$ and $\mathrm{(x,y)}$ is adjacent any nonzero element from M.

Theorem (2-2): Suppose R is a commutative ring. In this case, R is a field if and only if $\Gamma(\mathrm{M})$ for each $-\mathrm{R}$ modulus, the M is a complete graph.

$(\Rightarrow)$

Proof: Suppose that M is a field. If $\dim(M_R)=1$ then $\Gamma(\mathrm{M})=\phi$ and therefore, $\Gamma(\mathrm{M})$ is a complete graph. If $\dim(M_R)\geq 2$ for each $o\neq m\in M$, we have $(\mathrm{mR}:\mathrm{M})$, because there is $o\neq r\in(mR:M)$, that result is $\mathrm{Mr}\subseteq\mathrm{mR}$ and also $\mathrm{M}\subseteq\mathrm{mr}^{-1}\mathrm{R}\subseteq\mathrm{mR}$ that is inconsistency. Therefore, for each m,n element from M, we have $\mathrm{n}(\mathrm{mR}:\mathrm{M})=\mathrm{o}$. $(\Rightarrow)$ The N_o assumption is ideal maximal from R. Put $M=\frac{R}{N_o}\oplus R$. Then for each $x\in R/N_o$ and $\mathrm{o}\neq\mathrm{r}\in\mathrm{R}$, we have $(\bar{x},o)((o,r)R:\frac{R}{N_o}\oplus R)=o$. So for each distinct element of s, r apposing zero in R, we have $(o,r)((o,s)R:\frac{R}{N_o}\oplus R)=o$. Then for each $\mathrm{o},1\neq\mathrm{S}\in\mathrm{R}$, we have $(\mathrm{o},1)\mathrm{N_oSR}=\mathrm{o}$. Because $\mathrm{N_oSR}\subseteq((\mathrm{o},\mathrm{s})\mathrm{R}:\frac{\mathrm{R}}{\mathrm{N_o}}\oplus\mathrm{R})$. And this implies that for each $\mathrm{s}\neq\mathrm{o},1$, we have $\mathrm{N_oS}=(\mathrm{o})$ and $\mathrm{N_o}(1-\mathrm{s})=\mathrm{o}$ and therefore, $\mathrm{N_o}=(\mathrm{o})$ and so R is a field.

Theorem (2-4): Suppose M is a $-\mathrm{R}$ modulus of semi-simple finite generator, which homogeneous components is simple, then x, y is adjacent member of $M\setminus\{o\}$, if and only if $\mathrm{xR}\cap\mathrm{yR}=\mathrm{o}$.

Proof: Suppose that $M=\oplus_{i\in I}S_i$, which S_i are non-identical simple below modules from M. suppose that $\mathrm{x},\mathrm{y}\in\mathrm{Z(M)}^*$ are adjacent. We need to show that $\mathrm{xR}\cap\mathrm{yR}=\mathrm{o}$. Suppose $\mathrm{xR}\cap\mathrm{yR}\neq\mathrm{o}$, therefore, there is $\alpha\in\mathrm{I}$ that $\mathrm{S}_\alpha\subseteq\mathrm{xR}\cap\mathrm{yR}$, since y R and x R are below modules from M, there are subsidiaries of A and B from I that $M=xR\oplus(\oplus_{i\in A}S_i)$ and $M=yR\oplus(\oplus_{i\in B}S_i)$. (Lemma at the source [7]). Suppose $x(yR:M)=(o)$ then:

$$(yR:M)=(yR:yR\oplus(\oplus_{i\in B}S_i)=a_{nn}(\oplus_{i\in B}T_i)=\bigcap_{i\in B}a_{nn}(S_i)$$

And $xR\sim\oplus_{i\in I\setminus A}S_i$ as a result: $a_{nn}(xR)=a_{nn}(x)=a_{nn}(\oplus_{i\in I\setminus A}S_i)=\bigcap_{i\in I\setminus A}a_{nn}(S_i)$ .However, since $\mathrm{x}(\mathrm{yR}:\mathrm{M})=\mathrm{o}$, therefore, $(yR:M)\subseteq a_{nn}(x)$ and also we have $\oplus_{\mathrm{i\in B}}\mathrm{S_i}\subseteq\bigcap_{\mathrm{i\in I\setminus A}}\mathrm{a_{nn}}(\mathrm{S_i})$. However, since for each $\mathrm{i},\mathrm{j}\in\mathrm{I}$ ، $\mathrm{a_{nn}}(\mathrm{S_i})$ ، $\mathrm{a_{nn}}(\mathrm{S_j})$ are preliminary, so for each $\mathrm{r}\in\mathrm{I}\setminus\mathrm{A}$, we have $\bigcap_{i\in B}a_{nn}(S_i)=\prod_{i\in B}a_{nn}(S_i)\subseteq\bigcap_{i\in I\setminus A}a_{nn}(S_i)\subseteq a_{nn}(S_r)$. However, for $\mathrm{r}\in\mathrm{I}\setminus\mathrm{A}$, there is $j_r\in B$, which $a_{nn}(S_{jr})\subseteq a_{nn}(S_r)$ and so we have $a_{nn}(S_{jr})\subseteq a_{nn}(S_r)$. So $S_{jr}\simeq S_r$ and according to our assumption $\mathrm{S_{jr}}=\mathrm{S_r}$. Because, there is $\alpha\in\mathrm{I}$ that $\mathrm{S}_\alpha\subseteq\mathrm{xR}\cap\mathrm{yR}$. Since $S_\alpha\subseteq xR\simeq\oplus_{i\in I\setminus A}S_i$, therefore, there is $\mathrm{i}\in\mathrm{I}\setminus\mathrm{A}$, which $\mathrm{S}_\alpha\simeq\mathrm{S_i}$. However, the above equations, there is $\mathrm{j_j}\in\mathrm{B}$ that $S_\alpha=S_i=Sj_i$, which in result, we have $\mathrm{S}_\alpha\subseteq\mathrm{yR}\cap(\oplus_{\mathrm{i\in B}}\mathrm{S_i})=(\mathrm{o})$, which is an inconsistency. Proof the second side, as resulting using the following lemma.

Lemma (2-3): Suppose M is a $-\mathrm{R}$ module and m and n are in nonzero element from M.

(1) If m and n are adjacent, then for each $\mathrm{r},\mathrm{s}\in\mathrm{R}$ that $\mathrm{mr}\neq\mathrm{o}$ and $\mathrm{ns}\neq\mathrm{o}$, we have $mr_*ns=o$.

(2) If $\mathrm{mR}\cap\mathrm{nR}=\mathrm{o}$, then m and n are adjacent.

Proof (1): Suppose $\mathrm{m}(\mathrm{nR}:\mathrm{M})=\mathrm{o}$, easily it can be shown that for each $\mathrm{r}\in\mathrm{R}$, $\mathrm{mr}(\mathrm{nR}:\mathrm{M})=\mathrm{o}$ .On the other hand for each $\mathrm{S}\in\mathrm{R}$, we have $(nsR:M)\subseteq(nR:M)$. So $mr(nsR:M)\subseteq mr(nR:M)=o$.

(3) Since $(nR:M)\subseteq nR\cap mR$ is resulted $\mathrm{m}(\mathrm{nR}:\mathrm{M})=\mathrm{o}$ that can be concluded $m_*n=o$.

Result (2-4): Suppose $M = M_1 \oplus M_2$ that M1 and M2 are two non-identical simple sub graph from M. Then $\Gamma(\mathrm{M})$ is a complete bipartite graph.

Proof (proof of 1):Suppose $x, y \in Z(M)^*$ are adjacent, according to the before theorem $\mathrm{xR} \bigcap \mathrm{yR} = (\mathrm{o})$. We are showing that $\mathrm{x} \in \mathrm{M}_{\mathrm{i}}$ and $\mathrm{y} \in \mathrm{M}_{\mathrm{j}}$, which $\mathrm{i} \neq \mathrm{j}$ and $\mathrm{i}, \mathrm{j} \in \{1,2\}$. It is easy to show that xR and yR are two non-identical simple sub modulus From M, therefore, $xR = M_i$ and $yR = M_j$ for each $i \neq j$.

(Second proof):For each $x \in M_1$ and $y \in M_2$, we have $xR \bigcap yR = o$. Therefore, from lemma (2-5), x and y are adjacent. Show that any two elements from M_{i} are not adjacent. If $\mathrm{x}, \mathrm{y} \in \mathrm{M} \setminus \{\mathrm{o}\}$ so that $\mathrm{x}(\mathrm{yR} : \mathrm{M}) = \mathrm{o}$, then $\mathrm{xR}(\mathrm{yR} : \mathrm{M}) = \mathrm{M}_1(\mathrm{M}_1 : \mathrm{M}) = \mathrm{o}$. On the other $(M_1 : M) = a_{nn}(M_2)$ and therefore, $M_1 = (a_{nn}(M_2)) = o$ that is resulting $a_{nn}(M_2) \subseteq a_{nn}(M_1)$. Because $a_{nn}(M_2)$ is ideal maximal from R, so $a_{nn}(M_1) = a_{nn}(M_2)$, then $M_1 \cong \dfrac{R}{a_{nn}(M_1)} \simeq \dfrac{R}{a_{nn}(M_2)} \cong M_2$, which is a contradiction. As a result, we show that for $o \neq x \in M_1$ and $o \neq y \in M_2$, $(\mathrm{x} + \mathrm{y})$ are not adjacent of any element from M_{i} for $\mathrm{i} = 1{,}2$ and for each $\mathrm{Z} \in \mathrm{M}_2$, we have: $(z_R : M) = (M_2 : M) = a_{nn}(M_1)$ therefore, $(\mathrm{x} + \mathrm{y})(\mathrm{zR} : \mathrm{M}) = \mathrm{o}$ is resulted that $o = (x + y)(zR : M) = (x + y)(a_{nn}(M_1) = y(a_{nn}(M_1))$ and so $a_{nn}(M_1) \subseteq a_{nn}(y) = y(a_{nn}(M_2))$. Because $\mathrm{a}_{\mathrm{nn}}(\mathrm{M}_1)$ is ideal maximal from R. Therefore $a_{nn}(M_1) = a_{nn}(M_2)$, this is a contradiction. If $\mathrm{z}((\mathrm{x} + \mathrm{y})\mathrm{R} : \mathrm{M}) = \mathrm{o}$, then by lemma (2-9) in the source [7] one of the following conclusion:

(Case 1):If $M = (x + y)R \oplus M_2$, then $(x, y)R \simeq \dfrac{M}{M_2} \simeq M_1$, in the other words $(x, y)R \simeq \dfrac{R}{a_{nn}(x + y)} \simeq \dfrac{R}{a_{nn}(x) \bigcap a_{nn}(y)}$, which is not a simple module, because: $a_{nn}(x) \bigcap a_{nn}(y) = a_{nn}(x),\ a_{nn}(x) \bigcap a_{nn}(y) \subset a_{nn}(x)$, which result $\mathrm{a}_{\mathrm{nn}}(\mathrm{x}) = \mathrm{a}_{\mathrm{nn}}(\mathrm{y})$ and therefore $M_1 \simeq M_2$, which is a contradiction. Therefore, $\dfrac{a_{nn}(x)}{a_{nn}(x) \bigcap a_{nn}(y)} \neq< \dfrac{a_{nn}(x)}{a_{nn}(x) \bigcap a_{nn}(y)}$ is a contradiction.

(Case 2): If $M = (x + y)R \oplus M_1$ then similar case 1, a contradiction occurs.

(Case 3): Suppose $M = (x + y)R$ therefore, $((x + y)R : M) = (M : M) = R$, so $z((x + y)R : M) = o$ is resulting that $\mathrm{z} = \mathrm{o}$ that this is a contradiction. Similarly, by replacing M_2 instead M_1, also a contradiction occurs. Finally, about the $(x + y)((x' + y')R : M) = o$ is resulting that $y(x' + y')R : M) = o,\ x(x' + y')R : M) = o$, which is impossible. Suppose $M = \oplus_{i=1}^{n} M_i$ for $n \geq 3$ that its homogeneous components are simple. Can be predicted in this case the $\Gamma(\mathrm{M})$ is a complete n-part graph.(In theorem the source 4-2 [6]) has been shown to R reduction the displacement ring, we have that the non-empty $\Gamma(\mathrm{R})$ is with $gr(\Gamma(R)) = \infty$, if and only if $\Gamma(R) = K^{1,n}$ for each $\mathrm{n} \geq 1$, then, we generalize this result for $\Gamma(\mathrm{M})$.

Definition (2-7): The $-\mathrm{R}$ module of M called reduction, whenever for each $a \in R$ and $\mathrm{m} \in \mathrm{M}$ that we have $a^2.m = o$ then $aRm = o$.

Lemma (2-8): Suppose M is a reduction R module with $\mathrm{Z}(\mathrm{M})^* \neq \mathrm{M} \setminus \{\mathrm{o}\}$. If $\Gamma(\mathrm{M})$ is a bipartite graph by sector of V1 and V2then $\overline{V_i} = V_1 \bigcup \{o\}$ for each $i = 1{,}2$ is a sub module from M.

Proof: Suppose $r \in R,\ x_1 x_2 \in \overline{V_1}$. We must show $V_1 + V_2 \in \overline{V_1}$ and $rx_1 \in \overline{V_1}$. If $rx_1 = o$ then $rx_1 \in \overline{V_1}$. Now suppose $rx_1 \neq o$. From assumption, x_1 is adjacent of an element from V_2 in the name of y_1. If $rx_1 = y_1$ then from lemma (2-5), $y_1(y_1R:M)=o$ that is resulting for each $m\in M$, $r\in(y_1R:M)=o$, $mr^2=o$, since theM is a reduction $-R$ module, so $mr=o$, which is resulting m is adjacent y_1 that is a contradiction. So $rx_1\neq y_1$ and from lemma 2-5, rx_1 is adjacent y_1, since $y_1\in V_2$, therefore $rx_1\in V_1$. If x_1 or x_2 are equal to zero, then $x_1+x_2\in\overline{V_1}$, so it can be assumed that none of the x_1 or x_2 is not equal to zero. Since that $x_1,x_2\in V_1$, therefore, there is $y_1,y_2\in V_2$, which x_i are adjacent y_i, for each $i=1,2$. From lemma 2-5, we have $y_1R\bigcap y_2R\neq(o)$, therefore, $o\neq w\in y_1\bigcap y_2R$, science that for each $i=1,2$, we have $x_i(wR:M)\subseteq x_iR\bigcap wR\subseteq\overline{V_1}\bigcap\overline{V_2}=(o)$, we have $(x_1+x_2)(wR:M)=(o)$, now if $x_1+x_2=o$, that is belonging to $\overline{V_1}$ and if $x_1+x_2\neq o$, because $w\in V_2$ therefore, $x_1+x_2\in V_1$, similarly, can be shown that $\overline{V_2}$ is a sub module of M.

Lemma (2-9): If $m\notin Z(M)^*$, then mR is a fundamental sub module from M.

Proof: If mR is not fundamental, then there is non-zero sub module of K from M, which $mR\bigcap K=o$, from lemma 2-5, the m is adjacent each nonzero element from K, so $m\in Z(M)^*$, which is a contradiction, therefore, mR is a fundamental sub module from M.

Theorem (2-10): Suppose M is a reduction R module with $Z(M)^*\neq M\setminus\{o\}$. If $\Gamma(M)$ is a bipartite graph, then following items conclusion:

(1) $\Gamma(M)$ is a bipartite graph.

(2) $U.d_{in}M=2$

Proof (1): Suppose $Z(M)^*=V_1\bigcup V_2$ that $V_1\bigcap V_2=\phi$ and no element of V_i are not adjacent. From lemma 2-8, we have: $\overline{V_1}=V_1\bigcup\{o\}$ and $\overline{V_2}=V_2\bigcup\{o\}$ are sub modules of M. For each $Z\in V_1$ and $y\in V_2$, we have $ZR\bigcap yR\subseteq\overline{V_1}\bigcap\overline{V_2}=(o)$ and from lemma 2-5, z and y are adjacent.

Proof (2): Since $z(\overline{V_1})^*$ and $z(\overline{V_2})^*$ are empty, of lemma 2-9, each submodule of $\overline{V_1}$ and $\overline{V_2}$ are fundamental. So, $\overline{V_1}$ and $\overline{V_2}$ are uniform sub module from M. Now we show $\overline{V_1}\oplus\overline{V_2}$ in M is fundamental. Suppose K is a sub module from M, which $K\bigcap(\overline{V_1}\oplus\overline{V_2})=(o)$ and $(o)\neq y\in K$. Then, for each $o\neq z\in\overline{V_1}$ and $o\neq z\in\overline{V_2}$, we have $zR\bigcap yR=(o)=zR\bigcap wR$. Therefore, z is adjacent of y and w, so that $z\in\overline{V_1}\bigcap\overline{V_2}=(o)$, which is a contradiction, therefore, $\overline{V_1}\oplus\overline{V_2}$ in M is fundamental.

Lemma (2-11): Suppose M is a reduction R module with $Z(M)^*\neq M\setminus\{o\}$. Then $gr(\Gamma(M))=\infty$ if and only if $\Gamma(M)$ is a star graph.

Proof($\Rightarrow$):It is obvious.

($\Rightarrow$) Suppose $\Gamma(M)$ is not including a cycle, $\Gamma(M)$ is a tree and therefore is a bipartite graph. Now from theorem2-10, $\Gamma(M)$ is a complete bipartite graph. Suppose V_1 and V_2 are part of $\Gamma(M)$ graph. Since the $\Gamma(M)$ is not any cycle, we have $|V_1|=1$ or $|V_2|=1$, which concluded that $\Gamma(M)$ is a star graph.

Define (2-12): A half group graph is bipartite zero divisor, if and only if is not including any triangle. ([14])

Lemma (2-13): Suppose M is a R module. $\Gamma(M)$ is including a cycle of odd length, then $\Gamma(M)$ is a triangle.

Proof: By induction, we show for each cycle the odd length, $2n+1\geq 5$, there is a cycle of length of $2k+1$ for $k<n$. Suppose $x_1-x_2-x_3-...-x_{2n}-x_{2n+1}-x_1$ is a cycle of odd length of $2n+1$. If two non-consecutive vote of x_i and x_j are adjacent, then proof is complete. Otherwise element of $(o)\neq z\in x_1R\bigcap x_3R=(o)$ from lemma 2-5, $z\neq x_i$ for each $1\leq i\leq 2n+1$. So again z is adjacent to both elements of x_i and x_{2n+1}. Therefore, we have the cycle of $x_{2n+1}-z-x_4-x_5-...-x_{2n+1}$, that this is desired same distance.

Theorem (2-14): Suppose M is a R module. If is $gr(\Gamma(M)) = 4$, then $\Gamma(M)$ is a bipartite graph with parts of V_1 and V_2, so that $|V_1|, |V_2| \geq 2$. Conversely, it is true if the M is a reduction module with $Z(M)^* \neq M \setminus \{o\}$.

Proof: Suppose $gr(\Gamma(M)) = 4$. Using lemma 2-13, we show that the length of each cycle of $\Gamma(M)$ is even. Since $\Gamma(M)$ has cycle of length 4, this hypothesis is confirmed. On the contrary, also theorem 2-10can is proved. In the following, we explain the between generalization relationship from the definition of zero divisor graphs in [11], (it will show by Γ_b) and what, we have shown in this paper. First, it is worth noting that $\Gamma(M)$ is a sub graph from Γ_b that if $m, n \in Z(M)^*$ are two adjacent vertices in $\Gamma(M)$, Or such as equivalence $n(mR : M) = o$ or $m(nR : M) = o$ then $(nR : M)(mR : M) = o$. However, the reverse is not true as the following example.

Example (2-15): Suppose $M = Z_2 \oplus Z_4$ is a Z module. Then $\Gamma_b = K_v$, however $\Gamma(M)$ different from K_v , which we are showing in figure 2.2.

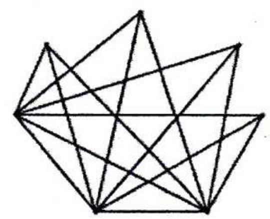

Figure2.2

However, when the M is a multiplicative R module (As for each sub modules of N from M, there is a ideal of I from R, which $N = MI$) , then $\Gamma(M) = \Gamma_b$. Suppose $(mR : M)(nR : M) = o$. Therefore, $(nR : M)M = nR$ and $(mR : M)M = mR$, so both of $m(nR : M) = o$ and $n(mR : M) = o$.

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