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Location of Service Centers with Multi-services (A Queuing Theory Approach)

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ABSTRACT

One of the influential factors in success of the service centers in providing appropriate and high quality services for their customers may be the short waiting time for receiving service and their easy access to the service center in order to receive their desired service. In this paper, a model of locating has been suggested for the service centers with the purpose of minimizing the travel and waiting time of customers for receiving service. This significant purpose is associated with the assumption of diversity in services. Since several types of services are presented in most of the service centers, the assumption of diversity in services has many applications and leads to precisely selection of appropriate location for service centers and choosing a number of servers from all kinds in each center. Proper selection of these two cases is very influential in elevating the quality of serving the customers. This issue has been modeled with this important purpose and assumption, and it is investigated and solved with the meta-heuristic algorithm of particle swarm optimization (PSO).

KEYWORDS: locating, queuing theory, service diversity, serving quality, particle swarm optimization algorithm

1. INTRODUCTION

Locating the service centers is considered from the most important management decisions in improving the quality of serving the customers; because the determination of proper location for service centers leads to improvement of serving quality to the customers. In other words, the customers spend shorter time in order to reach the service center from their source location to the service center and also shorter time to receive their desired service, which these factors increase the quality of serving of these centers and attracting more customers. In the past, these issues were modeled with the assumption of fixed demands and time of serving which the known middle –P model of Hakimi [7] can be referred from these models. But this assumption leaded to creation of congestion and queuing in the service center when facing demand higher than the average.

Consequently, in order for more precise modeling of these issues, now the demand and time of serving the customer are considered which they follow the principles of queuing systems. From the functions of these issues the locating of banks and their ATMs, health and treating centers and stores can be referred. There is an extensive research background in this regard.

Berman et al. [2] present a meta-heuristic algorithm for optimized locating of a server in swarm network with queuing system M/G/1. Then they [4] develop this meta-heuristic algorithm in order to find the optimal location of P server in the swarm network. Wang et al. [13] examine the issues of locating a server in service centers with the queuing system M/M/1. Their model's purpose is to increase the total travel time of the customers to the service center and their average waiting time in the center.

Berman and Zvi Drezner [4] &Aboolian et al. [1] developed their model and solved the issue of locating multi servers for the service centers in a random environment in modeling. Therefore, their model is modeled through M/M/m queuing system instead of M/M/1 queuing system. The customers choose the closest service center for receiving services. In these articles, the meta-heuristic algorithm is used in order to solve the problem. Also Buffy et al. [5] present a perfect review in the field of locating the service centers in random environment with fixed servers. Tammy Drezner and Zvi Drezner [6] locate the multi-servers service centers which the customers are allocated to the service centers according to the location attraction of the service centers and the customers' interest, and all customers of one demand node are not allocated to the closest service center.

Pasandideh & Niaaki [11] introduce a two-purpose model in order to reduce the total time customers spent for receiving services and also to reduce the unemployment of the servers. They model their issue with M/M/1 queuing

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system and solve the problem through utility function method and genetic algorithm. Seifbarghi et al. [12] introduce a model for locating the service centers from the viewpoint of customer with the M/M/m queuing center in which the customers of each node are allocated to the service centers based on the measures of distance from the center and the number of servers of each center. The purpose of this model is to minimize the average of queue length. They use meta-heuristic genetic algorithm (GA) and Simulated Annealinig algorithm (SA) in order to solve their model.

In all of the above researches, it has been assumed that just one service is presented in each service center. While most of the service centers present multiple kinds of services and there are multiple different waiting queues in them. For example, a gasoline service station may supply three different types of fuel, regular gasoline, super gasoline and diesel; or in the clinics, there may be several specialist doctors which each of the specialties has its own waiting patients. Various services differ from each other in terms of waiting time, serving rate, demands for the services, efficiency coefficient, price and the total number of providers, which these differences are very influential in the management decisions for selecting the appropriate location for the service center and determining the number of different providers and they must be paid attention to. Proper selection of the two above cases increases the quality of serving the customers.

In this paper, a new model is suggested in order to improve the quality of serving the customers through selecting appropriate location for the service centers, this model has been created with the assumption of service diversity. With the assumption of providing several services in each center, this model attempts to minimize the total travel and waiting time of the customers which is from the most important factors in their satisfaction from the center's serving quality. Allocating the customers to the service centers has been done likely and based on the distance from center, location's attraction and the number of providers of each service center. This issue has been modeled considering these cases and solved through the particle swarm optimization (PSO) algorithm. After that, we examine the results obtained.

The rest of the paper is followed by: in section 2, we express the description of methodology including modeling and problem solving through PSO algorithm. In section 3, we present the examples and the results obtained from the model, and finally in section 4, we express the conclusion and some suggestions for future researches.

2.MATERIALS AND METHODS

In this part, we first express the symbolizing and modeling the issue and after that explain the problem solving method in details. Modeling has been examined in a network with N nodes (N= {1, …, *N*}) and the network's nodes are the indicator of both the demand nodes and the potential locations for locating the service centers. The network's manes express the possible paths between the network's nodes. Also, M = {1,…, *M*} indicates all the different and diverse services which are provided in each center.

The following hypotheses were used in modeling the issue:

- Serving demand of each customer and PSO process are independent variables.
- Each service center has at least one provider with exponential serving time.
- In each service center, more than one type of service may be provided which they are independent from each other.
- Each service center acts in M/M/m_j^k queuing system ($k$indicates the type of service and $j$indicates the service location).
- The providers have steady location.

2.1. Symbolizing and Modeling the Issue

The following symbols and variables have been used in modeling:

N: The total nodes of the network

M: All different types of services

P^k: The total number of providers from the kind of k ($k \in M$)

h_i^k: Demand rate of i (i ∈ N) for the services of the kind of k ($k \in M$)

μ^k: serving rate of each provider of the kind of k ($k \in M$)

v: rate of the customer's movement from the *i* demand node ($i \in N$)

d_{ij}: the movement path length of the customer from *i* demand node ($i \in N$) to the service center located in *j* node ($j \in N$)

A_j: the extent of attractiveness of j node ($j \in N$) for the customers' reference

γ: the consonant parameter of exponential function of distance

R: the maximum coefficient of efficiency considered for all kinds of services in any service center ($0 < R < 1$)

x_{ij}^{k}: Possibility of each customer's reference from the *i* demand node ($i \in N$) to the service center located in *j* node ($j \in N$) to receive a service from the kind of $k (k \in M)$

w_j^k: The average of customer's waiting time in the service center located in *j* node ($j \in N$) to receive a service from the kind of $k (k \in M)$

λ_j^k: The rate of customers' entrances (demand) to the service center located in *j* node ($j \in N$) to receive a service from the kind of $k (k \in M)$

t_{ij}: the customer's travel time from the *i* demand node ($i \in N$) to the service center located in *j* node ($j \in N$)

m_j^k: The variable of decision making of the number of providers in the service center located in *j* node ($j \in N$) from the kind of $k (k \in M)$

y_j: decision making binary variable; if the service center is located in j node ($j \in N$), its value is 1, unless it is 0.

Therefore the mathematics model of the issue is expressed as following by the definition of the above symbols:

$$Min\ Z = \sum_{k\in M} \sum_{j\in N} \sum_{i\in N} \lambda_j^k\, t_{ij} + \sum_{j\in N} \sum_{k\in M} \lambda_j^k\, w_j^k \qquad (1)$$

s.t.:

$$\lambda_j^k = \sum_{i\in N} h_i^k\, x_{ij}^k\ ;\ \forall\, j \in N, \forall k \in M \qquad (2)$$

$$\sum_{j\in N} m_j^k = P^k; \forall k \in M \qquad (3)$$

$$x_{ij}^k = \frac{m_j^k A_j e^{-\gamma t_{ij}}}{\sum_{j\in N} m_j^k A_j e^{-\gamma t_{ij}}};\ \ \forall i, j \in N, \forall k \in M \qquad (4)$$

$$w_j^k = \left(\frac{1}{\mu^k}\right) + \left(\frac{\mu^k (\frac{\lambda_j^k}{\mu^k})^{m_j^k}}{\left(\mu^k - \lambda_j^k\right)^2 \left(m_j^k - 1\right)!}\right)\left(\left(\frac{1}{m_j^k!}\right) (\frac{\lambda_j^k}{\mu^k})^{m_j^k} \left(\frac{\mu^k m_j^k}{m_j^k \mu^k - \lambda_j^k}\right) + \sum_{r=0}^{m_j^k - 1} (\frac{1}{r!})(\frac{\lambda_j^k}{\mu^k})^r\right)^{-1};\ \forall\, j \in N, \forall k \in M \qquad (5)$$

$$t_{ij} = \frac{d_{ij}}{v}\ ; \forall i, j \in N \qquad (6)$$

$$\frac{\lambda_j^k}{\mu^k m_j^k} \le R\ ; \forall j \in N\ \ , \forall k \in M \qquad (7)$$

$$m_j^k \le P^k y_j\ ; \forall j \in N\ \ , \forall k \in M \qquad (8)$$

$$\sum_{k\in M} m_j^k \ge\ y_j\ ; \forall j \in N \qquad (9)$$

$$m_j^k \ge 0 \text{ and an integer};\ \forall j \in N, \forall k \in M \qquad (10)$$

$$y_j \in \{0,1\}\ ;\ \forall j \in N \qquad (11)$$

The objective function of the model in equation (1) indicates the total travel time of the customers to the service centers and their waiting time in the service center which this value must be minimized in order to improve the quality of serving the customers. The equation (2) defines the extent of demand rate reaches to the service center located in j node in order to receive a service of the kind of k. the (3) limitation equals the total number of providers from the kind of k of all the service centers with Pk. the equation (4) indicates the way of dedication of demand or the possibility of reference to the service centers based on the criteria of distance from the center, the node's (location's) attraction and the number of providers of each service center. This matter causes that always a proportion of population of each of the demand nodes is dedicated to one service center. The equation (5) expresses the average waiting time of the customers based on M/M/m queue system in the service center in j node in order to receive a service of the kind of k (Klienrock, [9]). The equation (6) calculates the travel time of the customer from i demand node to the service center in j node. The limitation (7) guarantees the efficiency coefficient of the service center to be lesser than a defined extent lesser than one in order to create balance in the system. The limitation (8) is a control limitation, the number of providers from the k m_j^k kind is not determined unless that service center is located in j node. If the service center is not created in j node, its provider number is equal to zero, and otherwise, the number of providers of the kind of k gets lesser than the total number of providers of the kind of k, this means that the limitation is confirmed and the number of providers is determined by solving the problem. The limitation (9) which is a control limitation expresses that if a service center is created in j node, i.e. yj=1, then there must be at least one provider of one kind in that center; and if yj=0, all mjks are equal to zero and the limitation is yet confirmed. The limitation (10) expresses the un-negativity and integer-ness of the variable of decision making of the number of providers of the kind of k in the service center located in *j* node. The limitation (11) is the binary variable of decision making of problem, if a service center is located in *j* node, its value is equal to *1* and otherwise it is equal to *0*.

2.2. Particle Swarm Optimization (PSO):

The problem model is from the kind of Nonlinear Integer Programming (NLP) and it is very time consuming to use the exact methods in order to solve it, especially if the problem size is very large. Berman and Drezner [4] and Aboolian et al. [1] use the meta-heuristic algorithm to solve the problem of locating multiple-providers service centers. Also in this paper we have used the meta-heuristic algorithm of particle swarm optimization or (PSO) in order to solve the problem.

PSO algorithm was introduced by Kennedy and Eberhart [8]. This algorithm is inspired by the natural collective life such as collective batch life of the birds and fish which has been useful in solving many problems.

Firstly, a population of particles with random location and movement direction are generated. In successive iterations of the algorithm, all the particles adjust their location and movement direction based on the inertia value of their movement direction, their own best personal experience and the information of the other particles of the collection or the best experience of the collection. Fitness function which is the same objective function in this paper determines how much good the location of each particle is.

Therefore, generally the location and movement direction of each particle can be expressed by the following two equations:

$$\overrightarrow{v_{k+1}} = \overrightarrow{w_k}.\overrightarrow{v_k} + \overrightarrow{c_1}.\overrightarrow{r_1}.(\overrightarrow{p_1}-\overrightarrow{x_k})+\overrightarrow{c_2}.\overrightarrow{r_2}.(\overrightarrow{p_2}-\overrightarrow{x_k}) \qquad (12)$$

$$\overrightarrow{x_{k+1}}= \overrightarrow{x_k} + \overrightarrow{v_{k+1}} \qquad (13)$$

In these equations, the dot symbol (.) is the indicative of multiplying the element to element vectors. $\vec{\boldsymbol{w}}$is the inertia coefficient the current location of each particle is given. $\vec{\boldsymbol{c}}1$ is the coefficient which the best location of each particle is given; and $\vec{\boldsymbol{c}}2$ is also the coefficient which the best location of each collection is given. $\vec{\boldsymbol{p}}1$ is the same best personal experience and also $\vec{\boldsymbol{p}}2$ is the best experience of the collection. Also, in order to randomize, the coefficients of $\vec{\boldsymbol{r}}1$ and $\vec{\boldsymbol{r}}2$ are orderly used for the best personal experience and the best experience of the collection which these coefficients are between zero and one. K indices indicate the repetition of algorithm and also $\vec{\boldsymbol{x}}$k is the location of particle in k repetition.

It must be noted that the value of inertia coefficient in each repetition is obtained from the equation (14):

$$\vec{w}_{k+1}=\vec{w}_k\times wdamp \qquad (14)$$

In the equation (14), wdamp is called the reduction inertia coefficient. Since the large inertia coefficient (w) is used for overall search and the small inertia coefficient is used for the local search, it is better that the value of inertia coefficient reduces gradually in order for the search to be done totally at first and then gradually tend to the local search.

Therefore, the overall outline of the meta-heuristic PSO algorithm used in this paper to solve the problem is as following:

Stage 1: initialization: generate a population of particles with random location and movement direction.

Stage 2: assessment: calculate the fitness function value for each particle.

Stage 3: comparison: compare the fitness function value of each particle with its best value of fitness function in the previous repetition. If the new location has better fitness function value, choose it as the best personal experience, otherwise the same location of particle in the previous repetition is chosen as the best personal experience. Also, compare the fitness function value of each particle with the value of the best experience of collection in the previous repetition. Select it as the best experience of the collection if it has better fitness function value, otherwise the best experience of the collection remains unchanged.

Stage 4: convergence: stop if the stop condition of algorithm which is the lack of improvement of the fitness function in several successive iterations of the algorithm is confirmed; otherwise, go to stage 5.

Stage 5: synchronization: calculate the inertia values, movement direction and the location of each particle using equations (12), (13) and (14) and go to stage 2.

In PSO algorithm of this paper, each answer has been indicated in the form of M×N matrix in which the matrix lines indicate the number of types of various services and the columns of matrix indicate the number of potential locations for placing the service center in them. The matrix's elements indicate the number of providers compatible with the kind and location of the service. If all the elements of a column are equal to zero, this means that no service center has been established in that node. In the figure 1, one example of answer with three types of services and four nodes for locating is indicated in which except the second node, in all nodesa service center has been established.

$$\begin{pmatrix} 1 & 0 & 2 & 1 \\ 2 & 0 & 0 & 3 \\ 3 & 0 & 3 & 0 \end{pmatrix}$$

Fig. 1.an example of an answer

In the suggested PSO algorithm, regarded to the different limitations of the model, it may produce an impossible answer. Therefore, the penalty function method has been used when facing the impossible answers. Penalty function is one of the most popular methods in order for dealing with the limited problems. For example, the penalty function of one limitation in form of g (x) ≤ b is added to the objective function of the problem in the form of the equation (15).

$$P(x)=U\times Max\ \{(\frac{g(x)}{b}-1),0\} \quad (15)$$

In this equation U is a large positive value. If the produced answer satisfies the limitation, the value of penalty or P(x) is equal to zero, otherwise, the penalty value is equal to a large positive number and this leads to the matter that value of problem's objective function be minimized for this impossible answer and this eliminates the impossible answer. In fact, we change the limited problems into the unlimited ones using the penalty function approach and this strategy is very useful in solving optimization limited problems.

In the next section we show several numeric examples obtained from problem solving using the suggested PSO algorithm.

3. Evaluation and reliability of the numeric results

Different numeric examples have been examined in order to evaluate the suggested solving method. For example, in table 1, the results of 10 calculative examples with different sizes with PSO algorithm are given. For each example in the table 1 we consider the values of the number of nodes as (N), types of services as (M) and the total number of providers of each kind (which are considered equal) as ($P=P_k$, $\forall k\epsilon M$). The values of the other parameters of the problem are as following:

The demand rate for any kind of serving follows a uniform distribution function, i.e. [1 & 15] uniform $\sim h_i^k$

Any kind of the serving rate follows a uniform distribution function, i.e. [50 & 80] uniform $\sim \mu^k$

The travel distance between two nodes follows a uniform distribution function, i.e. [50 & 100] uniform $\sim d_{ij}$

The movement speed between two nodes has been assumed equal to ten, i.e. v = 10

The attractiveness extent of each node for the customers follows a uniform distribution function, i.e. [1 & 0] uniform $\sim A_j$

The maximum efficiency coefficient has been assumed equal to 0.9, i.e. R = 0.9

The constant parameter in the exponential function of distance has been assumed equal to one, i.e. γ = 1

For instance, the values of parameters of PSO algorithm has been adjusted in the example 3 of table 1 with 10 nodes and 3 types of services through successive performances of the algorithm in values of **=0.9**w **=0.99** ، wdamp=**4**،C1=**2**،C2, the particles' population = 10 and the maximum number of successive repetitions without improvement in the value of the objective function. In the table 1, we have shown the value of objective function (Z) and also the time of problem solving in seconds.

Also, in figure 2, we have expressed the PSO algorithm convergence diagram for the example 6.

Table 1.the results of calculative examples with applying the PSO algorithm

Example number	Solving time (in seconds)	*Z (in PSO method)*	*P*	*M*	*N*
1	1.9670	121.8807	1	1	2
2	2.9934	1728.7009	3	2	5
3	4.1425	16664.4746	5	3	10
4	5.3738	83267.6776	10	4	20
5	10.5915	250734.7776	15	5	30
6	19.8663	810134.7916	25	6	50
7	26.9244	1882121.9692	35	7	70
8	40.7274	2887831.2624	40	8	80
9	68.4662	5240233.3339	50	9	100
10	314.1700	23063446.5193	100	10	200

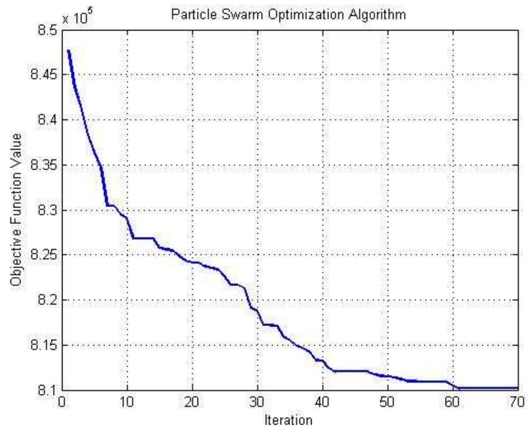

Fig. 2.convergence diagram for the example 6

The results obtained from numeric examples with different sizes indicate that the PSO algorithm is appropriate for solving the nonlinear model of the problem and it has been able to gain the answer in the desired time. PSO is from the popular and powerful algorithms for optimization mostly used for its high convergence speed. All the examples in table 1 have been solved in lesser than 6 minutes. For instance, the values of variables of the example number 2 with 5 potential location (nodes) and two types of services are indicated in table 2.

Table 2.the values of variables of the example number 2

Values of variable	Variable name
[0 1 0 1 0]	***y***
$\begin{bmatrix} 0 & 1 & 0 & 2 & 0 \\ 0 & 1 & 0 & 2 & 0 \end{bmatrix}$	***m***

The values of variables indicated in table 2 indicate that the management decisions for improving the serving quality to the customers must be made regarded to establishing service centers in location 2 and 4 in order for the customers to spend lesser time for accessing the service center.

In order to improve serving quality to the customers which lead to the reduction of their waiting time for receiving services, the management decisions related to the number of providers located in locations 2 and 4 are as following:

Location 2: one provider of type 1 and one provider of type 2

Location 4: one provider of type 1 and one provider of type 2

With these decision makings, the total travel and waiting time of customers for receiving service turns close or even equal to the minimum extent. This matter improves the serving quality to the customers and is considered as one of the most important factors in customers' satisfaction.

The PSO algorithm programming has been done through MATLAB software [10] version 2012 on a notebook with 4 gigabytes of RAM and a CPU of 2.30 GHz. In the next section, we express the conclusion obtained from multiple-services locating model.

4. Conclusion and future suggestions

In this paper, we introduced a new model for improving serving the customers with appropriate locating of service centers which has been modeled based on the assumption of service diversity, i.e. there may be more than just one services provided in each service center. The subject of service diversity in the service centers is very significant; because in most of the service centers usually more than one service is provided and there are several different waiting queues. Different services differ from each other in terms of different criteria and these differences are very influential in correct locating of service centers and determination of appropriate number of providers of each type and they must be paid attention to. Considering the service diversity, this model deals with the minimization of the purpose of total waiting and travel time of customers which are considered from the most important factors in customers' satisfaction from the serving quality in which the dedication of customers has been done based on the distance from center, the extent of location's attraction and the number of providers of each service center. This matter causes that always a proportion of population of each of the demand nodes is dedicated to one service center.

In this paper, the PSO meta-heuristic algorithm has been used in order to solve the model. The results of numeric examples with different sizes indicate that this algorithm as able to solve the problem in the desires time and access good answers in small sizes. The values of variables obtained from solving this problem indicate the management decisions in selecting the appropriate location for establishing service centers with appropriate number of providers of any kind; correct decision making of these two factors improves the quality of serving the customers and attracts their satisfaction; because it improves the travel and waiting time of customers for receiving service which are from the most important factors in the quality of serving them.

The following suggestions are presented for the future researches:

- Budget constraints to be added to the problem.
- Various providers to have different serving prices in different times.
- Other queue systems may also be used instead of M/M/m queue system.
- Some of the parameters of problem can be assumed phasic.

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