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Mathematical Programming of Operating Room Capacity Assignment: Robust Estimator Approach

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ABSTRACT

Assigning operating room capacity to surgical groups is one of the strategic decisions in the hospitals. In this problem capacity of operating room is assigned to surgical groups according to defined objective and constraints. This assignment is done according to limitation in recovery, ICU, and bed wards. In this article we developed a mathematical model to solve this problem. In addition, the objective function and the constraints are defined according to Shahid Madani hospital situations in order to make the problem as real as possible. According to this fact that in real data there are contaminations, we use M robust estimator to estimate the model parameters. Finally, the model is solved by GAMS with classic maximum likelihood estimation (MLE) and M robust estimator approach. Results show that there is a gap between solutions obtained by classic MLE and M robust estimator.

KEY WORDS: Operating Room Capacity Assignment, M-Robust Estimator, Mathematical Programming.

INTRODUCTION

According to the development of technology in the health services and its use in medicine diagnosis and increasing household access to medical facilities, along with the development of a culture of health, costs in the field of health services are increased. A high share of the health care cost and a steady increase in recent years, has made health care to one of the serious problems of the developed and developing countries.

The share of total health costs from the gross world production increase form 3 percent in 1948 to 9.7 percent in 1997. According to the statistics of the World Health Organization (WHO) in 2013, 17.9 percent of the gross domestic product in the United States dedicated to healthcare and can be anticipated that this will be increased in the year 2021 to 20 percent of the gross domestic product. Due to the fact that hospitals are one of the most important parts in healthcare system, proper management of healthcare system without managing hospitals and their resources is not possible. In Iran, as mentioned in WHO reports, a huge portion of the treatment cost is paid by the government. The portion in 2000 was about 37 percent in 2001, about 43.5 percent in 2007, as well as approximately 46.7 percent and in 2012 was more than 40 percent. As the government at year 2015 has a double emphasis on increasing the budget in the field of health in future, the need for proper management in this area seems to be essential.

Due to the importance of the health care to countries economic growth, the necessity of proper management and planning in this area is clearly visible. Also, among the different elements of the healthcare system hospitals are known as one of the most important component of these systems. In addition, the most important resource as hospital resources, which is known as the main center of costs and revenues, are operating room. This is why in most countries the question of the operating room management is a highly regarded researchers and managers. The formation of the various research groups, publish books and increase the ISI articles in this field confirm the importance of the issue [1, 2].

Hans et al. [1] introduce the framework of hospital management issues and classified them into strategic, tactical and operational level (Table 1).

As noted in Table 1 hospital management is significant in three levels of strategic, tactical and operational decisions and focusing on four areas of medical planning, resource capacity planning, material planning and financial planning.

In the field of resource capacity planning, determining type and composition of the patients, case mix planning, is one of the most important decisions to make and influences on other areas. With regard to the scope of application of this research which is operating room capacity assignment to the surgical groups, this study is a resource capacity planning strategic decision.

Determining operating room planning and scheduling strategy effects on the planning and scheduling of them in the mid and short term decision levels and influence and cause the formation of the planning and scheduling processes.

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Decision level	wiedical planning	Resource capacity planning	Material planning	r manciai pianning
Strategic	Research, development of medical protocols	Case mix planning, capacity dimensioning, workforce planning	Supply chain and warehouse design	Investment plans, contracting with insurance companies
Tactical	Treatment selection, protocol selection	Book planning, staffing, admission planning	Supplier selection, tendering	Budget and cost allocation
Offline operational	Diagnosis and planning of an individual treatment	Appointment scheduling, workforce scheduling	Material purchasing, determining order size	DRG billing, cash flow analysis
Online operational	Triage, diagnosing emergencies and complications	Monitoring, emergency coordination	Rush ordering, inventory replenishing	Billing complications and changes

Table 1. Example application of the framework for healthcare planning and control to a general hospital [1]

Access to elective surgery planning and scheduling is managed in one of three ways: block scheduling, open scheduling and modified block scheduling [3]. The choice between these scheduling paradigms is also a strategic decision. Under a block scheduling system, a set of time blocks is assigned to specific surgical groups in a cyclic schedule. In this way the entire operating room time series is divided into blocks of time. Below is the planning and scheduling steps in the block scheduling approach:

1- Operating room time assignment to surgeons or surgical groups (case mix planning) (strategic level)

2- Construction of a cyclic timetable that defines the surgeon groups available for each Operating room blocks (master surgical scheduling) (tactical level)

3- Filling up the blocks with surgical cases, which are then booked into the assigned time.

Under open scheduling strategy surgical cases are allowed to assign to an available OR, at the convenience of surgeons. An empty time table is filled up with surgical cases by following the order of arrival time (i.e., the first-come-first-served). First, at medium term the surgical cases schedule is constructed, and then a detailed OR scheduling concerns at short-term.

Finally, modified block scheduling strategy tries to modify block scheduling strategy in two ways to increase its flexibility. The first way is keeping some blocks open while others are booked and the second is releasing unused blocks at some times before surgery.

In practice there are some problems when open-scheduling systems are adopted in health care [4], whereas the blockscheduling systems are the most commonly used.

Generally in each hospital the type of scheduling strategy is dependent on the type of the hospital and its management. In Iran and in the training and public hospitals block or modified block strategy are used. Therefore, in this study block scheduling strategy is considered. As well as for the real-world conditions, constraints are defined and modeled according to Alborz Shahid Madani hospital condition. After modeling the problem based on the actual conditions of Shahid Madani hospital in order to obtain the optimum combination of operating room capacity assignment to surgical groups, the GAMS software is used to solving the problem. The developed linear programming model parameters have been estimated based on real data derived from Shahid Madani HIS system. It is worth mentioning that due to the presence of contaminants in raw data obtained from the HIS system, M robust estimator is used to estimate the average of desired data. The rest of the paper is organized as follows: section 2 provides a brief review of studies in this field. In the third section a brief description regarding the introduction of Shahid Madani hospital is presented and considered problem at the hospital is described. Mathematical linear programming model for the operating room capacity assignment to surgical groups is developed in the fourth section. M robust estimator is introduced in fifth part while part 6 provides computational results derived from the model and solutions of estimate approach. Finally, part 7 includes conclusions and scope for future studies.

LITERATURE REVIEW

In the past 60 years, a large amount of articles and studies has been done on the operating room management. Magerlein and Martin [5] survey the literature on surgical demand scheduling. They distinguish between advance and allocation scheduling. The process of fixing a surgery date for a patient is called Advance scheduling, whereas determining the operating room and the starting time of it on the specific day of surgery is called allocation scheduling.

Also in recent years, this is a fairly extensive studies by the Iranian researchers in problem regarding the operating room planning and scheduling. According to the uncertain nature of operating time Atighechian [6] developed two novel two-stage stochastic mixed-integer programming models under different assumptions to minimize total expected operating cost. Using these models, they determined several optimal or near optimal values, such as the allocation of operating room, surgeons and assistant surgeons to surgery, the sequence of surgeries within each operating room and the sequence of cases performed by each surgeon on a given day of surgery. ghazalbash et al., [7] examine the question of patients allocation to the operating room in a definite conditions. Akbarzadeh [8] examine the problem of operating room planning and scheduling at the operational level of resource capacity planning.

Stochastic and deterministic mathematical programing models, queuing theory, simulation and heuristic algorithm are developed and used to examine and overcome the problem of operating room planning and scheduling. These issues have been studies in the three strategic, tactical and operational level in the comments. A lot of writers such as Magerlein and Martin [5] classified the literature based on the solution method. Cardeon et al., [9] presented a comprehensive classification of the operating room planning and scheduling. They evaluated the literature on multiple fields that are

related to either the problem setting or the technical features. They proposed a literature review that was structured using descriptive fields. Manuscripts have been analyzed from a, which may be either problem or technically z oriented. In particular they distinguished between 6 fields: patient characteristics, performance measures, decision delineation, research methodology, uncertainty and applicability of research.

In order to simplify presentation of related articles in the rest of the paper we use this classification and present each related article according to Cardeon et al. [9] classification. But according to the fact that operating room planning and scheduling decisions affect downstream resources throughout the hospital, we separate this aspect from decision delineation and mention to this aspect independently. Different perspective by each field is as follow:

• Patient type (elective inpatient (1), non-elective inpatient (2), outpatient (3))

• Performance measures (waiting time (1), Throughput (2), Makespan (3), Leveling (4), utilization of resources (5), Patient deferral/refusal (6), financial (7), Preferences (8), emergency patient waiting time (9), overtime, undertime (10))

• Uncertainty (surgery duration (1), arrival (2), length of the stay (3), demand (4), other(5))

• Decision delineation (date (1), time (2), room (3), capacity (4) and decision on the patient (1) the surgeon (2), surgical group (3))

• Integrity (isolated (1), integrated (2))

• Type of analysis (optimization (1), decision problem (2), benchmarking (3), scenario analysis (4))

• Solution technique (mathematical planning (1), DEA (2), heuristic and meta heuristic algorithm (3), Analytical procedure (4), simulation (5))

Table 2 depicts manuscripts that consider resource capacity planning in strategic level of operating room planning and scheduling.

Author					<u>, , , , , , , , , , , , , , , , , , , </u>			
	year	Patient type	Performance measures	Uncertainty	Decision delineation	Integrity	Type of analysis	Solution technique
Lovejoy & Li [10]	2002	1	1,7	1,2	4-3	1	4	5
Kuo et al [11]	2003	1,2,3	7		4-3	1	1	1
Mulholland et al [12]	2005	1	7		4-3	2	1	1
Niu et al [13]	2007	1,2	1,5	1,2	4-3	1	4	5
VanBerkel & Blake [14]	2007	1,2,3	1,2	1,2	4-3	1	4	5
Gupta [15]	2007	1	7		4-3	1	1	1
Ma et al [16]	2009	1	7		4-3	2	1	1,3
Ma & Demeulemeester [17]	2013	1	7		4-3	2	1	1
Heng & Wright [18]	2013	1,2	1,2,5,6,9,10	2	Emergency room count	1	Queuing	4
Ferrand et al [19]	2014	1,2	1,5,9,10	1,2	Operating room count	1	4	5
Choi & Wilhelm [20]	2014	1,2	6,10	1,4	4-3	1	1	1
This article	2015	1,2,3	2		4-3	2	1	1

 Table 2. Resource capacity planning articles

Shahid Madani hospital

The hospital to be studied in this paper is a general surgery hospital, located in Azimiye, Karaj, Alborz, Iran. The surgery divisions consist of 36 surgeons and 6 surgical groups. Also there are two operating theater consist of 6 and 2 rooms. Shahid madani is a public hospital which is supported by Medical Science University of Alborz. This is now located in an area of about 8,000 square meters, with 180 beds. Tables (3) and (4) present Shahid Madani wards and its operating room characteristics respectively.

This is referred to as the reference emergency hospital of Alborz province. In addition, the only burn Center located in the province Alborz is Shahid Madani. Note that according to the operating rooms facility there are some restrictions using some operating rooms for some surgical groups and some procedure can execute only in some special rooms. Table 4 shows which operating room can be used for each surgical group.

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Wards	Room	Bed	Surgeon
Orthopedic	8	30	7
Eye and ENT	4	10	13
General surgery	6	25	9
Pediatric *	3	11	
Burn	5	18	3
Neurosurgery	5	18	4
Women*	8	30	
Recovery	1	11	
ICU 1	1	14	
ICU 2	1	8	
ICU 3	1	5	

*All operated women and pediatric hospitalize in their special wards

OR No.	Characteristic		
1	Only orthopedic and Neurosurgery operations is done in this room		
2	Only orthopedic and Neurosurgery operations is done in this room		
3	Neurosurgery operations limitation		
4	Neurosurgery operations limitation		
5	Laparoscopy surgery		
6	Eye surgery		
7 & 8	ENT and burn surgery (These rooms are located in second operating theater)		

Problem definition and mathematical formulation of the problem

The process of planning and scheduling of operating room in a block strategy consists of three steps. Resource capacity planning as long-term decision, master surgical scheduling as mid-term decision and planning and scheduling patients in operational level as short-term decision. As Table 2 shows many objective are considered by researcher for case mix planning problem. These objectives diverse from financial goals to patient waiting time reduction. According to referral nature of Shahid Madani hospital and management priorities maximizing hospital throughput with respect to covering emergency cases is defined as the objective of the study. So the aim of this article is to assign operating room capacity to surgical groups in such a way that maximize operated patients while considering real world constraints. In fact we search for the case mix of operated patients which helps hospital manager for strategic decision making while consider Shahid Madani real world constraints. The planning horizon is defined 6 months because the decision level is strategic long term planning. Two major constraints need to examine in the problem. The first and obvious one is operating room capacity. In Shahid Madani hospital operating room open at 7:30 and close at 16. In addition, some operating rooms are appropriate and facilitated for some special surgical groups or procedures and some of them are not suitable for some operating procedures or surgical groups. The second type of constraints is limitation of beds in surgery wards, recovery room and ICU. So length of stay in surgery wards, recovery and ICU beds are other important parameters should be consider when the case mix of patients is determined. As mentioned before operating room planning decisions affect downstream resources throughout the hospital. So, it seems to be useful to incorporate these resources, such as the ICU or ward beds or recovery beds, in the decision process. It helps to improve the global performance. Otherwise, improving the operating room planning solely may decreases the efficiency of those related resources and may cause demand fluctuation in other departments such as wards and ICU. So in this study we try to determine case mix of patients to maximize number of operated patients while operating room hour, recovery bed, ICU and surgical ward beds capacity are considered.

Here we describe a linear programing model for determining capacity assignment to surgical groups in strategic level of operating room planning and scheduling. Table 5 shows variables and parameters required for modeling the problem.

Table 5	. Model	variable and	parameters
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Variable/Parameters	Definition
x_i	Operating room time which is assigned to surgeon group <i>i</i>
w _i	Ward beds capacity for surgeon group <i>i</i>
${\cal Y}_{ij}$	Rate of <i>j</i> th surgical type of surgical group <i>i</i>
stw _{ij}	Average length of stay of patients who need <i>j</i> th surgical type of surgical group <i>i</i> in ward <i>i</i>
t _{ij}	Average time duration of <i>j</i> th surgical type of surgical group <i>i</i>
D	Planning time horizon
$LB_{i,j}$	Minimum demand for <i>j</i> th surgical type of surgical group <i>i</i>
R _{ijr}	If operating room r is appropriate for the <i>j</i> th surgical type of surgical group <i>i</i> ; is equal to 1; otherwise, 0
h _r	Opening hour of operating room r
λI_{ij}	If <i>j</i> th surgical type of surgical group <i>i</i> needs ICU; is equal to 1; otherwise, 0
λ_{ijk}	If <i>j</i> th surgical type of surgical group <i>i</i> needs ICU type k; is equal to 1; otherwise, 0.
stI _{ij}	Average length of stay of patients who need <i>j</i> th surgical type of surgical group <i>i</i> in ICU
I_k	Capacity of ICU type k
R	Recovery beds capacity
h	Opening hour of recovery room
stR _{ij}	Average length of stay of patients who need <i>j</i> th surgical type of surgical group <i>i</i> in recovery
over _r	Acceptable overtime of operating room r
wl _i	Number of patients of surgical group <i>i</i> in waiting list at the beginning of planning horizon

The LP model is then formulated as follows:

Max $\sum_{i} \sum_{j} \frac{y_{ij} \times x_{i}}{t_{ij}}$ (1)Subject to $\sum_{j} \left(\frac{y_{ij}}{t_{ij}} \times stw_{ij} \times x_{i} \right) \le D \times w_{i}$ $\forall i$ (2)(3) $\sum_{i} LB_{ij} \leq \sum_{i} \frac{y_{ij} \times x_{i}}{t_{ii}}$ $\forall i$ $y_{ij} \times x_i \leq z_{ij}$ $\forall i, j$ (4) $z_{ij} = \sum_{r} R_{ijr} \times D \times h_{r}$ (5) $\sum_{i} \sum_{j} \frac{y_{ij} \times x_{i}}{t_{ii}} \times \lambda I_{ij} \times stI_{ij} \times \lambda_{ijk} \leq D \times I_{k}$ (6) $\forall k$ (7) $\sum_{i} \sum_{j} \frac{y_{ij} \times x_{i}}{t_{ij}} \times stR_{ij} \leq D \times R \times h'$ (0)

$$\sum_{i} \sum_{j} R_{ijr} \times x_{i} \times y_{ij} \leq D(h_{r} + over_{r}) \qquad \forall r$$
⁽⁸⁾

In above formulation the objective function, Equation 1, looking for maximize number of operations performed during the planning horizon. Making a balance between the capacity allocated to surgery groups and inpatient department capacity is one of the challenging issues in real world's condition that should be considered. If you assign the operating room capacity to surgical groups regardless of their wards capacity during the planning time horizon, it may be possible that some operating rooms capacity remain empty because of there is no capacity in the related ward and operated patients cannot transfer to the ward. Hence, the Equation 2 ensures that the total length of stay of surgical group i patients do not exceed available time of ward i during planning horizon.

According to the model provided in this study based on actual limitations obtained from Shahid Madani hospital, and given the fact that this hospital is referred as referral center for emergency patient, taking into account patients who need to perform essential emergency operation or care is doubly necessary. In fact, there is a minimum capacity which must be

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assigned to each surgical groups according to their urgent and emergency demand. Constraint number 3 leads to assign of a minimum capacity equal to the time required to perform emergency surgery for each surgical groups.

Another tips that the problem model must take into consideration is the fact that in reality some operating rooms are equipped for some special operations and usually some rooms are not appropriate for some special surgical groups. On the other hand operating room capacity are limited. Equations 4 and 5 guaranty that each procedure is done in appropriate operating room and also operating room capacity limitation is met. Finally, the relationships 6, 7 and 8 represent the limits of the capacity of ICU, recovery beds and operating room respectively.

Univariate robust M-estimators

The univariate M-estimators are robust estimators used to estimate the parameters of univariate distributions. Two important parameters of univariate distributions are location and sale parameter. To define these two parameters let x denotes the random variable of interest. The c.d.f and p.d.f of x are shown by $F_X(X)$ and $f_X(X)$ respectively. If the following equation is satisfied then μ is called the location parameter.

$$f_X(X) = f_0(x - \mu) \tag{9}$$

Where $f_0(x)$ is a p.d.f.

Location model

Let $\{x_1, x_2, ..., x_n\}$ denote a random sample of size n with p.d.f introduced in Equation 9. Then the location model is defined as:

$$x_i = \mu + u_i, \ i = 1, 2, ..., \ n$$
 (10)

(10)

Where u_i 's are i.i.d random variables with p.d.f $f_0(x)$. In classical location models $f_0(x)$ is usually assumed to be known. As an example $f_0(x)$ is usually assumed to be standard normal p.d.f. Based on this assumption one may use the maximum likelihood estimation to estimate μ . However, in some practical application the probability distribution of u_i 's may not be known exactly Contaminated normal distributions are some examples of these situations. In this cases using classical estimators results in imprecise estimation of location parameter. Thus, it is better to use some other estimators which are not sensitive to departures from the assumed model. Those estimators are called robust estimators. One of the frequently used robust estimators of location parameter is the location M-estimator Maronna et al., [10] defined the location M-estimators as the following:

$$\hat{\mu} = \arg \sum_{i=1}^{n} \rho_i (x_i - \mu)$$
 (11)

Where $\rho_1(x)$ is a ρ function with the following properties.

- $\rho(x)$ is a non-decreasing function of |x| and $\rho(0) = 0$
- $\rho(x)$ Must be increasing for x > 0. So $\rho(x) < \rho(\infty)$
- If $\rho(x)$ is bounded, then $\rho(\infty) = 1$

By differentiating equation 11 with respect to μ , one may obtain the location M-estimator as follow:

$$\sum_{i=1}^{n} \psi_{i} \left(x_{i} - \mu \right) = 0$$

Where $\psi_i(x) = \frac{d\rho_i(x)}{dx}$. These functions are defined to have well performances in the absence of outliers as well

as being accurate in the presence of outliers.

COMPUTATIONAL RESULTS

As mentioned above, model parameters were obtained from Shahid Madani HIS system. This hospital contains 6 surgical suits and there are 11 recovery beds for patients who are operated. In addition, 3 ICU exist in the hospital with a capacity of 14, 8 and 5 beds. The developed model were solved by GAMS software. The model parameters are estimated based on information derived from the period of March 1, 2014 to September 22, 2014 of empirical data of the OR department and HIS system for inpatients in the Shahid Madani hospital. During this time operating information related to 6572 patients were registered. It is worth noting that, the arrival rate of emergency patients to this hospital is an important parameter should be considered, according to the fact that this hospital is known as referral center of emergency cases in Alborz province. We use M robust estimator to estimate the model parameters, due to existence of data contamination in real data obtained from HIS system. Average operating time duration and average patient length of stay in recovery, ICU and ward are estimated by M robust estimator. Opening hour of operating room and recovery is 8 hours a day and they are open 5 days a week. The minimum time required for each surgical group is equal to sum of the necessary operating time of emergency patients related to each group and derived from the HIS system during the period of March 1, 2014 to September 22, 2014. Planning horizon is defined 180 days and operating room overtime are allowed 1 hour a day. The

results of solving model by GAMS software using M robust estimator and MLE are summarized in Table 6 and Figure 1 and 2.

Table 6. Computational results for M robust estimator and MLE				
Surgical group	MLE estimator	M robust estimator		
Orthopedic	41% of first operating theater	47% of first operating theater		
Eye	6% of first operating theater	7% of first operating theater		
General surgery	37% of first operating theater	30% of first operating theater		
Neurosurgery	16% of first operating theater	16% of first operating theater		
Burn	55% of second operating theater	57% of second operating theater		
ENT	45% of second operating theater	43% of second operating theater		





Fig 1. Percent of first operating theater time assignment to surgical groups

Fig 2. Percent of second operating theater time assignment to surgical groups

CONCLUSION

This paper examines the problem of assigning operating room capacity to surgical groups as a resource capacity planning strategic decision. In this article a mathematical model is developed to solve the problem. In order to make the problem as real as possible the objective function and the constraints are defined according to Shahid Madani hospital limitations. The model parameters are estimated based on information derived from the empirical data of the HIS system of this hospital. According to this fact that in real data there are contaminations, we use M robust estimator to estimate the model parameters. Obtained computational results by GAMS software show differences between the answer obtained with any of the classic approaches MLE and M robust estimator. Considering uncertainty in estimate parameters is an important assumption in operating room planning and scheduling and is essential in many real world operating room management problems; thus a future research could be in corporation of uncertainty in the addressed problem. Also an analysis of the sensitivity of this study findings to changes in model parameters can helps the managers to achieve optimal solutions.

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