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Development of Reverse Logistic Network Model with Multiple Capacities and of Fuzzy Parameters (Resolution Model for Multi Product state)

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ABSTRACT

Many studies in the literature have assumed integrated logistics network design problem are Only an installed capacity of existing facilities Logistic network (such as production centers or repair, re- Distribution centers, collection centers etc) are available, This assumption is not realistic Facilities generally be different because of installed capacity development But the cost of installing these facilities increasing its capacity increases - increases. In the present study, a new model was introduced for the problem of designing an integrated logistics with multiple capacities and fuzzy parameters by consideration of more options of product recovery. To precisely solve this problem, a mathematical model was developed. To efficiently solve this model, a precise solution method based on Benders' analysis was suggested. Computational results of present study show efficiency of suggested analysis method because the result of Benders' analysis method and direct method have equal quality in two modes of single and multiple-product.

KEYWORDS: Network design, Integrated logistics, Multiple capacities, Recycling, Benderz analysis

1. INTRODUCTION

According to the American Reverse Logistics Executive Council (ARLEC), the reverse logistics is defined as follows: "The process of planning, implementing, and controlling the efficient, cost effective flow of raw materials, in-process inventory, finished goods, and related information from the point of consumption to the point of origin for the purpose of recapturing value or of proper disposal" [1]. Such a concept has been around for 20 years and several authors have studied it in terms of processes, objectives and complexity from different angles in different industries. In today's world, where the environment is a major concern for the public and the laws are enforceable, the field of reverse logistics is more important than ever. Among the many industries in which reverse logistics has become a necessity, electronics industry is facing a major challenge, which is shortening product life cycles. Therefore, designing an efficient reverse logistics network for the purpose of re-using the products that their life cycles has ended is very important.

As the product life cycles get shorter and governments make companies to recycle and restore their old or damaged products, many companies fall into the idea of a generating new return system or improving their existing systems in order to gain competitive advantage. In other words, many companies today are thinking about Reverse Logistics (RL) to optimize their return flows. This study examines the problem of designing an integrated logistics network with multiple capacities for different facilities and fuzzy parameters by consideration of more options of product recovery.

In many previous studies in the field of logistic network design, the design of forward and reverse logistics networks has been considered separately, whereas the reverse logistics network design has a great impact on the performance of forward network and vice versa. Thus, separating the design of reverse and forward logistics networks may result in sub-optimality, therefore the design of forward and reverse logistics networks should be integrated [2].

Today, one of the most important strategic problems in supply chain management is designing logistics network; because logistics network has a huge impact on the general performance of supply chain [3].

Jayaraman et al. [4] developed a mixed integer programming model for reverse logistics network design under a pull system based on customers' demand for recovered products. The purpose of the proposed model was to minimize the costs. They also discussed management aspects of their model and explained its application in decision-making.

Fleischmann et al. [5] suggested a mixed integer linear programming (MILP) model to analyze the impact of product recovery on logistics network design. They showed that the integrated approach' i.e. the simultaneous optimization of the reverse and forward networks is more cost-effective than the sequence design of reverse and forward networks. Realff et al. [6] offered a robust optimization model for carpet recycling. They studied a multi-product reverse network with single-capacity facilities.

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Zhou & Wang [1] studied an integrated general logistics network. They suggested a model of mixed integer programming and used an algorithm of branch and bound to solve this problem. Mutha et al. (2009) suggested a mathematical model for the design of a reverse logistics network with the assumption of reproduction, recovery and disposal as well as secondary markets.

Alumur et al. [7] suggest a multi-period model for the design of reverse logistics network. They introduced an integer programming model and stated that their suggested model can be used for existing real models. They considered a multiple-product network and tried to maximize profits. Finally, they performed computational tests for different scenarios and examined the advantage of their suggested model compared with other static models.

2. MATERIAL & METHODS

Library-based data collection was performed in this study and the data were obtained through searching the articles published in scientific journals. The available articles on the related literature were studies and examined, and then, the problem is formulated with the help and ideas from them and using mathematical modeling techniques. An algorithm analyzed to solve the problem and the results of the calculation of solving the model are analyzed.

The model presented in this study, in addition to considering capacity for all the centers and multi-product production, added one of the product recovery options that was not considered by almost any of the previous studies, i.e. recovery. It is the recovery option that is one of the most important aspects of the product life cycle due to its importance and profitability.

$$C^f_{mijl} = \left(c^{ij,m} \times t_{ij}\right) + \left(c^{jl,m} \times t_{jl}\right) + C_m$$
$$C^f_{mkjl} = \left(c^{kj,m} \times t_{kj}\right) + \left(c^{jl,m} \times t_{jl}\right) + C_m - CS_{rm}$$
$$C^r_{mljk} = \left(c^{lj,m} \times t_{lj}\right) + \left(c^{jk,m} \times t_{jk}\right) - CS_{rpm} + C_{km}$$
$$C^r_{mljko} = \left(c^{lj,m} \times t_{lj}\right) + \left(c^{jk,m} \times t_{jk}\right) + \left(c^{ko,m} \times t_{ko}\right) + C_o + C_{km}$$
$$C^r_{mljki} = \left(c^{lj,m} \times t_{lj}\right) + \left(c^{jk,m} \times t_{jk}\right) + \left(c^{ki,m} \times t_{ki}\right) + C_m + C_{km} - CS_{rm}$$
$$C^r_{mljkr} = \left(c^{lj,m} \times t_{lj}\right) + \left(c^{jk,m} \times t_{jk}\right) + \left(c^{kr,m} \times t_{kr}\right) + C_r + C_{km} - CS_{rc}$$

Objective function of the problem under study is expressed as follows. The first five terms indicate the costs of the production/reproduction centers construction (i), the mixed distribution-collection centers (j), CRC (k) centers, disposal centers (o) and recycle centers (e), respectively. The sixth term shows the cost of estimating demand from factories, whereas the seventh term shows the cost of estimating demand from CRC centers. The eighth, ninth, tenth and eleventh terms which are related to recovery options of returned products indicate respectively, recovery options, repair, reproduction, disposal and recycling. Finally, the last two terms show the un-estimated return and demand.

$$\begin{aligned}
\min & \sum_{i\in I}\sum_{h\in H} f^p_{ih} y^p_{ih} + \sum_{j\in J}\sum_{h\in H} f^d_{jh} y^d_{jh} + \sum_{k\in K}\sum_{h\in H} f^r_{kh} y^r_{kh} + \sum_{o\in O}\sum_{h\in H} f^x_{oh} y^x_{oh} + \sum_{r\in R}\sum_{h\in H} f^e_{rh} y^e_{rh} + \\
& \sum_{m\in M}\sum_{i\in I}\sum_{j\in J}\sum_{l\in L} C^f_{mijl} d_{ml} X^f_{mijl} + \sum_{m\in M}\sum_{k\in K}\sum_{j\in J}\sum_{l\in L} C^f_{mjkl} d_{ml} X^f_{mkjl} + \sum_{m\in M}\sum_{l\in L}\sum_{j\in J}\sum_{k\in K} C^r_{mljk} r_{ml} X^r_{mljk} \\
& + \sum_{m\in M}\sum_{l\in L}\sum_{j\in J}\sum_{k\in K}\sum_{i\in I} C^r_{mljki} r_{ml} X^r_{mljki} + \sum_{m\in M}\sum_{l\in L}\sum_{j\in J}\sum_{k\in K}\sum_{o\in O} C^r_{mljko} r_{ml} X^r_{mljko} \\
& + \sum_{m\in M}\sum_{l\in L}\sum_{j\in J}\sum_{k\in K}\sum_{r\in R} C^r_{mljkr} r_{ml} X^r_{mljkr} + \sum_{m\in M}\sum_{l\in L} C^u_{ml} d_{ml} U_{ml} + \sum_{m\in M}\sum_{l\in L} C^w_{ml} r_{ml} W_{ml}
\end{aligned} \qquad (\#)$$

3. Changing the fuzzy model of the problem into an absolute model

Suppose that $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$ are two triangular fuzzy numbers, the absolute equivalent of equation $\tilde{A} = \tilde{B}$ and inequality $\tilde{A} \le \tilde{B}$ are defined as follows:

$$\tilde{A} = \tilde{B} \iff a_1 = b_1,\ a_2 = b_2,\ a_3 = b_3$$
$$\tilde{A} \le \tilde{B} \iff a_1 \le b_1,\ a_2 \le b_2,\ a_3 \le b_3$$

Now, using the above definition, the absolute equivalent of equations and inequalities in the fuzzy model are written down. As a result, each of the limitations of the model which have fuzzy coefficient turn into three limitations with absolute coefficients. Thus, the model (#) is transformed as follows:

$$\min \sum_{i\in I}\sum_{h\in H}\tilde{f}^{p}_{ih}y^{p}_{ih}+\sum_{j\in J}\sum_{h\in H}\tilde{f}^{d}_{jh}y^{d}_{jh}+\sum_{k\in K}\sum_{h\in H}\tilde{f}^{r}_{kh}y^{r}_{kh}+\sum_{o\in O}\sum_{h\in H}\tilde{f}^{x}_{oh}y^{x}_{oh}+\sum_{r\in R}\sum_{h\in H}\tilde{f}^{e}_{rh}y^{e}_{rh}+\sum_{m\in M}\sum_{i\in I}\sum_{j\in J}\sum_{l\in L}C^{f}_{mijl}\tilde{d}_{ml}X^{f}_{mijl}$$
$$+\sum_{m\in M}\sum_{k\in K}\sum_{j\in J}\sum_{l\in L}C^{f}_{mjkl}\tilde{d}_{ml}X^{f}_{mkjl}+\sum_{m\in M}\sum_{l\in L}\sum_{j\in J}\sum_{k\in K}C^{r}_{mljk}\tilde{r}_{ml}X^{r}_{mljk}$$
$$+\sum_{m\in M}\sum_{l\in L}\sum_{j\in J}\sum_{k\in K}\sum_{i\in I}C^{r}_{mljki}\tilde{r}_{ml}X^{r}_{mljki}+\sum_{m\in M}\sum_{l\in L}\sum_{j\in J}\sum_{k\in K}\sum_{o\in O}C^{r}_{mljko}\tilde{r}_{ml}X^{r}_{mljko}$$
$$+\sum_{m\in M}\sum_{l\in L}\sum_{j\in J}\sum_{k\in K}\sum_{r\in R}C^{r}_{mljkr}\tilde{r}_{ml}X^{r}_{mljkr}+\sum_{m\in M}\sum_{l\in L}C^{u}_{ml}\tilde{d}_{ml}U_{ml}+\sum_{m\in M}\sum_{l\in L}C^{w}_{ml}\tilde{r}_{ml}W_{ml} \tag{1}$$

S.T.

$$\sum_{l\in L}\sum_{j\in J}X^{r}_{mljk}{r_{ml}}^{t}=\sum_{l\in L}\sum_{j\in J}X^{f}_{mkjl}{d_{ml}}^{t}\qquad \forall m\in M,k\in K,t=1,2,3 \tag{2}$$

$$\sum_{i\in I}\sum_{j\in J}X^{f}_{mijl}+\sum_{k\in K}\sum_{j\in J}X^{f}_{mkjl}+U_{ml}=1\qquad \forall m\in M,l\in L \tag{3}$$

$$\sum_{j\in J}\sum_{k\in K}\Big(\sum_{i\in I}X^{r}_{mljki}+\sum_{o\in O}X^{r}_{mljko}+\sum_{r\in R}X^{r}_{mljkr}+X^{r}_{mljk}\Big)+W_{ml}=1\quad \forall m\in M,l\in L \tag{4}$$

$$\sum_{j\in J}\sum_{k\in K}\sum_{l\in L}X^{r}_{mljki}{r_{ml}}^{t}\le\sum_{j\in J}\sum_{l\in L}X^{f}_{mijl}{d_{ml}}^{t}\qquad \forall m\in M,i\in I,t=1,2,3 \tag{5}$$

$$\gamma_m\left(X^{r}_{mljk}+\sum_{i\in I}X^{r}_{mljki}+\sum_{o\in O}X^{r}_{mljko}+\sum_{r\in R}X^{r}_{mljkr}\right)\le\sum_{o\in O}X^{r}_{mljko}\quad \forall m\in M,l\in L,j\in J,k\in K \tag{6}$$

$$\beta_m\left(X^{r}_{mljk}+\sum_{i\in I}X^{r}_{mljki}+\sum_{o\in O}X^{r}_{mljko}+\sum_{r\in R}X^{r}_{mljkr}\right)\ge X^{r}_{mljk}\quad \forall m\in M,l\in L,j\in J,k\in K \tag{7}$$

$$\eta_m\left(X^{r}_{mljk}+\sum X^{r}_{mljki}+\sum X^{r}_{mljko}+\sum X^{r}_{mljkr}\right)\ge\sum X^{r}_{mljk}\quad \forall m\in M,l\in L,j\in J,k\in K \tag{8}$$

$$\sum_{j\in J}X^{f}_{mijl}{d_{ml}}^{t}\le\sum_{h\in H}cap^{p,t}_{ih}Y^{p}_{ih}\,\forall m\in M,l\in L,i\in I,l\in L,t=1,2,3 \tag{9}$$

$$\sum_{j\in J}X^{f}_{mijl}{d_{ml}}^{t}+\sum_{k\in K}X^{f}_{mkjl}{d_{ml}}^{t}\le\sum_{h\in H}cap^{d,t}_{jh}Y^{d}_{jh}\,\forall m\in M,l\in L,j\in J,t=1,2,3 \tag{10}$$

$$\sum_{j\in J}X^{r}_{mljk}{r_{ml}}^{t}+\sum_{j\in J}\sum_{i\in I}X^{r}_{mljki}{r_{ml}}^{t}+\sum_{j\in J}\sum_{o\in O}X^{r}_{mljko}{r_{ml}}^{t}+\sum_{j\in J}\sum_{r\in R}X^{r}_{mljkr}{r_{ml}}^{t}\le\sum_{h\in H}cap^{r,t}_{kh}Y^{r}_{kh}\,\forall m\in M,k\in K,l\in L,t=1,2,3 \tag{11}$$

$$\sum_{k\in K}X^{r}_{mljk}{r_{ml}}^{t}+\sum_{k\in K}\sum_{i\in I}X^{r}_{mljki}{r_{ml}}^{t}+\sum_{k\in K}\sum_{o\in O}X^{r}_{mljko}{r_{ml}}^{t}+\sum_{k\in K}\sum_{r\in R}X^{r}_{mljkr}{r_{ml}}^{t}\le cap^{d,t}_{rj}Y^{d}_{k}\,\forall m\in M,j\in J,l\in L,t=1,2,3 \tag{12}$$

$$\sum_{j\in J}\sum_{k\in K}X^{r}_{mljki}{r_{ml}}^{t}\le\sum_{h\in H}cap^{p,t}_{rih}Y^{p}_{rih}\,\forall m\in M,i\in I,l\in L,t=1,2,3 \tag{13}$$

$$\sum_{j\in J}\sum_{k\in K}X^{r}_{mljko}{r_{ml}}^{t}\le\sum_{h\in H}cap^{x,t}_{roh}Y^{x}_{oh}\,\forall m\in M,i\in I,l\in L,t=1,2,3 \tag{14}$$

$$\sum_{j\in J}\sum_{k\in K}X^{r}_{mljkr}{r_{ml}}^{t}\le\sum_{h\in H}cap^{e,t}_{rh}Y^{e}_{rh}\,\forall m\in M,i\in I,l\in L,t=1,2,3 \tag{15}$$

$$0\le X^{f}_{mijl},X^{f}_{mkjl},X^{r}_{mljki},X^{r}_{mljk},X^{r}_{mljko},U_{ml},X_{ml}\le 1\quad \forall m\in M,i\in I,j\in J,k\in K,r\in R,l\in L \tag{16}$$

$$Y^{p}_{i},Y^{d}_{j},Y^{r}_{k},Y^{x}_{o},Y^{e}_{r}\in\{0,1\}\quad \forall m\in M,i\in I,j\in J,k\in K,r\in R,l\in L \tag{17}$$

However, by applying the definition, all of the limitations change from fuzzy to absolute mode, definitive, the objective function has not been yet converted into absolute mode. To convert the fuzzy objective function into absolute mode, the centroid method is used for de-fuzzification of the fuzzy coefficients, $\tilde{d}_{ml}=(d^{1}_{ml},d^{2}_{ml},d^{3}_{ml})$ and $\tilde{r}_{ml}=(r^{1}_{ml},r^{2}_{ml},r^{3}_{ml})$. According to this method, the absolute equivalent of the triangular fuzzy number is calculated as follows:

$$\bar{A} = \tfrac{1}{3}(a_1 + a_2 + a_3)$$

So, the absolute equivalent of fuzzy coefficients, $\tilde{d}_{ml} = (d^1_{ml}, d^2_{ml}, d^3_{ml})$ and $\tilde{r}_{ml} = (r^1_{ml}, r^2_{ml}, r^3_{ml})$ will be as follows by applying the centroid de-fuzzification method:

$$\bar{d}_{ml} = \tfrac{1}{3}(d^1_{ml} + d^2_{ml} + d^3_{ml})$$

$$\bar{r}_{ml} = \tfrac{1}{3}(r^1_{ml} + r^2_{ml} + r^3_{ml})$$

Other coefficients are similarly de-fuzzificated.

4. Solving the model by Benders' decomposition method

Benders' decomposition method transfers a mixed integer programming model into a main problem and a sub-problem which are solved iteratively using each other's solution [8].

The sub-problem includes continuous variables and related limitations related, while the main problem includes integer variables and a continuous variable which links the two problems. The optimal solution for the main problem provides a low limit for the desired goal. Using the solution obtained by solving the main problem, an upper limit can be defined for the general purpose of the problem through fixing the integer variables as inputs of sun-problem of a dual for the solved sub-problem. This solution is also used for generating Benders' cut which includes continuous variables added to the main problem. In the next iteration, this cut is added to the main problem; and using the solution of this problem, a new low limit is obtained for the general problem which is guaranteed not to be worse than the present low limit. Accordingly, the main problem and the sub-problem are solved iteratively until a stopping condition is achieved, i.e. lessening the difference between upper and lower limits from a small number. Benders' decomposition method provides optimal solutions in a finite number of iterations [8].

5. Benders' sub-problem

The sub-problem is a minimization problem which obtains the optimal value of continuous variables $(X^f_{mijl}, X^f_{mkjl}, X^r_{mljk}, X^r_{mljki}, X^r_{mljko}, X^r_{mljkr}, U_{ml}, W_{ml})$ for the fixed variables $(\hat{y}^p_{ih}, \hat{y}^d_{jh}, \hat{y}^r_{kh}, \hat{y}^x_{oh}, \hat{y}^e_{rh})$. This can be expressed as follows:

$$\begin{aligned}\min &\sum_{m\in M}\sum_{i\in I}\sum_{j\in J}\sum_{l\in L} C^f_{mijl}\tilde{d}_{ml}X^f_{mijl} + \sum_{m\in M}\sum_{k\in K}\sum_{j\in J}\sum_{l\in L} C^f_{mjkl}\tilde{d}_{ml}X^f_{mkjl} + \sum_{m\in M}\sum_{l\in L}\sum_{j\in J}\sum_{k\in K} C^r_{mljk}\tilde{r}_{ml}X^r_{mljk} \\ &+ \sum_{m\in M}\sum_{l\in L}\sum_{j\in J}\sum_{k\in K}\sum_{i\in I} C^r_{mljki}\tilde{r}_{ml}X^r_{mljki} + \sum_{m\in M}\sum_{l\in L}\sum_{j\in J}\sum_{k\in K}\sum_{o\in O} C^r_{mljko}\tilde{r}_{ml}X^r_{mljko} \\ &+ \sum_{m\in M}\sum_{l\in L}\sum_{j\in J}\sum_{k\in K}\sum_{r\in R} C^r_{mljkr}\tilde{r}_{ml}X^r_{mljkr} + \sum_{m\in M}\sum_{l\in L} C^u_{ml}\tilde{d}_{ml}U_{ml} + \sum_{m\in M}\sum_{l\in L} C^w_{ml}\tilde{r}_{ml}W_{ml}\end{aligned} \tag{18}$$

$$\sum_{l\in L}\sum_{j\in J} X^r_{mljk} r_{ml}{}^t \le \sum_{l\in L}\sum_{j\in J} X^f_{mkjl} d_{ml}{}^t \qquad \forall m\in M, k\in K, t \tag{19}$$

$$\sum_{l\in L}\sum_{j\in J} X^r_{mljk} r_{ml}{}^t \ge \sum_{l\in L}\sum_{j\in J} X^f_{mkjl} d_{ml}{}^t \qquad \forall m\in M, k\in K, t \tag{20}$$

$$\sum_{i\in I}\sum_{j\in J} X^f_{mijl} + \sum_{k\in K}\sum_{j\in J} X^f_{mkjl} + U_{ml} \le 1 \qquad \forall m\in M, l\in L \tag{21}$$

$$\sum_{i\in I}\sum_{j\in J} X^f_{mijl} + \sum_{k\in K}\sum_{j\in J} X^f_{mkjl} + U_{ml} \ge 1 \qquad \forall m\in M, l\in L \tag{22}$$

$$\sum_{j\in J}\sum_{k\in K}\Big(\sum_{i\in I} X^r_{mljki} + \sum_{o\in O} X^r_{mljko} + \sum_{r\in R} X^r_{mljkr} + X^r_{mljk}\Big) + W_{ml} \le 1 \quad \forall m\in M, l\in L \tag{23}$$

$$\sum_{j\in J}\sum_{k\in K}\Big(\sum_{i\in I} X^r_{mljki} + \sum_{o\in O} X^r_{mljko} + \sum_{r\in R} X^r_{mljkr} + X^r_{mljk}\Big) + W_{ml} \ge 1 \quad \forall m\in M, l\in L \tag{24}$$

$$\sum_{j\in J}\sum_{k\in K}\sum_{l\in L} X^r_{mljki} r_{ml}{}^t \le \sum_{j\in J}\sum_{l\in L} X^f_{mijl} d_{ml}{}^t \qquad \forall m\in M, i\in I, t=1,2,3 \tag{25}$$

$$\gamma_m \left(X^r_{mljk} + \sum_{i \in I} X^r_{mljki} + \sum_{o \in O} X^r_{mljko} + \sum_{r \in R} X^r_{mljkr} \right) \le \sum_{o \in O} X^r_{mljko} \quad \forall m \in M, l \in L, j \in J, k \in K \tag{26}$$

$$\beta_m \left(X^r_{mljk} + \sum_{i \in I} X^r_{mljki} + \sum_{o \in O} X^r_{mljko} + \sum_{r \in R} X^r_{mljkr} \right) \ge X^r_{mljk} \quad \forall m \in M, l \in L, j \in J, k \in K \tag{27}$$

$$\eta_m \left(X^r_{mljk} + \sum_{i \in I} X^r_{mljki} + \sum_{o \in O} X^r_{mljko} + \sum_{r \in R} X^r_{mljkr} \right) \ge \sum_{r \in R} X^r_{mljkr} \quad \forall m \in M, l \in L, j \in J, k \in K \tag{28}$$

$$\sum_{j \in J} X^f_{mijl} d_{ml}{}^t \le \sum_{h \in H} cap^{pt}_{ih} \hat{Y}^p_{ih} \, \forall m \in M, l \in L, i \in I, l \in L, t = 1,2,3 \tag{29}$$

$$\sum_{j \in J} X^f_{mijl} d_{ml}{}^t + \sum_{k \in K} X^f_{mkjl} d_{ml}{}^t \le \sum_{h \in H} cap^{d,t}_{jh} \hat{Y}^d_{jh} \, \forall m \in M, l \in L, j \in J, t = 1,2,3 \tag{30}$$

$$\sum_{j \in J} X^r_{mljk} r_{ml}{}^t + \sum_{j \in J} \sum_{i \in I} X^r_{mljki} r_{ml}{}^t + \sum_{j \in J} \sum_{o \in O} X^r_{mljko} r_{ml}{}^t + \sum_{j \in J} \sum_{r \in R} X^r_{mljkr} r_{ml}{}^t \le \sum_{h \in H} cap^{rt}_{kh} \hat{Y}^r_{kh} \, \forall m \in M, k \in K, l \in L, t = 1,2,3 \tag{31}$$

$$\sum_{k \in K} X^r_{mljk} r_{ml}{}^t + \sum_{k \in K} \sum_{i \in I} X^r_{mljki} r_{ml}{}^t + \sum_{k \in K} \sum_{o \in O} X^r_{mljko} r_{ml}{}^t + \sum_{k \in K} \sum_{r \in R} X^r_{mljkr} r_{ml}{}^t \le cap^{d,t}_{rj} \hat{Y}^d_k \, \forall m \in M, j \in J, l \in L, t = 1,2,3 \tag{32}$$

$$\sum_{j \in J} \sum_{k \in K} X^r_{mljki} r_{ml}{}^t \le \sum_{h \in H} cap^{p,t}_{rih} \hat{Y}^p_{rih} \, \forall m \in M, i \in I, l \in L, t = 1,2,3 \tag{33}$$

$$\sum_{j \in J} \sum_{k \in K} X^r_{mljko} r_{ml}{}^t \le \sum_{h \in H} cap^{x,t}_{roh} \hat{Y}^x_{oh} \, \forall m \in M, i \in I, l \in L, t = 1,2,3 \tag{34}$$

$$\sum_{j \in J} \sum_{k \in K} X^r_{mljkr} r_{ml}{}^t \le \sum_{h \in H} cap^{e,t}_{rh} \hat{Y}^e_{rh} \, \forall m \in M, i \in I, l \in L, t = 1,2,3 \tag{35}$$

$$0 \le X^f_{mijl}, X^f_{mkjl}, X^r_{mljki}, X^r_{mljk}, X^r_{mljko}, U_{ml}, X_{ml} \le 1 \quad \forall m \in M, i \in I, j \in J, k \in K, r \in R, l \in L \tag{36}$$

It should be noted that some limitations which are equally found in the proposed model are converted into two limitations of less-than-or-equal-to and more-than-or-equal-to with no changes in solution space and optimal solutions. This is to obtain the dual problem more easily.

Now, to generate Benders' cuts for the main problem, the dual BSP (.) is used. To obtain the dual of this problem, the dual variables π^1_{mkt} 'π^2_{mkt} 'π^3_{ml} 'π^4_{ml} 'π^5_{ml} 'π^6_{ml} 'π^7_{mit} 'π^8_{mljk} 'π^9_{mljk} 'π^{10}_{mljk} 'π^{11}_{mlit} 'π^{12}_{mljt} 'π^{13}_{mklt} 'π^{14}_{mjlt} 'π^{15}_{milt} ' π^{16}_{molt} 'π^{17}_{mrlt} are used for each of the limitations (19) to (36). Considering these variables, the dual sub-problem DBSP (.) will be as follows:

$$\begin{aligned} \max - \sum_{m \in M} \sum_{l \in L} \pi^3_{ml} + \sum_{m \in M} \sum_{l \in L} \pi^4_{ml} - \sum_{m \in M} \sum_{l \in L} \pi^5_{ml} + \sum_{m \in M} \sum_{l \in L} \pi^6_{ml} - \sum_{t=1}^{3} \sum_{m \in M} \sum_{l \in L} \sum_{i \in I} \sum_{h \in H} cap^p_{ih} \hat{Y}^p_{ih} \pi^{11}_{milt} \\ - \sum_{t=1}^{3} \sum_{m \in M} \sum_{l \in L} \sum_{j \in J} \sum_{h \in H} cap^d_{jh} \hat{Y}^d_{jh} \pi^{12}_{mljt} - \sum_{t=1}^{3} \sum_{m \in M} \sum_{l \in L} \sum_{k \in K} \sum_{h \in H} cap^r_{kh} \hat{Y}^r_{kh} \pi^{13}_{mklt} \\ - \sum_{t=1}^{3} \sum_{m \in M} \sum_{l \in L} \sum_{j \in J} \sum_{h \in H} cap^d_{rjh} \hat{Y}^d_{jh} \pi^{14}_{mjlt} - \sum_{t=1}^{3} \sum_{m \in M} \sum_{l \in L} \sum_{i \in I} \sum_{h \in H} cap^p_{rih} \hat{Y}^p_{rih} \pi^{15}_{milt} \\ - \sum_{t=1}^{3} \sum_{m \in M} \sum_{l \in L} \sum_{o \in O} \sum_{h \in H} cap^x_{oh} \hat{Y}^x_{oh} \pi^{16}_{molt} - \sum_{t=1}^{3} \sum_{m \in M} \sum_{l \in L} \sum_{r \in R} \sum_{h \in H} cap^e_{rh} \hat{Y}^e_{rh} \pi^{17}_{mrlt} \end{aligned} \tag{37}$$

$$-\pi^3_{ml} + \pi^4_{ml} + \sum_{t=1}^{3} d_{ml}{}^t \left(\pi^7_{mit} - \pi^{11}_{milt} - \pi^{12}_{mljt} \right) \le C^f_{mijl} \tilde{d_{ml}} \quad \forall m, i, j, l \tag{38}$$

$$-\pi^3_{ml} + \pi^4_{ml} + \sum_{t=1}^{3} d_{ml}{}^t \left(\pi^1_{mkt} - \pi^2_{mkt} - \pi^{12}_{mljt} \right) \le C^f_{mkjl} \tilde{d_{ml}} \quad \forall m, k, j, l \tag{39}$$

$$-\pi_{ml}^{5}+\pi_{ml}^{6}-\gamma_m\pi_{mljk}^{8}+(\beta_m-1)\pi_{mljk}^{9}+\eta_m\pi_{mljk}^{10}-\sum_{t=1}^{3}r_{ml}{}^{t}\left(\pi_{mkt}^{1}-\pi_{mkt}^{2}+\pi_{mkit}^{13}+\pi_{mjlt}^{14}\right)\le C_{mljk}^{r}\widetilde{r_{ml}}\quad\forall m,l,j \tag{40}$$

$$-\pi_{ml}^{5}+\pi_{ml}^{6}-\gamma_m\pi_{mljk}^{8}+\beta_m\pi_{mljk}^{9}+\eta_m\pi_{mljk}^{10}-\sum_{t=1}^{3}r_{ml}{}^{t}\left(\pi_{mit}^{7}+\pi_{mkit}^{13}+\pi_{mjlt}^{14}+\pi_{milt}^{15}\right)\le C_{mljki}^{r}\widetilde{r_{ml}}\quad\forall m,l,j,k,i \tag{41}$$

$$-\pi_{ml}^{5}+\pi_{ml}^{6}-(\gamma_m-1)\pi_{mljk}^{8}+\beta_m\pi_{mljk}^{9}+\eta_m\pi_{mljk}^{10}-\sum_{t=1}^{3}r_{ml}{}^{t}\left(\pi_{mkit}^{13}+\pi_{mjlt}^{14}+\pi_{molt}^{16}\right)\le C_{mljko}^{r}\widetilde{r_{ml}}\quad\forall m,l,j,k,o \tag{42}$$

$$-\pi_{ml}^{5}+\pi_{ml}^{6}-\gamma_m\pi_{mljk}^{8}+\beta_m\pi_{mljk}^{9}+(\eta_m-1)\pi_{mljk}^{10}-\sum_{t=1}^{3}r_{ml}{}^{t}\left(\pi_{mkit}^{13}+\pi_{mjlt}^{14}+\pi_{mrlt}^{17}\right)\le C_{mljkr}^{r}\widetilde{r_{ml}}\quad\forall m,l,j,k,r \tag{43}$$

$$-\pi_{ml}^{3}+\pi_{ml}^{4}\le C_{ml}^{u}\widetilde{d_{ml}}\quad\forall m,l \tag{44}$$

$$-\pi_{ml}^{5}+\pi_{ml}^{6}\le C_{ml}^{w}\widetilde{r_{ml}}\quad\forall m,l \tag{45}$$

6. Solving the model and computational results of multi-product mode:
The data of the studied instance are presented in Table 1.

Table 1. Network parameters for the single product instance

Description	Parameter	Value
Fixed cost of constructing each factory	f_{ih}^{p}	6,000,000+1,000,000*h
Fixed cost of constructing each mixed collection-distribution center	f_{jh}^{d}	12,000,+1000*(h-1)
Fixed cost of constructing each CRC center	f_{kh}^{r}	42,000+2,000*(h-1)
Fixed cost of constructing each disposal facility	f_{oh}^{x}	90,000+100,000*(h-1)
Fixed cost of constructing each recovery center	f_{rh}^{e}	2,550+200*(h-1)
Cost of transportation based on the distance between factory and mixed collection-distribution center per unit of product	$c^{pd,m}$	0.10
Cost of transportation based on the distance between factory and customer per unit of product	C^{dcm}	0.25
Cost of transportation based on the distance between CRC center and mixed collection-distribution center per unit of product	C^{rdm}	0.11
Cost of transportation based on the distance between mixed collection-distribution center and CRC center per unit of product	c^{drm}	0.15
Cost of transportation based on the distance between mixed collection-distribution center and customer location per unit of product	c^{cdm}	0.19
Cost of transportation based on the distance between CRC center and disposal facility per unit of product	c^{rxm}	0.30
Cost of transportation based on the distance between CRC center and recycle center per unit of product	c^{rem}	0.12
Minimum percentage of waste disposal	γ_m	0.50
Maximum percentage of recycling	η_m	0.40
Maximum percentage of repairs	β_m	0.00
Value of savings per unit of product for CRC repairs	CS_{rp}	18
Value of savings per unit of product for recycling in recycling center	CS_{rc}	20
Value of savings per unit of product for reproduction in factory	CS_{rm}	23
Cost of disposal in disposal facility	C_O	10
	C_1	35
Cost of reproduction in factory	C_2	45
Cost of repair in CRC center per unit of product	C_{rp1}	10
	C_2	14
Cost of recycling in recycling center per unit of product	C_r	5.20
Cost of fines of an uncollected unit of product	C_{ml}^{w}	∞
Cost of fines of an unsatisfied unit of demand	C_{ml}^{u}	∞

Now, with regard to the above date, this problem is solved after 17 Benders' iterations and the network cost is obtained 8589342671353 units of currency. The convergence of Benders' method is shown in Figure 1. To compare the performance of the proposed solution, the intended instance has been solved using a mathematical model. The results of the two methods are shown in Table 2.

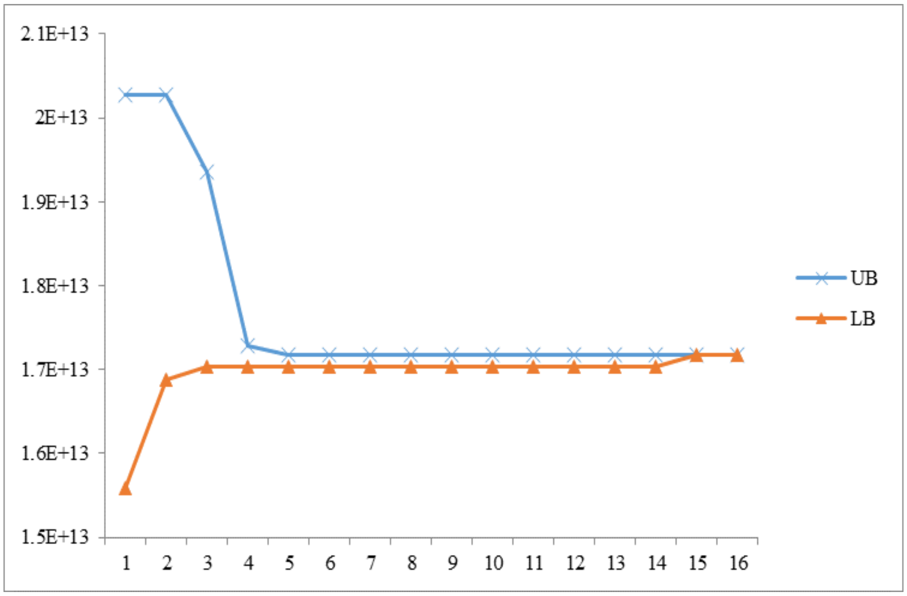

Fig. 1. The convergence of Bender' decomposition method for multi-product example

Table 2. Comparison of performance of Bender' decomposition method and mathematical model

Objective function	Duration of solution	Method
17178626108393	96.678	Mathematical model
17178626108393	29.167	Benders' decomposition method

In the following, Table 3 shows the results of the activation of different sites in which the activated centers cells are gray.

Table 3. Active units in the optimal solution

Recycling center	Disposal facility	CRC center	Collection and Distribution Centers	Factory
1	1	1	1	1
2	2	2	2	2
3		3	3	3
		4	4	4
		5	5	5
		6	6	6
		7	7	
			8	
			9	
			10	

Now, the way the existing facilities in the network respond to customers' returns and demand are discussed. In other words, the percentage of customers' demand satisfaction for the products and the rate of return flow are presented in the following figures. In the next figures, the diagram of percentage of customers' demands which are satisfied by factories (P) and CRC Centers (K) are drawn for products 1 and 2.

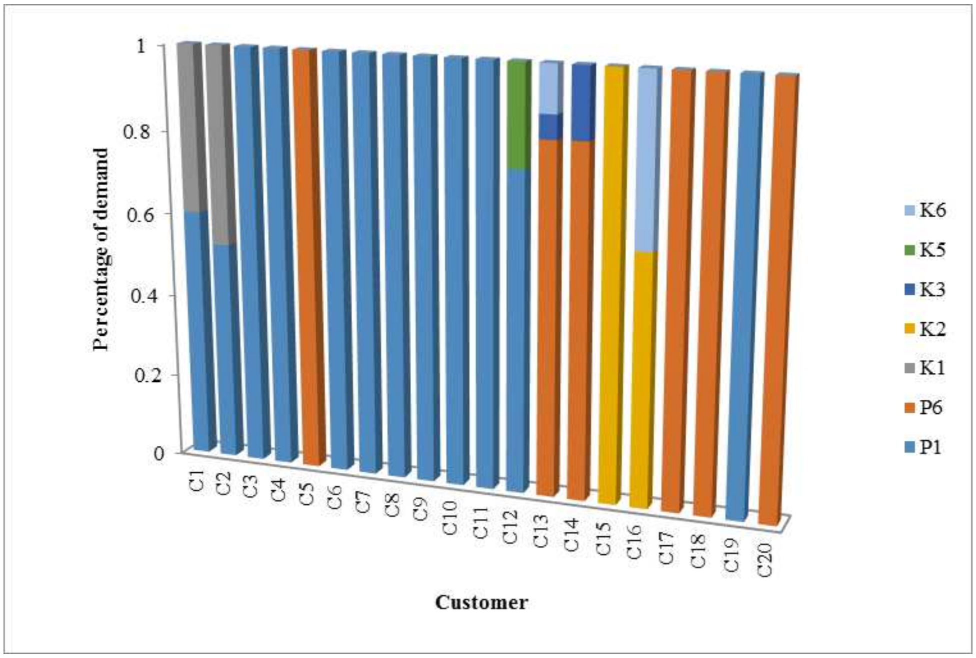

Fig. 2. Percentage of customers' demand satisfaction for product 1

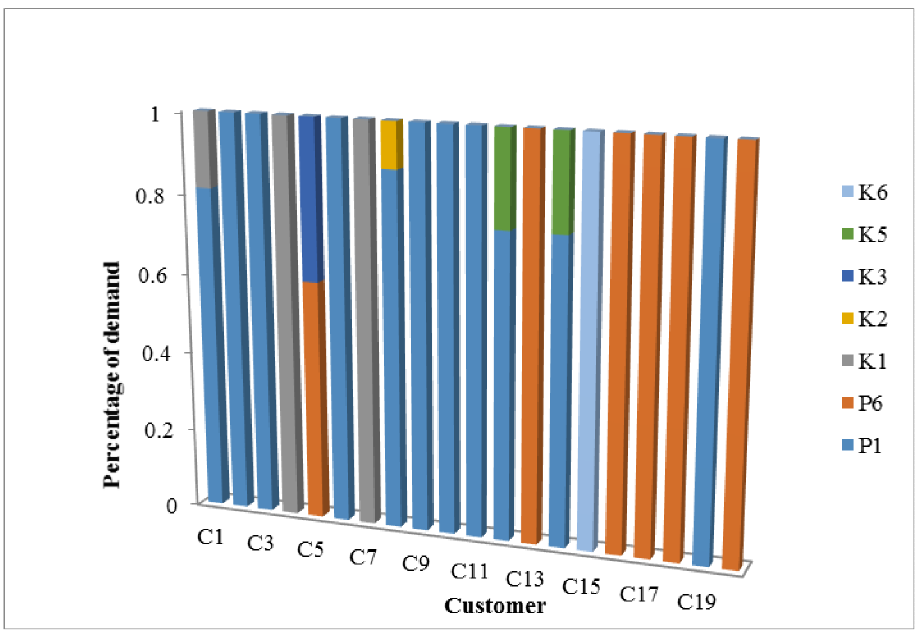

Fig. 3. Percentage of customers' demand satisfaction for product 2

According to Figure 2, it is clear that, for customer C1 for instance, demand for the product is satisfied 60% by the factory P1 and 40% by the CRC Center K1. Also, according to Figure 3, demand for second product by customer C15 is satisfied 99.6% by the CRC Center K6 and 0.4% by CRC Center K2. In the figure below, the percentage of product returns by customers for products 1 and 2 is presented.

According to the diagram of product returns in Figure 4, for the first product and customer C12 for instance, 30% of product returns are sent to CRC center K5, 30% is sent to factory P1 for reproduction, and the remaining 40% is sent to the disposal facility O1.

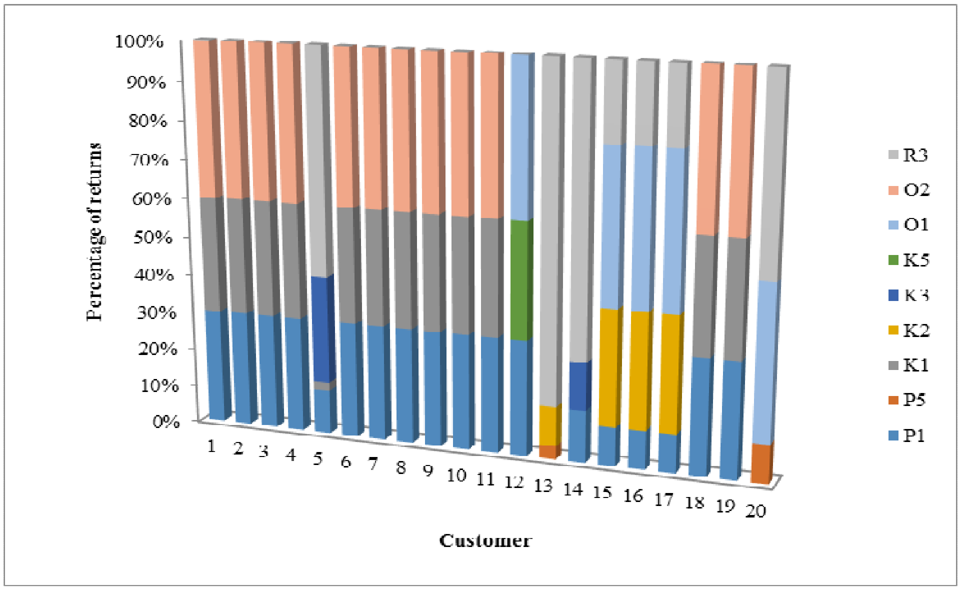

Fig. 4. Percentage of customers' returns for product 1

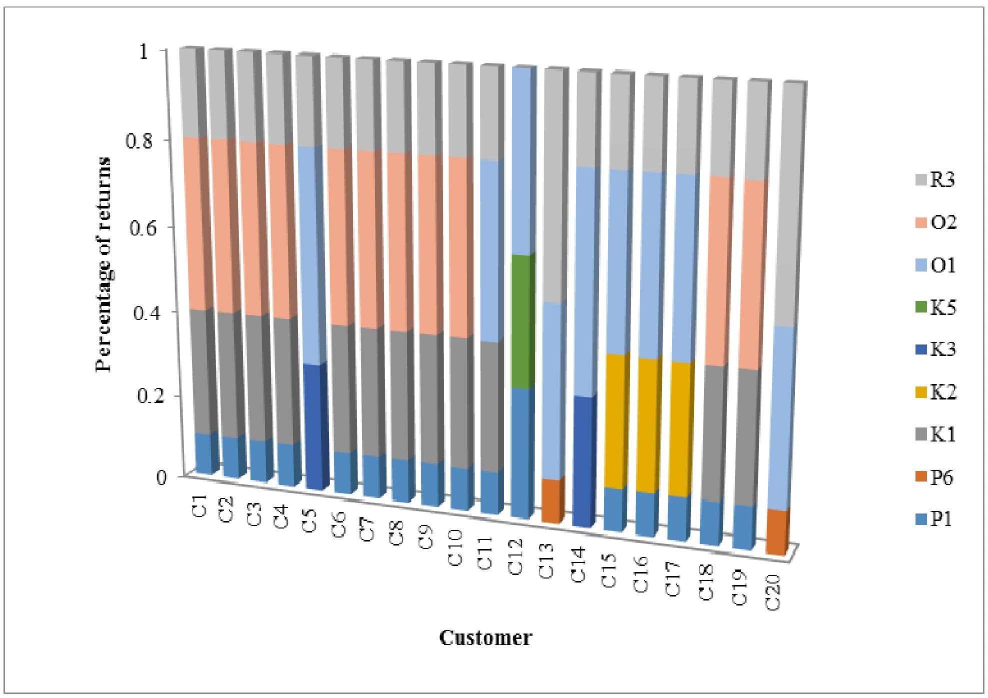

Fig. 5. Percentage of customers' returns for product 2

Conclusions and recommendations:

With regard to the development of reverse logistics network and customers' increasing awareness about environmental factors, designing an efficient logistics network is very important; because the results of network design may take years to affect the organization. If the return products are not effectively managed, it might be likely that the product cost is higher than its production. In this study, a new model is presented for the problem of designing an integrated reverse logistics network with multiple-capacity facilities and more options of product recovery. In general, researches in the field of integrated logistics network design have not considered all options for product recovery. Many studies in the literature of logistic network design only considered the option of reproduction for product recovery and other options such as repair and recycling have not been considered. On the other hand, many existing studies on the logistic network design have assumed that there is only one capacity for the installation of existing facilities in the logistics network (such as production or re-repairing centers, distribution centers, collection centers, etc.) This assumption is not realistic, because generally different facilities with different capacities can be installed. The cost of installing these facilities increases as their capacity increases. In this study, a new model was presented for the problem of designing an integrated logistics network with multiple capacities and more options for product recovery. A mathematical model was developed to produce precise solutions and an accurate solution method was analyzed based on Benders' decomposition method to produce efficient solutions. The computational results show the efficiency of the proposed analysis method, because the answers of Benders' decomposition method and the direct method have equal quality in the multi-product mode. In the multi-product mode, Benders' method obtained the optimal solution after 17 iterations in 29 seconds, whereas the direct method obtained the optimal solution in 96 seconds. Thereupon, it can be said that the computational efficiency of Benders' decomposition method is higher than that of the direct method.

REFERENCES

1. Zhou, Y. & Wang, S. (2008) "Generic model of reverse logistics network design" *Journal of Transportation Systems Engineering and Information Technology,* Vol.8**,** pp**.** 71-78.
2. Lee, D.-H. & Dong, M. (2008) "A heuristic approach to logistics network design for end-of-lease computer products recovery" *Transportation Research Part E: Logistics and Transportation Review,* Vol.44**,** pp**.** 455-474.
3. Amiri, A. (2006). "Designing a distribution network in a supply chain system: Formulation and efficient solution procedure. "*European Journal of Operational Research*, Vol. 171, PP. 567–576.
4. Jayaraman, V., Guide Jr, V., Srivastava R. (1999). "A closed-loop logistics model for manufacturing. "*Operational Research Society*, Vol. 50, PP. 497-508.
5. Fleischmann, M., Beullens, P., Bloemhof-Ruwaard, J. M. & Wassenhove, L. N. (2001) "The impact of product recovery on logistics network design" *Production and Operations Management,* Vol.10**,** pp**.** 156-173.
6. Realff, M. J., Ammons, J. C. & Newton, D. J. (2004) "Robust reverse production system design for carpet recycling" *IIE Transactions,* Vol.36**,** pp**.** 767-776.
7. Alumur, S. A., Nickel, S., Saldanha-Da-Gama, F. & Verter, V. (2012) "Multi-period reverse logistics network design" *European Journal of Operational Research,* Vol.220, pp. 67-78.
8. Benders, Jacques F. (1962). "Partitioning procedures for solving mixed-variables programming problems. Numerische mathematik. "4(1), 238-252.