

Genetic algorithm for supplier selection and optimal purchase pricing problem in a two-echelon supply chain given the Demand Curve

Hamed Hassanvand¹- Jamal Farahmand Harami²-Mohammad Saleh Sohrabi³

1-Master degree in industrial engineering Raja University - Department of Financial Engineering, Qazvin, Iran

2-Payame Noor University – Asaloyeh– Iran

3-Phd - industrial engineering Department - Razi University – Kermanshah – Iran

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ABSTRACT

This paper investigated the operational issues of a two-echelon supply chain with multiple suppliers and multiple retailers under linear demand function/curve and retailer fixed markup pricing policy for each buyer/retailer. The operational decisions of the model are purchase price and quantity which determine the channel cost of the supply chain. To find out the optimal purchase price and quantity, a mathematical model is formulated for each buyer from several suppliers. Also, for using the genetic algorithm method to solve the formulated problem, the model was modified and then the best value of the parameters was derived. This research anticipates completing optimal parameter combination in genetic algorithm using Taguchi method. Finally, a numerical example is given to illustrate the model.

KEYWORDS: Genetic algorithm; Two-echelon supply chain; Pricing; Linear demand function/curve; Taguchi method

1. INTRODUCTION

A two echelon supply chain model, involved some suppliers and buyers that have relationship with each other. Each buyer has demands that can be explained with demand function/curve or constant value that is satisfied by suppliers.

From 20th century until now, supplier selection problem has been an important problem in business management literature that is called supply chain management.

Today, some problems are discussed in supply chain management such as: (cf. Burton [1], Degraeve et al. [2], Dickson [3], Jayaraman et al. [4], Weber et al. [7–9]). supply base reduction, Internet and e-commerce and so forth, strategic buyer–supplier relationship, cross functional purchasing program, core-competence outsourcing strategy.

Regarding the competitive atmosphere in firms and markets, and as a result of increasing technology improvements, increasing productivity and maximization of profit in businesses are very important in these days. For this purpose best supplier must be selected with firms for optimization of their profit. Supplier selection in supply chains is often leading to an operational research model that models demand and supply in business environment. The goal for this is to determine the number and location of suppliers, and products price and purchase quantity to maximization of the profit or minimization costs of entire chain total cost.

Buffa and Jackson [8] presented a goal programming for a scheduled purchase problem from mix of vendors over a defined planning horizon. Bender et al. [9] studied, a mixed integer optimization model to minimize the sum of purchasing, transportation and inventory costs in a multiple period planning horizon and, constraints of vendor capacity and policy, but they did not develop their model.

The supplier selection and purchase problem are governed by three main decisions: how much should be ordered from the selected supplier, what is optimum price for purchased product and which supplier should be selected. The objective of our paper is to determine the best suppliers for product and best price for them and optimizing profit of retailers.

Furthermore, the past studies in business management on supply chain coordination consider the price of products determined from firms and business environment. In this study we suppose that the supply chain had a perfect market structure due to competition or unlimited substitution. But Under a retailer fixed markup policy (RFM), the retailer agrees to a fixed markup before any operational transactions occur. After the wholesale price is announced by the manufacturer, the retailer has no control over the retail price (since he has committed to a markup), but he/she does choose his/her order quantity. Although in this study market and market demand curve determine the wholesale price of the supplier. [10]

In some researches, (Kotler, [11]; Ray, Gerchak & Jewkes, [12]) demands and prices are determined by using known demand functions and maximization of supply chain profit. Nachiappan and Jawahar [13] used genetic

algorithm (GA) to find out the prices and demands under VMI assumptions. Likewise, in our study pricing problem is a nonlinear integer programming (NIP) and this problem is an NP problem. According to the literature (Costa & Oliveria, [14]; Exler, et al [15]; Schlüter, Egea&Banga [16]), genetic algorithm (GA), particle swarm optimization (PSO), ant colony optimization (ACO) and tabu search (TS) have been adopted to resolve the NIP problem.

In this paper, we introduce two-echelon supply chain with multiple suppliers and multiple retailers under linear demand function/curve and retailer fixed markup pricing policy for each buyer/retailer with supplier selection problem and formulate the problem as a nonlinear integer programming model. Then the model is solved with a Genetic Algorithm.

The paper is organized as follows: in Section 2, we describe assumptions, notations and formulate the optimization problem. In Section 3, we present GA method to solve the problem, and derive the best level of the GA parameters using Taguchi experimental design. Section 4 presents a numerical example to apply our methodology and discusses the output. After that, in section 5 we make a comparison between our propose method and Lingo software. Finally, Section 6 concludes the paper and suggests some possible future research.

2. Model formulation for the inventory routing and pricing problem

2.1. Assumptions and notations

Assumptions

We used some assumptions in this paper about pricing and supply chain as follows:

1. A single product is considered and distributed to retailers.
2. The pricing cannot be zero.
3. Planning horizon is annual.
4. The demand function is linear (The most popular demand function in the literature [12, 13]).
5. The retailers cannot hold inventory for next period and must sell all of ordered product.
6. The retailer fixed markup is mandated by government or economic equilibrium

Notation:

D_j	Demand of retailer 'j'
a_j	Positive constant
b_j^+	Upper bound of purchase price of retailer 'j'
b_j^-	Lower bound of purchase price of retailer 'j'
p_j	Retailer price of retailer 'j' (a decision variable)
x_{ij}	Quantity of product that retailer 'j' purchase from supplier 'i' (a decision variable)
X	Parents chromosome for quantity of product
P	Parents chromosome for price of product
O	Offspring of X matrix
L	Offspring of vector P
R_j	Total revenue of retailer 'j' in each period
θ	Retailer fixed markup under RFM ($0 \leq \theta \leq 1$)
h_{ij}	inventory cost include transportation and holding cost per unit of product between retailer 'j' and supplier 'i'
C	Total inventory cost
K	Total purchase cost of retailers
M	Large positive number (penalty coefficient)
η	Signal to noise ratio
S_i	Capacity of supplier 'i'
ψ_{ij}	Crossover mask for uniform crossover of X ($\psi_{ij} \in \{0, 1\}$)
λ	A random variable for linear combination ($0 \leq \lambda \leq 1$)
ζ	Penalty function
ϕ	Fitness of objective function
ϕ^*	Compensate fitness

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Before the model that we present for this problem, is formulated, firstly we discuss the revenue and cost function in section 2.2 and 2.3 based on above assumptions.

2.2. Revenue

Demand function defines the relationship between product price and demand quantity. The planning horizon usually is 1 year or half year. The demand function for retailer j is given by $D_j = a_j(b_j^+ - (1+\theta)p_j)$, a linear function typically used in similar studies, eg [12,13]. Then according to assumption (6), total revenue of product j is

$$R_j = (1 + \theta)p_j D_j = (1 + \theta)^2 a_j b_j^+ p_j - (1 + \theta)^2 a_j p_j^2.$$

2.3. Supply chain cost

The supply chain cost C includes the transportation and holding costs as $C = \sum_i \sum_j h_{ij} x_{ij}$, and purchase cost is $K = \sum_j \sum_i p_j x_{ij}$.

2.4. Modeling

The model for the supplier selection and pricing in hand to maximize the total profit can be written as:

$$\text{Max} \sum_j ((1 + \theta) a_j b_j^+ p_j - (1 + \theta)^2 a_j p_j^2) - \sum_i \sum_j h_{ij} x_{ij} - \sum_j p_j \sum_i x_{ij}$$

Subject to

$$\sum_i x_{ij} = a_j (b_j^+ - (1 + \theta) p_j) \quad \forall j \quad (1)$$

$$\sum_j x_{ij} \leq S_i \quad \forall i \quad (2)$$

$$b_j^- \leq p_j \leq b_j^+ \quad \forall j \quad (3)$$

$$p_j \in N \quad \forall j \quad (4)$$

$$x_{ij} \in N \quad \forall j \quad (5)$$

$$x_{ij} \geq 0 \quad \forall ij \quad (6).$$

Here, N is the set of integer numbers including zero. According to assumption (5), constraint (1) indicates the purchase for retailer j equal to demand of retailer j . Constraint (2) indicates that the sum of i -th supplier sell must be less than or equal to supplier capacity s_j . The constraint (3) indicates the boundary of price (p_j) and integer constraint. Better performance of algorithm and making wide sprite space of all feasible solutions (the set of solutions among which the desired solution resides) is called search space we considered p_j as an integer number. The constraint (5) and (6) indicate integer constraint and non-negative for x_{ij} . This is a quadratic optimization problem.

3. The proposed study methodology

Many optimization problems such as the problem of this study are very difficult and complex to be modeled or solved by conventional methods, for example simplex or dynamic programming. Therefore, metaheuristics methods such as naturally inspired algorithms were developed to solve them [19]. In that approaches, GA is a powerful method for solving many optimization problems. GA Originally was proposed by Holland in 1970, GA belongs to stochastic searching technique classes based on Darwin theory that is called natural selection. The procedure of a proposed GA is described in figure 1

3.1. Coding and decoding

In GA literature, solution is called chromosome. Also, in this paper we have two kinds of chromosomes: transportation matrix and price vector that are defined as follow:

$$P = (p_1, p_2, \dots, p_j, \dots, p_n)$$

$$X = \begin{pmatrix} x_{11} & \dots & x_{1n} \\ \vdots & x_{ij} & \vdots \\ x_{m1} & \dots & x_{mn} \end{pmatrix}$$

3.2. Crossover

The purpose of crossover operator is to generate two suitable offspring individuals from two existing solutions (parents). In addition, to select the parents from the mating pool of existing chromosome, we use roulette wheel selection method. This approach uses cumulative distribution function (CDF) for selecting parents, if fitness value of a chromosome is higher than another probability of rise up and they can participate on next generation. The GA in this study employs a different evolution mechanism from conventional ones. In this paper we compensate fitness function for best efficiency of roulette wheel selection. There are several crossover operators proposed in the literature. For crossover operator in this paper we use two kinds of crossover approaches.

For transportation quantity matrix, we use uniform crossover, and multiplication of matrices is element – by – element multiplication.

Another crossover operator is uniform crossover and is totally different from the others. In this approach we have a crossover mask that is a random binary matrix with the same size as the chromosomes. For applying this operator when, we have 1 in crossover mask gene is copied from the first parent. In other hand, when we have 0 in crossover mask gene is copied from the second one. As a result of this operator, new offspring contains a mixture of genes based on crossover mask density.

$$O_1 = \psi X_1 + (1 - \psi) X_2$$

$$O_2 = (\mathbf{1} - \psi)X_1 + \psi X_2$$

Where $\mathbf{1}$ is unique matrix (fill with one) with same size as crossover mask.

And for price vector we use linear combination for crossover operation as follows:

$$L_1 = \lambda P_1 + (1 - \lambda)P_2$$

$$L_2 = (1 - \lambda)P_1 + \lambda P_2$$

X_1 , X_2 , P_1 and P_2 are candidate chromosomes for crossover that are selected by roulette wheel selection technique, O_1 , O_2 , L_1 and L_2 are offspring after crossover.

Mutation

For diversifying the feasible region and prevent the algorithm obstacle in a local optimum in GA, we use mutation approach. In nature mutation, recovers the lost genetic materials. Likewise that, in GA algorithms mutation randomly changes the value of some gens of a chromosome for diversifying in the population. Crossover operates in current population to find parents to make new individuals, but mutation explores the whole feasible region. [17]

For mutation, we use one point mutate for both transportation quantity matrix and price vector. In this operator we chose a random position and change it randomly.

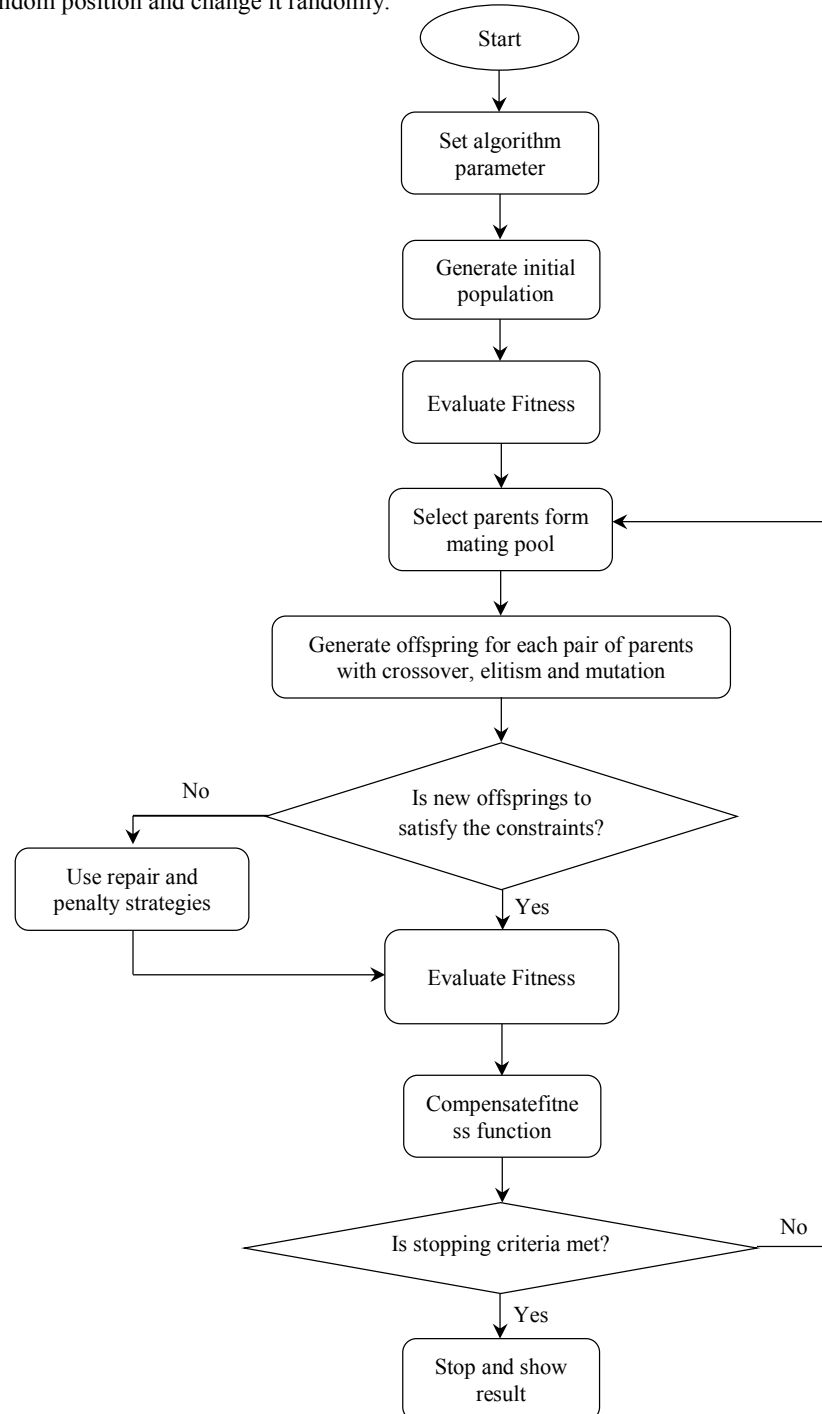


Fig 1 The flow chart for proposed genetic algorithm

Elitist strategy:

When we use other operators (cross over and mutation) the best fitness individual might change in next generation and fitness of problem decreases. Elitist strategy forces GA to keep a copy of a more fit point of the solutions in population. This approach can help GA for better performance and faster convergence.[18-19] In this study, we use this strategy with probability of $1-(P_m+P_c)$ and we keep $popsizex(1-(P_m+P_c))$ of more fit point of the population.

Feasible solution:

One of the problems concerned in modeling of an optimization problem with GA is how to deal with the constraints. There are reject, modifying genetic operators, penalty strategy and repairing. First, in reject strategy, in each generation after new individual offspring is made by genetic operators, if the new solution is infeasible this strategy immediately excludes this offspring. Second, strategy of modifying genetic operators, specialize the genetic operators to generate feasible solution in each generation. Third, the penalty strategy, uses a suitable penalty function in order to penalize infeasible individual decrease their fitness and chance of selection of this penalized solutions decrease, hoping this new offspring evolve in the next generations during the algorithm runs. Finally, the repairing strategy, transforms the infeasible solution to a feasible one throughout mathematical processes that are called repairing processes.

Repairing strategy for demand constraints are defined as follows.

$$x'_{ij} = \frac{x_{ij}}{\sum_j x_{ij}}$$

$$x^*_{ij} = x'_{ij} \times D_j$$

x^*_{ij} is quantity of product that retailer j purchase, from supplier i after repairing the solution for satisfying demand constraints.

The penalty function that impels the solutions to satisfy supply constraint is formulated as follows.

$$\xi = \begin{cases} \sum_i x_{ij} - S_i & \text{if } \sum_i x_{ij} > S_i \\ 0 & \text{otherwise} \end{cases}$$

Fitness evaluation:

Incorporating the objective function and the penalty function, the target function for model can be defined as

$$\varphi = \sum_i (a_i b_i p_i - p_i^2 + \theta a_i b_i p_i - \theta p_i^2) - \sum_i \sum_j h_{ij} x_{ij} - \sum_i p_i \sum_j x_{ij} - M \times \xi$$

The large positive number M forces the solution to meet the supply constraint before maximizing the fitness function. Then we compensate fitness for best effect for roulette wheel selection as follow formulate.

$$\varphi^* = \begin{cases} \frac{1}{|\varphi|} & \varphi \leq -1 \\ 0.1|\varphi| & -1 < \varphi \leq 0 \\ \varphi & \text{other wise} \end{cases}$$

φ^* is fitness function after compensate process

Stop criteria:

The algorithm stops when the specified number of generations is achieved.

Parameter setting:

In 50's, Dr. Genuchi Taguchi developed a method for optimization of complex systems that called Taguchi methods. Base of Taguchi method is analyzing data for simplifying these systems. This method calculating the best level of each parameter by peculiarity of orthogonal arrays and this operation reduces the number of experiments to find the best one. [20]

To practice the experimental design for selecting the most suitable level (or scheme) or combination of control factors, we select one problem instance corresponding to different levels of the factors. In this method we have 4 factors and 3 levels for these factors; we show factors and levels in Table 1. We select orthogonal array $L_9 (3^4)$ design for this problem and we run 9 scenarios 5 times and collect the fitness and computation time (CPU time). For best efficiency of algorithm and robust answers we select parameters level with main effect plot for mean and signal-to-noise ratio (S/N) of objective function and CPU time.

S/N ratio for factors defined as:

$$\eta = -10 \log\left(\frac{\sum (1/y_{lk}^2)}{n}\right) \quad \text{For larger is better}$$

$$\eta = -10 \log\left(\frac{\sum y_{lk}^2}{n}\right) \quad \text{For smaller is better}$$

Where y_{lk} is the performance characteristic of observation k at trial l averaged after 5 replications, and n is the number of factors.

4. Empirical study:

The main effect plot for mean and S/N ratio of objective function and CPU time are shown in Fig. 2 to Fig. 5. Regarding objective function that is maximization for calculate S/N ratio we used “the larger is better equation” and for CPU time S/N ratio we used “the smaller is better equation”.

The retailers' parameters for problem are shown in Table 2 and inventory cost between suppliers and retailers are shown in Table 1. The values of the M were set 100, the percentage of the sell profit for retailers was set 0.1, maximum supply for supplier A is 1000 and supplier B is 1500, respectively. Also, Fig 2 shows the GA convergence plot for problem.

Table 1. Parameters for problem			
Retailer Name	a	b_j^+	b_j^-
I	20	100	50
II	30	120	50
III	10	140	50

Table2. Inventory cost between suppliers and retailers				
Retailer Name		I	II	III
Supplier Name	A	5	7	4
	B	3	6	2

The experiments were conducted on a PC with an Intel ® Core™ 2 Duo E7500 @ 2.93GH CPU, 2 GB of RAM and Windows 7 Ultimate and implemented in MATLAB 7.10.0.499 (R2010a) and taguchi DOE solved with Minitab 16 .

For this problem the optimum solution after solving the model are shown below:

Maximum of objective function: 8200

Average of CUP time: 47.6587 Sec

Optimum price and inventory quantity are shown in Table.4. With this analyses the best level of factors is:

Percentage of crossover (Pc) = 0.8, Percentage of Mutation (Pm) =0.1, Population size= 50, Number of iterations =5000

Table 3. Factors and Levels .				
Level	Crossover rate	Mutation rate	Population Size	Number of iterations
I	0.6	0.05	30	300
II	0.7	0.1	50	1000
III	0.8	0.15	100	5000

Table4. Optimum price and inventory quantity

Retailer Name		I	II	III
Supplier Name	A	0	400	0
	B	600	350	550
	Price	70	95	85

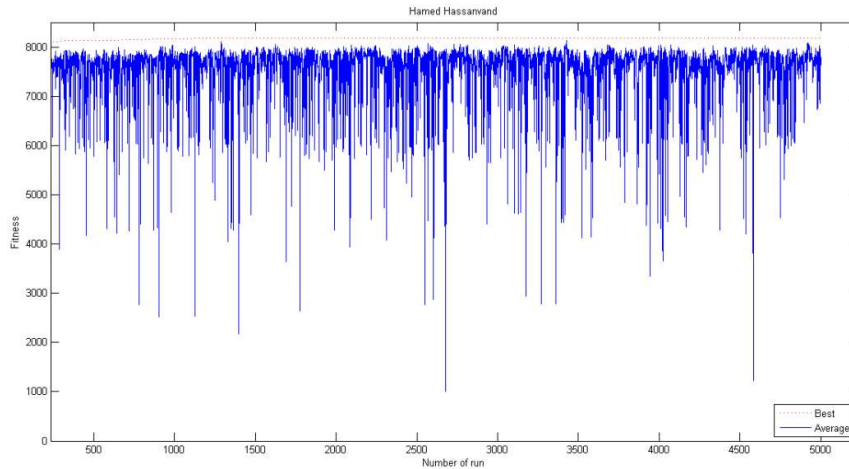


Fig2. The GA convergence plot

5. Performance evaluation

For evaluating performance of the proposed algorithm, we compare 3 small problem and one bigger problem that is solved with LINGO11 and our proposed algorithm. Also, the LINGO11 can solve small size problems but for high dimension time of solving rise up dramatically and for some high dimension problems Lingo stops on a local optimum. Furthermore, the result of this comparison is shown in table 5.

Table 5. Comparison between proposed GA and LINGO

Number of retailer	Number of supplier	GA	Lingo
6	4	359957.8	-76694.75*
2	2	135685.8	137237.5
1	3	128948.16	131358.4
3	2	142506.4	144365.0

* Lingo stopped in a local optimum.

6. Conclusions

In this study, we modeled a two echelon supply chain with linear demand function and found out the optimal solution using genetic algorithm. To achieve the best efficiency of the GA, we set the algorithm parameters using Taguchi method which resulted in a robust solution and better performance for GA. In addition, regarding the result of comparison between GA solutions and LINGO in table5 we found out that the proposed method has a high performance. For future work, we advise using other metaheuristic methods such as harmony search algorithm, scatter search, etc. can be helpful. Another extension can be inserting forward and future contracts in the model instead of spot contracts utilized in this study.

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