# A mathematical programming model for location-routing problem and solve it using an efficient meta-heuristic method 

Hooman Mohammadpour ${ }^{1}$ and Alireza Alinezhad ${ }^{2}$<br>${ }^{1}$ Msc. Student, Faculty of Industrial and Mechanical Engineering, Qazvin branch, Islamic Azad University, Qazvin, Iran<br>${ }^{2}$ Assistant Professor, Faculty of Industrial and Mechanical Engineering, Qazvin branch, Islamic Azad University, Qazvin, Iran

Received: July 24, 2015
Accepted: September 31, 2015


#### Abstract

Locational decisions, routing and allocation in supply chain management philosophy is undoubtedly one of the most important issues that affect the frequency of the supply chain cost reduction and customer satisfaction. Selection of suitable sites for construction of the facility from a specific location and design of distribution network in the levels of the supply chain is very important, and life. In recent years the issue of location and allocation decisions in the supply chain has been stressed. Attention to both categories of vehicle routing and placement within the urban distribution depots optimal solution to both problems can provide. This paper, a new mathematical model location routing consider a facility with multiple product types to minimize total costs related to facility location and transportation costs and maximizing population coverage of the demand has been presented. Because in the real world as well as some of the model parameters are uncertain in nature, in terms of the fuzzy model is developed. Also, genetic algorithms NSGA-II and NRGA was used to solve the proposed model and the results of each algorithm to solve the model through comparison and analysis are presented.


KEYWORDS: Location, Routing, Mathematical Programming, Objective meta-heuristic algorithm, fuzzy

## 1- INTRODUCTION

Supply chain is a network of suppliers, manufacturers, distributors, and customers that are linked to each other based on the flow of information and materials in order to provide good quality products and services to end customers in the most effective and economic situation. In today's competitive markets, the success of a supply chain relies on its scientific management. So far, researchers have studied different aspects of the supply chains. Planning for the distribution of goods and services in cities is one of the most applied issues in the supply chain.

Physical distribution is one of the key activities of manufacturing companies, because more than $20 \%$ of the finished price of products, on average, is spent on physical distribution of them [1]. Customers expect today's distribution systems to meet their needs at a specific time and with the minimum possible cost. Planning related to site selection for distribution centers and routing the service-providing channels is of particular importance, as the future success of medium and even small units depends on this decision. Planning should be done in a way that not only minimizes the costs imposed on the system but also address increased quality of service delivery to customers, which has always been one of the main objectives of public and private services. According to previous studies conducted on service-delivering systems, it can be found that the location of distributors (warehouses) and the way of routing for providing services have a direct impact on the costs of supply chain and especially costs of goods transportation. These locations should be selected in a way that shorten the required movements from the supplier to the warehouse and from the warehouse to customers. On the other hand, selection of suitable sites for the establishment of distribution centers in cities can be a great help to the citizens for quick and easy access to the goods and provide customer satisfaction.

According to the views expressed about service providing systems, it can be concluded that location of warehouses and selection of the routes for good or service delivery have a direct impact on the level of services provided and play an important role in the performance of service provider systems. Hence, site selection and touting will be simultaneously taken into account in the present study and it will be attempted to propose an efficient post-innovative algorithm for taking better decision in this sector.

[^0]
## 2- REVIEW OF RESEARCH LITERATURE

The issue of site selection of maximum coverage with the aim minimizing the costs of establishment of service provider centers was firstly raised in 1974. Afterward, considerable improvements have been made to this topic based on various studies conducted on this issue. The major objective of site selection-routing is minimization of total cost, including costs of warehousing and transportation. In this regard, a limited number of studies have chosen a different objective function or functions with multiple objectives. Averbakh and Berman [2], Aksen, and Altinkemer [3], and Alumur and Kara [4] are some of the most eminent researchers on this issue. Minimization of harmful material handling costs, fixed costs of facilities, and risks of exposure to discovery of harmful substances was the objective function in the study conducted by Alumur and Kara [4]. Ghiani and Laporte [5] studied a structure in which transport vehicles start from the warehouse, move among different vertices, deliver the products to customers, and return to the same warehouse after the inventory is finished. Instead of moving to the vertices, transportation vehicles would pass through the edges between the vertices. Ahn, and Ramakrishna [6] proposed the application of genetic algorithms for solving the issue of shortest path. Computer simulation showed that the proposed algorithm acts better than other conventional algorithms in terms of the convergence. Tai-Hsi et al. [7] studied the site selection of warehouses and proposed an innovative model in which the number of available trucks have been considered without limitation and only the general costs of transportation have been taken into account. Barahona, and Jensen [8] proposed a model for determining the place of warehouses customer allocation. In this model, both the costs of warehousing and limitations on the level of service providing were considered. This model that was related to computer spare parts distribution network expected to meet at least $95 \%$ of demands and orders in less than 2 hours.

Barreto et al. [9] used an innovative method based on customer classification for solving the issue of locationrouting. He used some hierarchical and non-hierarchical methods for classification of customers. Yu et al. [10] applied an innovative annealing stimulated algorithm to solve the issue of location-routing and used three neighborhood structures for improving the performance of this algorithm. He claimed that the use of this structure improves the performance of annealing stimulated algorithm in solving the issue of location-routing. Yao et al. [11] proposed a non-linear mixed model and combined the issues of site selection, allocation, and inventory in which. Ahmadi and Azad [12] proposed an integrated model for the design of distribution networks in the possible situations and considered the issue of location, inventory, and routing in which. Ahmadi and Seddighi [13] again the issues of location-inventory-routing by making some changes in the assumptions.

## 3- Statement of the problem:

In the present paper, a three-level supply chain system was studied. The first level includes several factories or suppliers that each of them offers multiple different products. The second level involves $I$ warehouses with specified capacities that are used for temporary storage of products. Therefore, storage cost is not included in these warehouses. There are $V$ vehicles in each of these active warehouses that deliver products to customers. Each of these warehouses receives the inventory of products and distribute them in order to cover part of entire demand. The third level includes the end customers and customer demand has been considered as fuzzy. In order to allocate and distribute the products from warehouses to customers, transport vehicles start from the warehouse, move among different vertices, deliver the products to customers, and return to the same warehouse after the inventory is finished. The aim of this paper is to study maximizing the customer coverage while minimizing the costs imposed on the supply chain such as cost of construction of warehouses and transport costs from suppliers to warehouses and from warehouses to customers. In order to achieve these objectives, some decisions should be made about where to build the warehouses and the routes of transport vehicles. Mathematically, this decision-making requires 4 binary variables and one contributing variables as follows:

$H_{i, v}^{p} \quad\{$
1 If product $p$ is transported to the warehouse $i$ by the vehicle $v$.
0 Otherwise
$M_{i v} \quad\{\quad 1$ The contributing variable for to prevent repetitive motion paths.
0 Otherwise

Other assumptions considered in the given supply chain are as follows:

- Location of establishment of warehouses are some predetermined points with fixed costs of establishment.
- Demand of each customer is considered a triangular fuzzy.
- The distance between suppliers, warehouses and customers is fixed.
- Capacity of vehicles is specified based on the type of product.


## 3-1- Material \& methods

For modeling the above mentioned issues, the following indices and parameters were firstly defined:

## Indices:

I: Collection of potential warehouses ( $i=1, \ldots, m$ )
$J$ : Collection of customers ( $) j=1, \ldots, n$ )
$V$ : Collection of transportation vehicles $(\nu v=1, \ldots, p)$
$P$ : Collection of products ( $p=1, \ldots, q$ )
$N_{i}$ : Collection of possible levels for establishment of a warehouse at the node $i$

## Parameters:

CAF ${ }^{2 \pi}$ The capacity of the warehouse $i$ which has been established at the level $n$ for the product $p$
$Q_{v}^{F}$ The capacity of the vehicle $v$ for the product $p$
$\tilde{d}_{j}^{p}$ The demand of customer $j$ for the product $p$ (triangular fuzzy number)
$\operatorname{CoST}_{\mathrm{i}}{ }^{2}$ The cost for building a warehouse at the level $n$ in the node $i$
$B$ The budget available for establishment of warehouses
prod " The production rate of product $p$
$d_{i, j}$ The distance between node $i$ and node $j$
$d_{i p}$ The distance between node $i$ and the place of production of product $p$
$c_{v}$ The operational cost of the vehicle $v$ per unit
The mathematical expression of the objective functions and restrictions are as follows. It should be noted that decisions on the location of warehouses are taken at the macro and strategic level, while decision-making about the way of products transportation is tactical. Due to the different nature of these two types of decision-making, the results of these decisions will be assessed separately in the objective functions in order to achieve a higher level of freedom of action for making decisions.

$$
\begin{align*}
& \max O F_{2}=\sum_{i \in l} \sum_{j \in j} \sum_{p \in F^{i}} Z_{i, j}^{p} \times d_{j}^{p}  \tag{2}\\
& \sum_{i \in(2 \cup \mathcal{D})} \sum_{v \in V} X_{i / j v}^{p} \leq 1  \tag{3}\\
& \sum_{i \in(M)} x_{i, j v}^{p}-\sum_{i \in(\lambda j)} x_{j i_{i v}}^{p}=0  \tag{4}\\
& \sum_{i=\mathbb{R}} \sum_{j \in(M /)} X_{i, j v}^{p} \leq 1  \tag{5}\\
& \forall j \in J, p \in P \\
& \forall j \in I \cup J, v \in V, p \in P \\
& \forall v \in V, p \in P
\end{align*}
$$

$$
\begin{align*}
& \sum_{v=Y} H_{i, V}^{p} \times q_{v}^{D} \leq C A F_{i}^{n D} \times Y_{i}^{n} \quad \forall i \in I, p \in P, n \in N_{i}  \tag{6}\\
& \sum_{i \in(N) j} \sum_{j=j} x_{i / j v}^{p} \times d_{j}^{p} \leq q^{F}  \tag{7}\\
& \sum_{j=j} Z_{i, j}^{p} \times \tilde{d}_{j}^{p} \leq \sum_{x=F} H_{i_{i} p}^{p} \times Q_{x}^{F}  \tag{8}\\
& \sum_{u \leq \Gamma \nu \eta} X_{i, k, v}^{p}+\sum_{u \leq T M /} X_{w, j, v}^{p}-Z_{i, j}^{p} \leq 1 \quad \forall i \in I, j \in J, v \in V, p \in P  \tag{9}\\
& \begin{array}{l}
\sum_{n \in N_{i}} Y_{i}^{n} \leq 1 \\
\sum_{i=1} Y_{i}^{n} \operatorname{COST}_{i}^{n} \leq B
\end{array}  \tag{10}\\
& \begin{array}{c}
\sum_{n \in N_{i}} Y_{i}^{n} \leq 1 \\
\sum_{\mathrm{m} \in \mathbb{N}_{1}} \sum_{i \in 1} Y_{i}^{n} \operatorname{COST}_{i}^{n} \leq B
\end{array}  \tag{11}\\
& \sum_{i \in I}^{=} \sum_{v i r}^{*} H_{i, v}^{p} \times Q_{v}^{p} \leq \operatorname{prod}^{*} \quad \forall i \in I, p \in P  \tag{12}\\
& M_{i_{i} v}^{p}-M_{j v}^{p}+N M_{i_{j v}}^{p} \leq N-1  \tag{13}\\
& X_{i p v}^{p} \in\{0,1\}  \tag{14}\\
& Y_{1}^{n} \in\{0,1\}  \tag{15}\\
& Z_{i, j}^{p} \in\{0,1\} \\
& H_{i, p}^{p} \in\{0,1\}  \tag{17}\\
& M_{i v}^{p} \geq 0 \\
& \forall i \in l \\
& \forall i, j \in J, v \in V, p \in P \\
& \forall i \in(I \cup J), j \in(I \cup J), \quad p \in P, v \in V \\
& \forall i \in I, n \in M_{i} \\
& \forall i \in I, j \in J, p \in P  \tag{16}\\
& \forall i \in I, v \in V, p \in P \\
& \forall i \in J, v \in V, p \in P \tag{18}
\end{align*}
$$

The first objective function includes three parts. The first part aims to minimize the total cost of establishment of warehouses and the second and third parts try to minimize the transport costs from supplier to warehouse and from warehouse to customers. The second objective function aims to maximize the total population coverage. The limitations collection 3 warrants that each customer receives service for any product. The limitations collection 4 ensures that if a vehicle enters a node, it certainly exists it. The limitations collection 5 warrants that a vehicle starts its movement from one warehouse. Given the limitations collection 4 that establishes the balance in the entry and exit in the nodes and the limitations collection 5 , it can be concluded that any vehicle starts its movement from a warehouse and finishes at the same warehouse. The limitations collection 6 ensures that the entry to the warehouse does not exceed the warehouse capacity. The limitations collection 7 warrants that the input of any product to any vehicle is less than or equal to the capacity of that vehicle. The limitations collection 8 controls the amount of any product that is entered or existed from a warehouse. The limitations collection 9 establishes the relationship between allocation and routing in the model: customer $j$ is allocated to the warehouse $i$ if the vehicle $v$ that passes through the node related to customer $j$ had started its movement from warehouse $i$. The limitations collection 10 warrants that each warehouse can be established at a single capacity level. The limitations collection 11 controls the total budget. The limitations collection 12 ensures that the amount product $p$ carried to warehouses does not exceed its production. The limitations collection 13 prevents the establishment of Duplicate route in the route of vehicle $v$ that carries product $p$. The limitations collection 14 to 18 control the values of variables. Since location and routing issues themselves are of NP-Hard issues [14], the location-routing issues is considered a NP-Hard issue. Hence, post-innovative algorithms should be used for solving such issues.

## 4- Conversion of a fuzzy model into a corresponding deterministic model:

To solve the proposed model, the proposed fuzzy model should be first changed into a deterministic model. The method for changing the fuzzy location-routing issue to a deterministic model is as follows:
According to Lemma [15], if $\bar{m}$ and $\bar{n}$ are two fuzzy values with continuous membership function, at a confidence level of $\eta \in[0,1]$ we will have:

$$
\begin{equation*}
\operatorname{Pos}\{\bar{m} \geq \bar{n}\} \geq m_{\eta}^{R} \geq n_{\eta}^{L} \tag{19}
\end{equation*}
$$

$\left[m_{\eta}^{L}, m_{\eta}^{R}\right]$ and $\left[n_{\eta}^{L}, n_{\eta}^{R}\right]$, respectively, are the left and right end points of $\bar{m}$ and $\bar{n}$ at the confidence level of $\eta$ and $\operatorname{Pos}\{\bar{m} \geq \bar{n}\}$ is the degree of possibility that shows $\bar{m}$ is greater than or equal to $\bar{n}$.
Theorem: It is assumed that the fuzzy variable of $\tilde{d}_{j}$ is specified by three levels of left, right, and middle points $\left(\alpha_{j}, \gamma_{j}, \beta_{j}\right)$, then we will have:

$$
\begin{align*}
& \operatorname{Pos}\left\{\sum \sum x_{i j v}^{n} \tilde{d}_{j} \leq Q_{v}\right\} \geq \delta  \tag{20}\\
& (1-\delta) \sum \alpha_{i} \sum X_{i j k}^{l}+\delta \sum \gamma_{i} \sum X_{i j k}^{l} \leq Q_{l}
\end{align*}
$$

Proving: For Equation 7, we have:

$$
\begin{align*}
& \sum_{i \in(I \cup,)} \sum_{j \in J} X_{i, j, v}^{p} \times \tilde{d}_{j}^{p} \leq Q_{v}^{p}  \tag{21}\\
& \sum_{i \in(I \cup J) j \in J} \sum_{j \in J} X_{i, j, v}^{p} \times \tilde{d}_{j}^{p}=\left(\sum_{i \in(I \cup J)} \sum_{j \in J} X_{i, j, v}^{p} \alpha_{j}^{p}, \sum_{i \in(I \cup)} \sum_{j \in J} X_{i, j, v}^{p} \gamma_{j}^{p}, \sum_{i \in(I \cup \mathcal{Y})} \sum_{j \in J} X_{i, j, v}^{p} \beta_{j}^{p}\right) \\
& \rightarrow=\sum_{i \in(I \cup N)} \sum_{j \in J} X_{i, j, v}^{p} \alpha_{j}^{p}+\delta \sum_{i \in(I \cup J)} \sum_{j \in J} X_{i, j, v}^{p}\left(\gamma_{j}^{p}-\alpha_{j}^{p}\right) \\
& \rightarrow=(1-\delta) \sum_{i \in(I \cup} \sum_{j \in J} X_{i, j, v}^{p} \alpha_{j}^{p}+\delta \sum_{i \in(I \cup)} \sum_{j \in J} X_{i, j, v}^{p} \gamma_{j}^{p} \\
& \text { final } \Rightarrow(1-\delta) \sum_{i \in(I \cup J) j \in J} \sum_{i, j, v}^{p} \alpha_{j}^{p}+\delta \sum_{i \in(I \cup J) j \in J} \sum_{i, j, v}^{p} \gamma_{j}^{p} \leq Q_{v}^{p}
\end{align*}
$$

Similarly, for Equation 8 we have:
$(1-\delta) \sum_{i \in I} Z_{i, j}^{p} \times \alpha_{i}^{p}+\delta \sum_{i \in I} Z_{i, j}^{p} \times \gamma_{i}^{p} \leq \sum_{v \in V} H_{i, v}^{p} \times Q_{v}^{p}$

## 5- Algorithm for solving multi-objective issues:

Over the past decade, population-based algorithms such as genetic algorithms have been widely used for optimization of multi-objective issues. The most important reason for this development is the ability of these algorithms in finding a set of Pareto solutions at only one time. According to Konak et al. [16], other conventional methods of optimization can reach such set of solutions after multiple sequential and separate run times.

Multi-Objective Genetic Algorithm was firstly proposed by Schaffer in 1985. Although VEGA ${ }^{1}$ algorithm had good results, the problem of excessive focus on dome of the Pareto solutions justified its inefficiency. Srinivas and Deb [17] developed NSGA $^{2}$ algorithm based on the proposal of Golberg [18]. NSGA is a conventional method for solving the problems with multiple objective functions based on genetic algorithm. This algorithm has weaknesses in selection of the dominant particles and computational complexities. Non-elitist approach and the need for determination of the sharing parameter by the user (Sharing parameter is used to maintain the diversity of answers).

[^1]Accordingly, Deb et al. [19] introduced a modified method named NSGA-II which has a better performance than NSGA.

## 5-1- NSGA-II algorithm:

This algorithm uses the total members the population dominated by particle M and the number of time particle M has been dominated by other particles. In addition, a quick non-recessive sorting approach is also used in this algorithm. In this method, children population ( Qt ) is created by parents' population (Pt). After integration of populations, non-recessive sorting method is used for classification of population members that are shown in rows $F_{1}, F_{2}, \ldots, F_{n}$. Members of row $F_{l}$ are those who have dominated the members of other rows and the members of the last row are those who have been dominated in competition with members of other rows. In order to generate a new population $\left(P_{t+1}\right)$, members of the first rows are placed in the population $P_{t+l}$. When the number of member of this new population reaches N , the process of placing the members of row in the new population is stopped. If there is a row that its members can increase the number of members of new population to more than N , members of that row should be sorted based on the congestion distance and the member with higher congestion distance should be put at the first priority for being placed in the new population. The main steps in the implementation of NSGA-II are as follows.

## 5-1-1- Answer structure (Chromosome)

In order to display the answers, a chromosome is shown as a p-dimensional structure. P shows the number of products. Each of the dimensions forms a matrix that the number of their rows and columns are varied. The first and the second columns show the vehicles for the transport of goods to the warehouse and vehicles for the transport of goods from warehouse to customers. The third column denotes the warehouse and the fourth column and other show the customers.
The proposed chromosome is formed by the following steps:
1- Suppose the first product.
2- The customers that are going to receive the first product are selected randomly.
3- Some break points are created among the customers and they are divided into some certain groups.
4- A vehicle and a warehouse are allocated to each group of customers.
As a result, the limitation that each customer receives its ordered product from at least one warehouse is observed. In addition, since the transport vehicles selected at each step are eliminated from the list of available vehicles, the limitation of selecting one vehicle for carrying each product is also observed.

5- A number of vehicles is allocate to each warehouse to carry the first product to that warehouse.
6- All the above steps are repeated for the next product.
7- Total products entered into each of the warehouses is calculated and the capacity level of that warehouse is determined. If the input products to the warehouse exceeds the highest capacity of the warehouse or if there is not enough budget for building a new warehouse at that capacity level, warehouses with high capacity are built as many as possible and the chromosome will be fined commensurate with the violation of warehouse capacity.

## 5-1-2- Chromosome evaluation and selection of parents:

Selection strategy in genetic algorithm determines the chromosomes which can transfer their traits to the next generation. In NSGA-II, Binary Tournament Selection strategy is used. To select each of parents in this strategy, some members of the population are randomly selected and the one with lower rank is selected. If the ranks are the same, selection would be based on congestion distance. To rank the answers available in the population, the following steps should be done.

## 5-1-3- Non-recessive ranking of chromosomes:

For non-recessive sorting, the population members which have never been dominated are identified and given the first rank. Then, non-recessive sorting is done for the rest of the member, regardless of the effect of members with the first rank on the population. The second rank is given to the members that have never been dominated in this step. Regardless of the effect of members with the first and second ranks on the population, non-recessive sorting is done again for the rest of the members and the members which have never been dominate in this step are given the third rank. This will be continued until the rank if all members is determined.

## 5-1-4- Crowding Distance:

Congestion distance is used to maintain the diversity of answers in the Pareto optimal set. In fact, elimination of some members from a collection is done in a way that there are regularly answers in any range. In order to calculate the congestion distance corresponding each point on a certain side, all points before and after the objective functions are selected to form a rectangular. Obviously, if the number of objective functions is more then 2 , these points form a cube. The less the congestion distance, the higher the density of answers. If the issue has more than
two objective functions, the points $i+1$ and $\mathrm{i}-1$ are not the same for all objective functions. The steps for calculating the congestion distance for the answers on the side F are as follows:

The number of answers on the side $F$ is calculated and named as $1(|F|=1)$. For any i-th member in this set, the initial value of congestion distance is considered zero $\left(d_{i}\right)$.

Answers are sorted based on each objective function.
In each objective function, a large distance of congestion is given to answers on the boundaries (the first and the last points) $\left(d_{I_{1}}=d_{I_{1}} m=\infty\right)$.To calculate this indicator for the rest of the answers, Equation 23 is used.

In this equation, $\mathbb{I}^{\mathrm{m}}$ denotes the answer $i$ in the list of answers sorted based on the objective function $m$. The numerator in the right side of the equation shows the difference of objective function $m$ for two adjacent answers. The denominator shows the difference between the minimum and maximum value of objective function $m$ in the population.

## 5-1-5- Intersection operator:

Intersection operator is the main strategy for generation of new chromosomes in the genetic algorithm. The important point in this regard is the use of a suitable operator that does not withdraw the answer from the justified state. In this study, two types of intersection operators were used. For doing the intersection by the first type of operator, the following steps should be done:

- Two chromosomes are selected from the population considering the selection strategy.
- Generate a random $r$ number between 1 and $p . P$ and $r$ denote number of products and cross point, respectively.
- Make pairs of the selected chromosomes.
- The bits from 1 to cross point existing in the chromosomes of the first parent are directly copied to genes of the first child.
- The bits from cross point +1 to $N$ existing in the chromosomes of the second parent, according to their arrangement in the second parent, are transferred to the first child.
- The above steps are repeated for generation of other child chromosomes.

| Father 1 |  |  |
| :---: | :---: | :---: |
| $\mathrm{P}=1$ | $\mathrm{P}=2$ | $\mathrm{P}=3$ |
| Movement of <br> product 1 | Movement of <br> product 2 | Movement of <br> product 3 |



| Movement of |  |  |
| :---: | :---: | :---: |
| product 1 | $\begin{array}{c}\text { Movement of } \\ \text { product 2 }\end{array}$ | $\begin{array}{c}\text { Movement of } \\ \text { product 3 }\end{array}$ |

Figure 1. Intersection operator Type 1
Intersection operator Type 1 greatly influences the answer structure.
For doing the intersection by the second type of operator, the following steps should be done:

- Two chromosomes are selected from the population considering the selection strategy.
- A product pattern is randomly selected from each chromosome.
- The genes related to customers and their sequence are opened and put together.


## 5-1-6- Mutation operator:

The main task of mutation operator is to avoid convergence and local optimization and also searching in the intact spaces of the issue. A chromosome mutation means to change its gene and there different methods to do this depending on the type of coding. To apply mutation on chromosomes, a chromosome existing in the population is selected. Then, the genes related to a certain product are extracted from the chromosome and are arranged in all rows inversely. It should be noted that the mutation operator keeps the justification of a chromosome.

| Father-Product 1 |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Vehicles | dock | customer |  |  |  |  |
| 12 | 3 | 3 | 1 | 8 | 5 | 2 |
| 17 | 5 | 7 | 9 |  |  |  |


| Son-Product 1 |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Vehicles | dock | customer |  |  |  |  |
| 12 | 3 | 2 | 5 | 8 | 1 | 3 |
| 17 | 5 | 9 | 7 |  |  |  |

Figure 2. Mutation operator

## 5-2- NRGA $^{3}$ algorithm:

In order to improve the performance of NSGA-II algorithm, Rajamani et al. [20] used a modified algorithm of selection based on the roulette wheel in which each answer is given a fitness figure equal to the rank of that answer in the population. The proposed algorithm provides a two-layer ranking through selection based on the selection operator of roulette wheel. In this method, the new generation is selected from the parent generation based on the selection of the best answers (regarding to the fitness and extension). NRGA algorithm is similar to NSGA-II algorithm with a difference in selection strategy, population sorting, and selection for the next generation.

As explained above, each member of the population is characterized by two indices of rank and congestion distance. Therefore, to select an answer, a non-recessive boundary should be selected and the answer should be found within that boundary. The possibility for selection of the non-recessive boundary of $i$ is calculated by Equation 24:

$$
\begin{equation*}
p_{\mathrm{i}}=\frac{2 \times \mathrm{rank}_{\mathrm{i}}}{\mathrm{~N}_{\mathrm{f}} \times\left(\mathrm{N}_{\mathrm{f}}+1\right)}=\frac{\operatorname{rank}_{\mathrm{i}}}{\sum_{\mathrm{i}=1}^{\mathrm{p}} \mathrm{rank}_{\mathrm{i}}} \tag{24}
\end{equation*}
$$

In this equation, $\operatorname{rank}_{i}$ denotes the rank of boundary $i$ and $N_{f}$ shows the number of boundaries specified in nonrecessive sorting. It is clear that the answers on better boundaries are more likely to be selected. The possibility for selection of the answer $j$ existing in the non-recessive boundary $i$ is calculated by Equation 25:

$$
\begin{equation*}
p_{\mathrm{ij}}=\frac{2 \times \operatorname{rank}_{\mathrm{ji}}}{\mathrm{~N}_{\mathrm{j}} \times\left(\mathrm{N}_{\mathrm{j}}+1\right)}=\frac{\operatorname{rank}_{\mathrm{ji}}}{\sum_{\mathrm{j}=1}^{\mathrm{p}} \operatorname{rank}_{\mathrm{ji}}} \tag{25}
\end{equation*}
$$

In this equation, $N_{j}$ denotes the number of answers existing in the boundary $i$ and rank $_{i j}$ shows the rank of answer $j$ in the boundary $i$. According to this equation, the answers with higher congestion distance are more likely to be selected.
In the roulette wheel, $S_{1}=\sum_{\mathrm{i}=1}^{\mathrm{n}} \mathrm{p}_{\mathrm{i}}$ and $S_{2}=\sum_{\mathrm{j}=1}^{\mathrm{m}} \mathrm{p}_{0 \mathrm{j}}$ are defined in two intervals of $\left[0, s_{1}\right]$ and $\left[0, s_{2}\right]$. Then, the answers in each boundary occupy part of the intervals of $\left[0, s_{1}\right]$ and $\left[0, s_{2}\right]$ based on their possibility of selection. Then, two random numbers between 0 and 1 are selected. The first and second random numbers are used for selection of boundary in the interval $\left[0, s_{1}\right]$ and selection of one of the answers existing in the selected boundary in the interval $\left[0, s_{2}\right]$.

## 5-3- The criteria for stopping the algorithms:

The last step in genetic algorithms is to study the conditions of stopping. As the evolutionary algorithms are based on the production and testing, the answer of problem is not known and we do not know that which of the produced rows has the optimum answer to define it as the condition of stopping the algorithm. Hence, number of repetition is considered as the criterion for the termination condition. The number of optimal repetitions for algorithm can be determined by using the techniques of design and analysis of experiments.

6- Computational tests:
Computational tests aim to evaluate the efficiency of the proposed solution. For this purpose, it is described that how the parameters of proposed algorithms are set. Then, comparison criteria for evaluation of algorithms are

[^2]presented. Finally, the results obtained from the implementation of algorithms are investigated. All algorithms used in the present study were programmed by Matlab R2010a software and run on a computer with 4 gigabytes of RAM and a CPU of Core i5 2.2 GHz.

## 6-1- Setting the parameters by Taguchi method:

The aim if Taguchi tests is to find a combination of controlling factors level, as the objective function is maximized and the standard deviation is minimized for the answer variable. This is known as optimization of controlling factors level. To achieve this objective, the answer variable is converted into the performance criteria proposed by Taguchi, that is, "signal to noise". The $\mathrm{S} / \mathrm{N}$ ratio should be greater as much as possible. Since the considered answer variables is the number of Pareto answers, the corresponding formula is selected for the larger-better mode. Accordingly, the $\mathrm{S} / \mathrm{N}$ ratio for this variables is calculated as follows:

$$
\begin{equation*}
\left(\frac{S}{N}\right)=-10 \log \left(\frac{1}{n}\right) \sum_{i}\left(\frac{1}{y}^{2}\right) \tag{26}
\end{equation*}
$$

Where, $n$ and $y_{i}$ denote the number of tests and the desired answer in the test i-th, respectively. Table 1 shows the range of searching for the levels of parameters entering into three algorithms. In this study, 4 levels were considered for each variable. Selection of the initial levels is based on preliminary tests that provide a general view of the quality of algorithms in different levels of parameters.

Table 1. Controllable factors and their levels

| Algorithms | parameters | Description | Level 1 | Level2 | Level 3 | Level 4 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| NSGA-II | $n P o p$ | The number of initial population | 50 | 100 | 200 | 300 |
|  | $P_{c}$ | Cross rate | . 5 | . 6 | . 7 | . 8 |
|  | $P_{m}$ | Mutation rate | . 05 | . 1 | . 2 | . 3 |
|  | Iteration | The number of iterations of the algorithm | 150 | 250 | 350 | 450 |
| NRGA | $n P o p$ | The number of initial population | 100 | 200 | 300 | 100 |
|  | $P_{c}$ | Cross rate | . 4 | . 5 | . 6 | . 7 |
|  | $P_{m}$ | Mutation rate | . 1 | . 2 | . 3 | . 4 |
|  | Iteration | The number of iterations of the algorithm | 200 | 300 | 400 | 500 |

Structure of the designed tests and their results for NSGA-II and NRGA algorithms are presented in Table 2.
Table 2. Orthogonal arrays and values of NOS and S/N

| Exp No. | nPop | $\mathbf{P}_{\text {c }}$ | $\mathbf{P}_{\mathrm{m}}$ | Iteration | NSGA-II |  | NRGA |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  | NOS | S/N | NOS | S/N |
| 1 | 1 | 1 | 1 | 1 | 12 | 21,584 | 9,000 | 19.085 |
| 2 | 1 | 2 | 2 | 2 | 18 | 25,105 | 12,000 | 21.584 |
| 3 | 1 | 3 | 3 | 3 | 21 | 26,444 | 29,000 | 29.248 |
| 4 | 1 | 4 | 4 | 4 | 18 | 25,105 | 19,000 | 25.575 |
| 5 | 2 | 1 | 2 | 3 | 48 | 33,625 | 50,000 | 33.979 |
| 6 | 2 | 2 | 1 | 4 | 33 | 30,370 | 37,000 | 31.364 |
| 7 | 2 | 3 | 4 | 1 | 42 | 32,465 | 42,000 | 32.465 |
| 8 | 2 | 4 | 3 | 2 | 42 | 32,465 | 50,000 | 33.979 |
| 9 | 3 | 1 | 3 | 4 | 39 | 31,821 | 69,000 | 36.777 |
| 10 | 3 | 2 | 4 | 3 | 45 | 33,064 | 44,000 | 32.869 |
| 11 | 3 | 3 | 1 | 2 | 57 | 35,117 | 66,000 | 36.391 |
| 12 | 3 | 4 | 2 | 1 | 33 | 30,370 | 30,000 | 29.542 |
| 13 | 4 | 1 | 4 | 2 | 22 | 26,848 | 34,000 | 30.630 |
| 14 | 4 | 2 | 3 | 1 | 44 | 32,869 | 95,000 | 39.554 |
| 15 | 4 | 3 | 2 | 4 | 88 | 38,890 | 67,000 | 36.521 |
| 16 | 4 | 4 | 1 | 3 | 44 | 32,869 | 48,000 | 33.625 |

Each of the predesigned tests are done and the value of objective function is changed into $\mathrm{S} / \mathrm{N}$ ratio. Figure 3 and Figure 4 show that how the values of $\mathrm{S} / \mathrm{N}$ ratio are changed at different levels of algorithms. The levels in which $\mathrm{S} / \mathrm{N}$ ratio reaches its maximum can be selected as the optimum levels.


Figure 3: How the values of $\mathrm{S} / \mathrm{N}$ ratio are changed at different levels of NSGA-II

Main Effects Plot for SN ratios
Data Means


Signal-to-noise: Larger is better
Figure 4: How the values of S/N ratio are changed at different levels of NSGA

## 6-2- Comparison indices:

As mentioned, the proposed multi-objective algorithms search and work based on Pareto. On the other hand, we know that the final Pareto found by algorithms must have two feature of convergence to the optimum solution and acceptable diversity. For this purpose, it is needed to study various measures in order to ensure a comprehensive understanding of the performance of a multi-objective algorithm. Some of these measures and criteria are as follows:

## 6-2-1- The highest expansion:

This measure, measures the diameter of a spatial cubic that is used by the end values of objectives for a set of non-recessive answers.

$$
\begin{equation*}
D=\sqrt{\sum_{j=1}^{m}\left(\max _{i} f_{i}^{m}-\min f_{i}^{m}\right)^{2}} \tag{27}
\end{equation*}
$$

In two-objective issues, this measure is equal to Euclidean distance between two final answers in the answer space. Higher values of this measures show better results.

## 6-2-2- NOS:

The algorithm which can provide more numbers of non-recessive answers in Pareto archives would be more successful in outlining the optimum level of real Pareto and can provide the decision-maker with more options.

## 6-2-3- MID:

This criterion measure the mean distance of Pareto answers from an ideal answer. The ideal answer would be determined based on opinions and comments of experts. It can be inferred from Equation 28 that the lower the values of this criterion, the more efficient the algorithm.

$$
\begin{equation*}
M I D=\frac{1}{N O S} \sum_{i=1}^{N O S} c_{i} \text { where } c_{i}=\sqrt{\sum_{i=1}^{m} f_{j i}^{2}} \tag{28}
\end{equation*}
$$

## 6-2-4- Spacing:

This index has been developed by Scott in 1995 and measures the relative distance between successive answers. This index is calculated as shown in Equation 29.

$$
\begin{align*}
& s=\sqrt{\frac{1}{n-1} \sum_{i=1}^{n}\left(d_{i}-\bar{d}\right)^{2}} \text { where } d_{i}=\min _{k m, k+i} \sum_{m=1}^{2}\left|f_{m}^{i}-f_{m}^{k}\right| \text { and }  \tag{29}\\
& \bar{d}=\frac{1}{n} \sum_{i=1}^{n} d_{i}
\end{align*}
$$

The distance measured is equal to the lowest sum of absolute difference in values of objective functions between the answer $i$ and the answers in the final non-recessive set. This index measures the standard deviation of different value of $d_{i}$. When the answers are uniformly next to each other, then the value of $S$ is small. Therefore, the algorithm that its final non-recessive answers have smaller values of $S$ is better.

## 6-2-5- Algorithm time:

The shorter the time of an algorithm, the more favorable the performance of that algorithm.

## 6-3- Comparison of algorithms:

In order to describe the efficiency of algorithms, numerical examples were randomly created and run on a PC. These examples include issues with different number of warehouses, customers, and products. Three numerical examples were created for each size of issues. The results obtained from two algorithm are shown in Table 3. All computations were done by a computer with 2 gigabytes of RAM and a PCU of Core i4 2 duo 2.2 GHz and setting the parameters was done by Minitab 16 software.

Table 3: Results of computation of proposed algorithms for the example issue

| Num | SIZE | NSGA-II |  |  |  | NRGA |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | D + | NOS + | MID | TIME | D + | NOS + | MID | TIME |
| 1 | 3-10-4 | 3026800 | 29 | 1369100 | 220 | 3183851 | 28 | 1386035 | 288 |
| 2 | 3-10-4 | 3019525 | 33 | 1254157 | 254 | 2563264 | 31 | 1480344 | 332 |
| 3 | 3-10-4 | 3125415 | 25 | 1152369 | 262 | 3159474 | 23 | 1354634 | 294 |
| 4 | 5-10-6 | 4698522 | 34 | 2614400 | 244 | 4921244 | 33 | 2519081 | 300 |
| 5 | 5-10-6 | 4854259 | 35 | 2752141 | 242 | 4831947 | 36 | 2843736 | 280 |
| 6 | 5-10-6 | 4574800 | 30 | 2354145 | 244 | 4439468 | 26 | 2646722 | 280 |
| 7 | 8-15-4 | 8170200 | 30 | 2547521 | 256 | 7759802 | 28 | 2915991 | 278 |
| 8 | 8-15-4 | 7252336 | 33 | 2456390 | 254 | 7366289 | 27 | 2672734 | 276 |
| 9 | 8-15-4 | 8152098 | 38 | 2894300 | 638 | 80022286 | 40 | 3105267 | 706 |


| 10 | $8-15-6$ | 7729400 | 31 | 2971900 | 624 | 7354002 | 31 | 3036126 | 510 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 11 | $8-15-6$ | 7987110 | 36 | 3125477 | 504 | 7632248 | 37 | 3144953 | 602 |
| 12 | $8-15-6$ | 7741253 | 40 | 3054125 | 560 | 7777930 | 40 | 2924569 | 874 |
| 13 | $10-20-4$ | 6619600 | 20 | 3379300 | 630 | 5381519 | 18 | 4049545 | 518 |
| 14 | $10-20-4$ | 7524120 | 38 | 3245827 | 716 | 6368142 | 39 | 3551415 | 766 |
| 15 | $10-20-4$ | 6285170 | 41 | 2379300 | 748 | 6593546 | 41 | 2539634 | 702 |
| 16 | $10-20-6$ | 9051200 | 36 | 3872400 | 1163 | 7956759 | 30 | 4335216 | 1252 |
| 17 | $10-20-6$ | 845245 | 44 | 3965141 | 1672 | 7920028 | 38 | 3940730 | 1340 |
| 18 | $10-20-6$ | 7895146 | 41 | 3802044 | 1786 | 7757840 | 35 | 3778402 | 1569 |
| 19 | $10-30-4$ | 6438100 | 40 | 3902900 | 1798 | 6569216 | 34 | 4013327 | 1466 |
| 20 | $10-30-4$ | 6985214 | 39 | 3854120 | 1590 | 5859603 | 38 | 4598185 | 1318 |
| 21 | $10-30-4$ | 7566214 | 48 | 4210589 | 1848 | 7456843 | 49 | 4910562 | 1250 |
| 22 | $10-30-6$ | 8975900 | 37 | 3243800 | 1964 | 8430844 | 31 | 3557637 | 1344 |
| 23 | $10-30-6$ | 7521445 | 47 | 3987452 | 1906 | 6024516 | 39 | 4562314 | 1962 |
| 24 | $10-30-6$ | 7025883 | 40 | 3852114 | 2052 | 7173179 | 35 | 4534889 | 1714 |
| 25 | $15-40-4$ | 1045200 | 26 | 3629600 | 11212 | 8559750 | 22 | 3601172 | 8320 |
| 26 | $15-40-4$ | 9874256 | 33 | 3985221 | 7985 | 8602900 | 27 | 4283553 | 6248 |
| 27 | $15-40-4$ | 8395472 | 29 | 4215774 | 8700 | 8687477 | 26 | 4705792 | 7768 |
| 28 | $15-40-6$ | 7923900 | 39 | 4803300 | 10932 | 7331397 | 33 | 51190283 | 8472 |
| 29 | $15-40-6$ | 9785412 | 40 | 4754112 | 11200 | 8237512 | 35 | 5053283 | 10508 |
| 30 | $15-40-6$ | 10857123 | 49 | 4352948 | 9866 | 8934979 | 46 | 4911728 | 8294 |


(a)

(b)


Figure 5: Graphical display of algorithms performance in the studied indicators
Figure 5-a shows the efficiency of NSGA-II and NSGA algorithms in the criterion of the highest expansion. It can be generally stated that none of these two algorithm are superior over each other in this indicator. Figure 5-b depicts the efficiency of these algorithms in the criterion of number of Pareto answers. As observed in this figure, the number of Pareto answers of the issue increases with the increase in the number of warehouses and customers. This is due to increased number of modes of the use and allocation of customers to warehouses and the path of vehicles and thereby the probability of finding Pareto answers increases more. Figure 5-c shows the efficiency of NSGA-II and NSGA algorithms in the criterion of distance from the ideal answer. According to this figure, NSGAII has a better efficiency than NSGA in this criterion. Figure 5-d is related to the measures of the distance from the ideal answer. It can be stated the efficiency of algorithms is the same in these measures. Figure 5-e shows the efficiency of these two algorithms in the criterion of computations time.

Statistical analyses were applied for a more accurate assessment and comparison. The results of algorithms in different sizes were normalized by Relative Division Percentage (RDP). The values of RDP show that how far the answers in an algorithm are from the ideal answer. This index can be calculated by Equation 30.

$$
\begin{equation*}
R P D_{i j}=\left(\operatorname{sol}_{i j}-\operatorname{sol}_{j, \text { min }}\right) / \operatorname{sol}_{j, \text { min }} \tag{30}
\end{equation*}
$$

Where, $i$ and $j$ denote algorithm number and size of example, respectively. Then, algorithms were tested by variance analysis for each criterion. If $p$-value is less than 0.05 , it shows that the difference between the answers of an algorithm in relation to a specific criterion is significant. Otherwise, it can be stated that there is no significant difference between the performances of algorithms in relation to that criteria. First of all, it should be examined that whether the mean of algorithms in any of the criteria is different or not (the null and alternative hypotheses).
$H_{0}: \mu_{1}=\mu_{2}=\mu_{3}$
$H_{1}: \mu_{i} \nLeftarrow \mu_{j} \quad i, j=1,2,3$ and $i \neq j$
According to F-statistic and P-value, it can be concluded that there is a significant different between algorithms in all criteria and this necessitates the use of Tukey test. The results obtained from Tukey test are presented in figures $6,7,8$, and 9 . According to these figures, it can be stated that NRGA algorithms has a better efficiency than NRGA-II in spacing and the highest expansion. According to Figure 11, it can be concluded that NRGA-II algorithm has a high ability in the number of Pareto answers, while MOHS and NRGA, respectively, have the similar and much weaker performance compared with NRGA-II in this criterion.


Figure 6. A confidence level of $95 \%$ for relative standard deviation in the criterion of the highest expansion


Figure 7. A confidence level of $95 \%$ for relative standard deviation in the criterion of the number of Pareto answers


Figure 8. A confidence level of $95 \%$ for relative standard deviation in the criterion of distance from the ideal answer


Figure 9. A confidence level of 95\% for relative standard deviation in the criterion of time

## 7- Conclusion and recommendations:

Given the model is non-linear, NP Hard, and multi-objective, post-innovative multi-objective algorithms were proposed for solving the model. To evaluate the efficiency of algorithms, 32 problems in different sizes were randomly selected and solved. The results of the presented measures were tested on the proposed algorithms. It was found that none of the algorithms are superior to each other in the criterion of the highest expansion and NSGA-II algorithm has a better performance than NSGA in the distance from the ideal answer.

The following items can be proposed as recommendations for further studies:

- Other assumptions can be added to the issue, such as operational costs of warehousing for each product, time limitations, and the distance of each route.
- Taking into account the parameters such as travel time, cost of route between two nodes, capacity of vehicles, and costs of warehouses establishment and operation and also modeling them can provide us with methods closer to reality.
- Considering other models of uncertainty such as gray fuzzy and so on.
- Solving the issues with more efficient innovative and post-innovative algorithms.


## 8-REFRENCES

[1] Srivastava, H,. Benton, WC, (1990). The location-routing problem: considerations in physical distribution system design, Computers \& operations research, 17:5, 427-435.
[2] Averbakh I, Berman O, (2002), Minimax p-traveling salesmen location problems on a tree, Annals of Operations Research, 110, 55-62.
[3] Aksen, D, Altinkemer, K. (2008). A location-routing problem for the conversion to the "click-and-mortar" retailing: The static case. European Journal of Operational Research 186:2, 554-575.
[4] Alumur,S. and B. Y. Kara, 2007. A new model for the hazardous waste location-routing problem.Computers\& Operations Research, 34: 1406-1423.
[5] Ghiani G, Laporte G., (1999), Eulerian location problems, Networks, 34:4, pp.291-302.
[6] Ahn, C. W. and Ramakrishna, R. S. (2002). A genetic algorithm for shortest path routing problem and the sizing of populations., IEEE Transaction on Evolutionary Computation, 6:6, 566-579.
[7] Tai-Hsi Wu, Chinyao Low, Jiunn-Wei Bai, "Heuristic solutions to multi-depot location-routing problems", Management Science, 2001.
[8] Barahona, F., Jensen, D., (1998). Plant Location with Minimum Inventory, Mathematical Programming, 83, 101111.
[9] Barreto, S. S., Ferreira, C., Paixa, J. and Santos, B. S. (2007). Using clustering analysis in capacitated locationrouting problem. European Journal of Operational Research, 179, 968-977.
[10] Yu, V. F., Lin, S-W., Lee, W. and Ting, C.-J. (2010). A simulated annealing heuristic for the capacitated location-routing problem. Computers \& Industrial Engineering, 58: 288-299.
[11] Yao, Z., Lee, L. H., Jaruphongsa, W., Tan, V., Hui, C. F. (2010) Multi - source facility location - allocation and inventory problem, European Journal of Operational Research, 207, 750-762.
[12] Ahmadi, J. A., Azad, N. (2010). Incorporating location, routing and inventory decisions in supply chain network design, Transportation Research, 46, 582-597.
[13] Ahmadi, J. A., Seddighi, A. H. (2012) A location-routing-inventory model for designing multisource distribution networks, Engineering Optimization, 44, 637-656.
[14] Owen Sh,Daskin Ms, (1998). Strategic facility location: A review, European Journal of Operational Research 111:3, 423-447.
[15] Sakawa, M., (1993), Fuzzy sets and interactive multi-objective optimization. Plenum Press. New York.
[16] A Konak, DW Coit, AE Smith, (2006). Multi-objective optimization using genetic algorithms: A tutorial, Reliability Engineering \& System Safety 91 (9), 992-1007.
[17] N. Srinivas and K. Deb, "Multiobjective function optimization using nondominated sorting genetic algorithms," Evol. Comput., vol. 2, no. 3, pp. 221-248, Fall 1995.
[18] Golberg,D. E. (1989). Genetic algorithm in search, optimazaion, and machine learning. Reading, MA:Addison Wesley.
[19] K. Deb, A.Pratap, S.Agarwal and T. Meyarivan, (2002). A Fast and Elitist Multiobjective Genetic Algorithm: NSGA-II, IEEE TRANSACTIONS ON EVOLUTIONARY COMPUTATION, VOL. 6, NO. 2, APRIL 2002
[20] L.Rajamani O.Al Jadaan, C.R. Rao,(2005). Ranked based roulette wheel selection method. In International Symposium on Recent Advances in Mathematics and its Applications: (ISRAMA 2005), Calcutta Mathematical Society at AE-374, Sector-1, Salt Lake City Kolkata (Calcutta) 700064, India.


[^0]:    * Corresponding Author: Hooman Mohammadpour, Msc. Student, Faculty of Industrial and Mechanical Engineering, Qazvin branch, Islamic Azad University, Qazvin, Iran, hooman.mohammadpour@yahoo.com

[^1]:    ${ }^{1}$ Vector Evaluated Genetic Algorithm
    ${ }^{2}$ Non Dominated Sorting Genetic Algorithm

[^2]:    ${ }^{3}$ Non-dominated Ranked Genetic Algorithms

