J. Appl. Environ. Biol. Sci., 5(12S)602-607, 2015
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ISSN: 2090-4274
Journal of Applied Environmental and Biological Sciences
www.textroad.com

Presenting an Enhanced Approach for Economic Assessment of Projects with Multiple Rates

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Received: July 24, 2015
Accepted: September 31, 2015

ABSTRACT

The economy is the engineering of a set of mathematical techniques for economic evaluation of investment plans and projects; present value method and internal rate of return method are among the most important ways for such assessment. Although present value method provides reliable results, but directors and investors have a higher tendency to employ internal rate of return method for various reasons. This is the case while applying internal rate of return method is concomitant with serious issues. During recent years, different papers have been published in order to troubleshoot the problems of this method; it may be stated conclusively that Hazen's method is the basis of many of these papers. The aim of the present study is to demonstrate Hazen's approach through Horner's method by means of an easily comprehensible process which leads finally in reduced calculations.
KEY WORDS: Capital Budgeting, NPV, IRR, Project Analysis, Horner.

1. INTRODUCTION

The internal rate of return (IRR) is often used by managers and practitioners for investment decisions. Unfortunately, it has serious flaws: among others, (i) multiple real-valued IRRs may arise, (ii) complex-valued IRRs may arise, (iii) the IRR is, in general, incompatible with the net present value (NPV) in accept or reject decisions (iv) the IRR ranking is, in general, different from the NPV ranking, (v) the IRR criterion is not applicable with variable costs of capital (vi) it does not measure the return on initial investment, (vii) it does not signal the loss of the entire capital, (viii) it is not capable of measuring the rate of return of an arbitrage strategy. The efforts of economists and management scientists in providing a reliable project rate of return have generated over the decades an immense bulk of contributions aiming to solve these shortcomings Magni [1].One reason why debate persists is that financial practice has not caught up with theory. Large corporations and banks use both criteria, but often prefer IRR, perhaps because of the ease of comparison with the cost of capital. Surveys from the last decade include Payne [2] and Kester et al., [3], who report on practices that vary from country to country; sometimes IRR is preferred, sometimes NPV. Ryan and Ryan [4] report on studies between 1960 and 1996 that show IRR dominated company practice during that period while their own study of Fortune 1000 companies shows NPV is narrowly preferred to IRR. Surveys by Graham and Harvey [5] and Liljeblom and Vaihekoski[6] show NPV used more than IRR. Brounen et al. [7] apply Graham and Harvey's methodology to European companies and show NPV is used more in Germany and the Netherlands but IRR is used more in the UK and France.If the cash flow of a project has a singular condition in terms of brand change, then the project being single rate or multiple rate may be recognized in (-1, +∞) interval Norstrom [8] multiple or nonexistent internal rates are not contradictory, meaningless or invalid as rates of return. it does not matter which rate is used, we mean that regardless of which rate is chosen, the cash-flow acceptance or rejection decision will be the same, and consistent with net present value[9].It should be noted that Hazen's approach is the basis of the methods proposed in papers [1, 10,11, 13]; thus, in what follows, we first touch on the internal rate of return on investment method and also Hazen's technique and then demonstrate this approach using Horner's method in order to achieve the main goal of this paper (that is, the simplification of the calculations via a schematic model, i.e., the very Horner's table).

2.Internal Rate of Return

A cash flow stream is a finite or infinite sequence $X=(x_0, x_1, \ldots)$ of monetary values. The monetary amount received initially is x_0, and the amount received after period t is x_t. For a finite stream $X=(x_0, x_1, \ldots, x_n)$, we assume the horizon n is chosen so that $x_n \neq 0$. The net present value $PV(X|r)$ of a cash flow stream X at interest rate r is given by:

$$PV(X|r)=\sum_{t=0}^{n}\frac{x_t}{(1+r)^t} \qquad (1)$$

Defined for proper interest rates $r>-1$. For a cash flow stream X, let IRR(X) be the set of all interest rates r which make $PV(X|r)=0$. (Note that IRR(X) cannot contain -1 because $PV(X|r=-1)$ is undefined.) For finite

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streamsX $= (x_0, x_1, \dots, x_n)$, the present value function PV(X|r) is a degree-n polynomial in$(1 + r)^{-1}$, so IRR(X) can contain anywhere from 0 to n distinct values. If r ∈ IRR(X) , then we will call r an internal rate of return for X.

As is well known, for conventional cash flows X that are negative for the first few periods but positive thereafter, the internal rate of return exists and is unique. Moreover, the internal rate of return is the largest interest rate at which the cash flow shows a discounted net profit. So if IRR(X) exceeds the available market rate of interest r, then $PV(X|r) > 0$ and the investment which generates the cash flow X is worthwhile. Conversely, if the internal rate of return is smaller than the market rate r, then one is better off investing at the market rate r. This is the fundamental justification for the use of internal rate of return[9].

3.Hazen's Approach

While any IRR may be used for decision-making, one still does not know which one of the IRRs is the economically correct rate of return. Hazen say that $C = (c_0, c_1, \dots, c_{n-1})$ is an investment stream yielding cash flow streamX $= (x_0, x_1, \dots, x_n)$, at (constant per-period) rate of return k if the following equations hold[9]:

$$\begin{cases} x_0 = -c_0 \\ x_t = (1+k).c_{t-1} - c_t \quad for\ t = 1, \dots, n-1 \\ x_n = (1+k).c_{n-1} \end{cases} \qquad (2)$$

In other words, X is the cash flow stream obtained when the investor sinks an amount c_t into a project at the end of each period $t < n$ and receives return $(1+k).c_t$ after period $t+1$. The quantity c_t is the increment of capital invested at time t, and $(1+k).c_t - 1$ is the increment of capital recovered at time t. We allow the quantities c_t to be negative as well. In this case, $-c_t$ is the increment of capital borrowed at time t and $(1+k).c_t - 1$ is the increment of capital repaid at time.

Theorem 1: If investment stream $C = (c_0, c_1, \dots, c_{n-1})$ yields $X = (x_0, x_1, \dots, x_n)$ at constant per-period rate of return k, then for $r \neq -1$,

$$PV(X|r) = \frac{k-r}{1+r} PV(C|r) \qquad (3)$$

Theorem 2: The quantity k is an internal rate of return for a cash flow stream Xif and only if there exists an investment stream Cwhich yields Xat constant per-period rate of return k.

Theorem 3: Suppose Xis the yield of a pure investment or pure borrowing stream Cat a proper internal rate of return k. Then k is the only proper internal rate of return for X, and

a) If Xis a pure investment stream, then $PV(X|r) \geq 0$ if and only if $k \geq r$.

b) If Xis a pure borrowing stream, then $PV(X|r) \geq 0$ if and only if $k \leq r$.

Theorem 4: Suppose k is an internal rate of return for the cash flow stream X, and let Cbe the investment stream yielding Xat constant per-period rate of return k.

a) If $PV(C|r) > 0$ (that is, Cis a net investment) then $PV(X|r) \geq 0$ if and only if $k \geq r$.

b) If $PV(C|r) < 0$ (that is, Cis a net borrowing) then $PV(X|r) \geq 0$ if and only if $k \leq r$.

c) If $PV(C|r) = 0$ then $PV(X|r) = 0$.

4.Determining the economic quality of a project through using Hazen's approach to the actual internal rate(s) of return on investment

STEP (1): solve the IRR equation and pick any one of the IRRs.

STEP (2):compute the corresponding capital stream $C^{k_i}$and calculate its present value $PV(C^{k_i}|r)$to ascertain its financial nature (investment or borrowing).

STEP (3): if the project is an investment (borrowing), accept the project if and only if the IRR is greater (smaller) than the market rate.

For observation proof of the Theorems, You can see Hazen [9]

5.Presentation of a new methodology

Before introducing the new methodology, it seems necessary to give a simple definition for distinguishing plans (projects); so, after defining the classification, we will analyze Horner's theorem and then demonstrate the new approach according to this very theorem.

6.Classification of the economic plans and projects

Without precise scientific definitions, human knowledge may not be developed. It is crystal clear that those definitions originating in superficial attitudes not only do not play a role in promoting and developing knowledge but may disrupt the encoding of the concepts in most cases. In many of the economic studies including books and articles, scholars employ the term "economic projects" for analyzing projects from the perspective of economics. In fact, this term signifies that the projects are being studied economically. Also, once evaluations are made, profitable projects are again called by the same term "economic projects"; thus, in trying to demystify these ambiguities, we will follow with providing a classification for the projects via giving various definitions. Generally speaking, projects and plans investigated in economic studies could be classified in three different classes which will be introduced now.

6.1. The economic class: It refers to that group of projects (plans) with every member being profitable; we represent this class (or set) by $\boldsymbol{C^0}$ symbol.

6.2.The neutral class: It refers to that group of projects or plans with every member being neither profitable nor loss-making; we represent this class (or set) by $\boldsymbol{C^-}$ symbol.

6.3. The non-economic class: It refers to that group of projects or plans with every member being loss-making; we represent this class (or set) by $\boldsymbol{C^+}$ symbol.

In order to identify to which of these classes the projects or plans belong, it is sufficient to use balance methods such as PV. Put it more simply, if the cash flow X is an economic project (or plan) with interest rate r, it could be stated:

$$\begin{cases} C^+ = \{X \mid PV(X|r) > 0\ \} \\ C^0 = \{X \mid PV(X|r) = 0\ \} \\ C^- = \{X \mid PV(X|r) < 0\ \} \end{cases} \tag{4}$$

Regarding that the foundation of this new approach is Horner's method in decomposition of polynomials, Horner's theorem is first explained and then the new approach, i.e., the enhanced approach of Hazen, will be dealt with

7. Horner's theorem

If P is a polynomial of degree n as the following:

$$P_n(x) = p_n x^n + p_{n-1}x^{n-1} + \ \dots \ + p_3x^3 + p_2x^2 + p_1x + p_0 \ ; \ p_n \neq 0 \tag{5}$$

And a is a given number (whether real or complex), then P could be decomposed as:

$$P_n(x) = (x-a)Q_{n-1}(x) + R \tag{6}$$

Where $R = P_n(a)$ is remainder and $Q_{n-1}(x)$ is a polynomial of $(n-1)$ degree [14].

8. Horner's algorithm

One application of Horner's algorithm is to compute Q polynomial coefficients in equation (1). By drawing a table consisting of three rows and $(n+2)$ columns as table (1) and assuming equations (7), Q polynomial coefficients could be easily computed.

Table1

	$\boldsymbol{p_n}$	$\boldsymbol{p_{n-1}}$	…	$\boldsymbol{p_2}$	$\boldsymbol{p_1}$	$\boldsymbol{p_0}$
$x = a$	t_n	t_{n-1}	…	t_2	t_1	$\boldsymbol{t_0}$
$q_n = 0$	q_{n-1}	q_{n-2}	…	q_1	q_0	$\boldsymbol{R = P_n(a)}$

Source: Current research computing

The following equations illustrate the trend of calculations:

$$\begin{cases} t_j = a * q_j \\ \\ q_{j-1} = p_j + t_j \end{cases} \qquad j=n,n\text{-}1,\dots,2,1 \tag{7}$$

Note that if a is the root of P, then value of remainder (R) will be equal to zero.

Numerical examples (1): Assuming $P_3(x)$, calculate the coefficients of Q polynomial existing in equation (6) according to $x = 2$, which is one root for P.

Solution: Tables 2 to 6 illustrate the steps of running Horner's algorithm in a schematic and step-by-step manner.

Table 2.

	$\boldsymbol{p_3 = 1}$	$\boldsymbol{p_2 = -10}$	$\boldsymbol{p_1 = 31}$	$\boldsymbol{p_0 = -30}$
$x = 2$				
$q_3 = 0$				

Source: Current research computing

Table 3.

	1	-10	31	-30
2	0			
0	1			

Source: Current research computing

Table 4.

	1	-10	31	-30
2	0	2		
0	1	-8		

Source: Current research computing

Table 5.

	1	-10	31	-30
2	0	2	-16	
0	1	-8	15	

Source: Current research computing

Table 6.

	1	-10	31	-30
2	0	2	-16	30
0	1	-8	15	0

Source: Current research computing

According to table (6), it may be easily stated that:
$Q_2(x) = x^2 - 8x + 15.$

9. Theorem (Hazen's enhanced approach)

If X is a cash flow with lifespan n as $X = (x_0, x_1, \dots, x_n)$ and k is a given internal rate of return of X and also r is assumed as the interest rate (cost of capital) constant in different periods, then there exists a cash flow such as C with lifespan of $(n-1)$ so that:

$$PV(X|r) = \frac{r-k}{1+r} PV(C|r) \tag{8}$$

Proof: According to the definition of future value:

$$FV(X|r) = x_0(1+r)^n + x_1(1+r)^{n-1} + \cdots + x_{n-2}(1+r)^2 + x_{n-1}(1+r) + x_n \tag{9}$$

By modifying the variables in $1 + r = x$, $FV(X|r)$ is converted to a polynomial of degree n like P as the following $p_j = x_{n-j}\ ; j = 0,1, \dots n$:

$$FV(X|r) =$$
$$P_n(x) = x_0 x^n + x_1 x^{n-1} + \ \dots\ + x_{n-2}x^2 + x_{n-1}x + x_n \tag{10}$$

Note: It is clear that $\mathbf{a = 1 + k}$ is its root ($\mathbf{P_n(a) = 0}$).

Now, according to Horner's theorem, it may be stated that there is a polynomial as $Q_{n-1}(x) = q_{n-1}x^{n-1} + q_{n-2}x^{n-2} + \cdots + q_1 x^1 + q_0$ so that:

$$P_n(x) = (x - a)Q_{n-1}(x) \tag{11}$$

Now, by restoring variables into their original states ($x = 1 + r\ and\ a = 1 + k$), we have:

$$P_n(1+r) = \big((1+r) - (1+k)\big)Q_{n-1}(1+r) \tag{12}$$

$$FV(X|r) = (r-k)(q_{n-1}(1+r)^{n-1} + q_{n-2}(1+r)^{n-2} + \cdots + q_1(1+r) + q_0) \tag{13}$$

It is obvious that the result of $Q_{n-1}(1+r)$ is equal to the future value of a flow like $C = (c_0, c_1, \dots, c_{n-1})$ (it is adequate to assume $c_j = q_{n-j-1}\ ; j = 0,1, \dots, n-1$):

$$FV(X|r) =$$
$$(r-k)(c_0(1+r)^{n-1} + c_1(1+r)^{n-2} + \cdots + c_{n-2}(1+r) + c_{n-1}) \tag{14}$$

$$FV(X|r) = (r-k)FV(C|r) \tag{15}$$

By multiplying both sides of the previous relation by $\frac{1}{(1+r)^n}$, we have:

$$FV(X|r) = \frac{(r-k)FV(C|r)}{(1+r)(1+r)^{n-1}} \tag{16}$$

$$PV(X|r) = \frac{\mathrm{r-k}}{\mathrm{1+r}} PV(C|r) \tag{17}$$

This is the end of proof.

9.1result's previous Theorem

$$\begin{cases} \text{if } \mathrm{r} > k \,\&\, C \in \mathrm{C}^{+} \Rightarrow \mathrm{X} \in \mathrm{C}^{+} \\ \text{if } \mathrm{r} > k \,\&\, C \in \mathrm{C}^{0} \Rightarrow \mathrm{X} \in \mathrm{C}^{0} \\ \text{if } \mathrm{r} > k \,\&\, C \in \mathrm{C}^{-} \Rightarrow \mathrm{X} \in \mathrm{C}^{-} \\ \text{if } \mathrm{r} = \mathrm{k} \Rightarrow \mathrm{X} \in \mathrm{C}^{0} \\ \text{if } \mathrm{r} < k \,\&\, C \in \mathrm{C}^{+} \Rightarrow \mathrm{X} \in \mathrm{C}^{-} \\ \text{if } \mathrm{r} < k \,\&\, C \in \mathrm{C}^{0} \Rightarrow \mathrm{X} \in \mathrm{C}^{0} \\ \text{if } \mathrm{r} < k \,\&\, C \in \mathrm{C}^{-} \Rightarrow \mathrm{X} \in \mathrm{C}^{+} \end{cases} \tag{18}$$

In Figure 1 the mentioned result could be well observed. In fact, this figure illustrates that instead of deciding according to the present value in one level, internal rate of return and the present value of a smaller flow could be used in two levels.

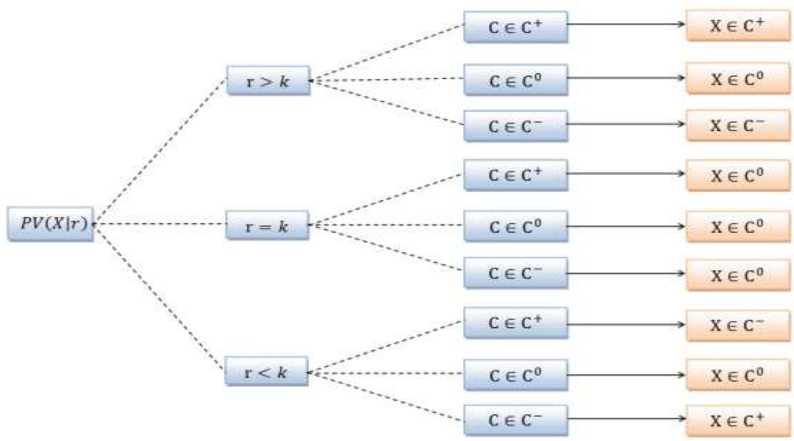

Fig 1.[**Source:** Current research computing]

Numerical examples (2): Consider a project with the cash flow of $X = (-1\$, 6\$, -11\$, 6\$)$ and a constant interest rate equal to $r = 10\%$; it is observed that this project has three actual internal rates of return ($r_1 = 0\%, r_2 = 100\%, r_3 = 200\%$). Now, using Horner algorithm, we calculate the investment flow related to the Internal rate of return (r_2) (according to $x = 1 + r_2$). Table (7) incorporates the whole calculations.

Table 7.

	−1	**6**	**−11**	**6**
2	0	−2	8	**−6**
0	−1	4	−3	**0**

Source: Current research computing

In the next step of this example, it may be decided whether to confirm or reject the relevant project through presenting a final table (Table(8)) involving all the X-dependent *cash flow streams*(C^{k_i}). Figure (2) illustrates the diagram of the *cash flow stream* present value and the relevant sub-cash flows.

Table 8. [**Source:** Current research computing]

	t_0	t_1	t_2	t_3	PV	If		
$\boldsymbol{X}$	**−1**	**6**	**−11**	**6**	**−0.128**			
$C^{\%0}$	−1	5	−6		-1.413	$(10\% > 0\%)$ $\&(C^{\%0} \in \mathrm{C}^{-})$	$\mathrm{X} \in \mathrm{C}^{-}$	reject
$C^{\%100}$	−1	4	−3		0.157	$(10\% < 100\%)$ $\&(C^{\%100} \in \mathrm{C}^{+})$	$\mathrm{X} \in \mathrm{C}^{-}$	reject
$C^{\%200}$	−1	3	−2		0.074	$(10\% < 200\%)$ $\&(C^{\%200} \in \mathrm{C}^{+})$	$\mathrm{X} \in \mathrm{C}^{-}$	reject

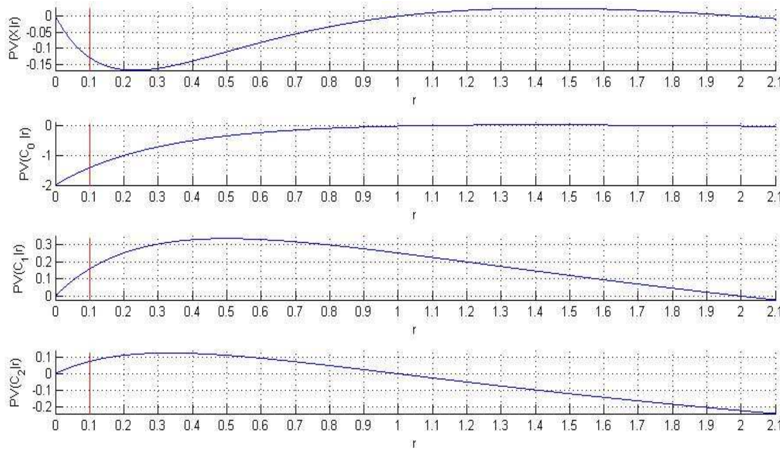

Fig 2.[**Source:** Current research computing]

10. CONCLUSION

Although Hazen's approach provided a new perspective for demystifying the ambiguities and contradictions within the internal rate of return method, but this method has not seen much practical use in papers and it is cited by scholars only in theoretical ways; thus, in this paper, by approaching to this method from another viewpoint we tried to encourage the students of engineering and economics to use this method more frequently. Moreover, this approach proposes a two-criterion decision-making method for assessing plans and projects with multiple rates. It doesn't recognize the rate of return as an adequate criterion for correct decision making. It should be noted that by using this new approach, we could only evaluate the quality of the projects by means of the actual rates of return; so, mixed rates of return and also ranking of the projects were totally excluded in this study. Then, a suggestion for further studies is to generalize this approach for economic evaluations of projects through mixed rates of return; the second suggestion would be to generalize this approach in order to incorporate ranking projects and plans.

11. REFERENCES

[1] Magni, C.A. Average internal rate of return and investment decisions: a new perspective. The Engineering Economist, 2010, 55(2), 150-180
[2] Payne, J., Carrington, H. W., & Gale, L. Comparative financial practice in the US and Canada: Capital budgeting and risk assessment techniques. Financial Practice and Education, 1999, 9(1).
[3] Kester, G., Chang, R., Echanis, E., Haikal, S., Isa, M., &Skully, M. Capital budgeting practices in the Asia-Pacific region: Australia, Hong Kong, Indonesia, Malaysia, Philippines and Singapore. Financial Practice and Education, 1999, 9 (1).
[4] Ryan, P., & Ryan, G. Capital budgeting practices of the Fortune 1000: How have things changed? Journal of Business and Management, 2002, 8 (4).
[5] Graham, J., & Harvey, C. The theory and practice of corporate finance: Evidence from the field. Journal of Financial Economics, 2001, 60.
[6] Liljeblom, E., & Vaihekoski, M. Investment evaluation methods and required rate of return in Finnish publicly listed companies. Finnish Journal of Business Economics, 2004, 53 (1).
[7] Brounen, D., De Jong, A., & Koedijk, K. Corporate finance in Europe: Confronting theory with practice. Financial Management, 2004, 33 (4).
[8] Norstrom, C.J. A sufficient condition for a unique non-negative internal rate of return. Journal of Financial and Quantitative Analysis, 1972, 7(3), 1835–1839.
[9] Hazen, G. A New Perspective on Multiple Internal Rates of Return. The Engineering Economist, 2003, 48 (1), 31-51.
[10] Hazen, G. An extension of the internal rate of return to stochastic cash flows. Management Science, 2009, 55(6), 1030–1034.
[11] Liu, W .Y. Zhou, R. X., & Wang, S. H. (2014). The model of cash evaluation based on the Svensson model. Systems Engineering Theory & Practice, 34(1): 61-66.
[12] Moradi, B., Shakeri, H ., & Namdar Zangeneh, S. Solving the Paradox of Multiple IRR's in Engineering Economic Problems by Choosing an Optimal α-cut . International Journal of Industrial Engineering & Production Research, 2012, 1 (23), 45 -52.
[13].Weber, T.A. On the (Non-) Equivalence of IRR and NPV. Switzerland: Economics and Strategy, Ecole Polytechnique Fédérale de Lausanne, 2013.
[14] Burden, R. L., Faires, J. D. Numerical Analysis. Canada: Richard Stratton, 2011.