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Analytical Solution of Unsteady MHD Blood Flow and Heat Transfer through Parallel Plates when Lower Plate Stretches Exponentially

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ABSTRACT

This article looks into the mathematical solution of unsteady, viscous, incompressible, electrically conducting blood flow and heat transfer through a parallel plate channel when the lower plate is stretching. A uniform magnetic field has been applied perpendicularly. Similarity transformations of exponential form have been used to convert the governing higher order non-linear partial differential equations of motion to ordinary differential form. The resulting equations have been solved analytically. The effects of flow parameters namely Grashof number G_r, Prandtl number P_r, heat source parameter N, Hartmann number M and decay parameter λ have been observed on velocity and heat functions. The mathematical analysis and graphical presentations of velocity and temperature distributions are very simple and help to understand physiological fluid dynamics.

AMS Subject Classification: **76M20**

KEY WORDS: Blood flow, Similarity solution, Hartmann number, Grashof number, Prandtl number, Decay parameter, stretching channel,

I INTRODUCTION

Several authors have been working to explain the behavior of blood flow in the presence of external magnetic field. As it is reported by many authors like Eldesoky [1] that blood is an electrically conducting fluid and the Lorentz force acts on opposes the motion of blood and thereby flow of blood is impeded, so that the external magnetic field can be used in the treatment of some kinds of diseases like cardiovascular diseases and in the diseases with accelerated blood circulation such as hemorrhages and hypertension.

Singh et al. [1] and Zamir et al. [3] analyzed the analytical solution of two- dimensional model of blood flow with variable viscosity through an indented artery due to LDL effect in the presence of magnetic field. The effect of heat source on MHD blood flow through bifurcated arteries is discussed by Prakash et al. [4]. Tzirtzilakis [5] studied the mathematical model for blood flow in magnetic field. Sanyal et al. [6] investigated the effect of magnetic field on pulsatile blood flow through an inclined circular tube with periodic body acceleration. The effects of magnetic field and hematocrit on blood flow in an artery with multiple mild stenosis is studied by Verma et al. [7]. The arterial MHD pulsatile flow of blood under periodic body acceleration is presented by Das and Saha [8]. Jain et al. [9] studied the mathematical analysis of MHD flow of blood in very narrow capillaries. Dulal and Ananda [10] investigated the pulsatile motion of blood through an axisymmetric artery in presence of magnetic field. Adhikary et al. [11] discussed the unsteady two-dimensional hydromagnetic flow and heat transfer for viscous fluid. The Influence of blood flow in large vessels on the temperature distribution in hyperthermia is presented by Lagendijk [12]. The blood flow in small tube and heat transfer was modeled by Wang [13].

The present study focuses on the mathematical formulation of MHD blood flow in a stretching parallel plate channel to extend the work of Eldesoky [1]. We considered this problem involving stretching phenomenon and introduced new similarity functions. The resulting equations have been solved analytically and the results have been obtained in a simple way using MS Exel. The curves for velocity and temperature distribution have been plotted and presented for some representative values of the physical parameters involved in this work.

II MATHEMATICAL ANALYSIS

Consider flow of blood between two parallel plates, the lower one is a stretching sheet at $y=-\sqrt{\frac{\upsilon}{b}}$ and upper one is fixed at y= $\sqrt{\frac{\upsilon}{b}}$. Blood is to be considered as electrically conducting, incompressible,

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homogenous and Newtonian fluid. A magnetic field of strength B_0 is applied perpendicular to the flow of blood. The velocity components are u and v in x and y directions respectively at time t. The equations of motion and energy equation are given as:

$$\partial u/\partial t+\frac{1}{\rho}dP/dx=\frac{\mu}{\rho}\partial^2 u/\partial y^2-\sigma B_0^2 u/\rho+g\beta T \tag{1}$$

$$\partial u/\partial x+\partial v/\partial y=0 \tag{2}$$

$$\partial T/\partial t=\frac{K}{\rho C_p}\partial^2 T/\partial y^2+\frac{Q'}{\rho c_p}T \tag{3}$$

where B_0 is strength of magnetic field, P is the pressure, g is acceleration due to gravity and β is the volumetric coefficient of thermal expansion, ρ is fluid density, μ is the coefficient of viscosity, υ is kinematic viscosity, σ is electrical charge density, T is the temperature inside the boundary layer, K is the thermal conductivity and c_p is the specific heat at constant pressure.

The associated boundary conditions are:

$$\begin{aligned} &u=bxe^{-\lambda^2 t},\ v=0,\ T=e^{-\lambda^2 t}\\ &\text{as } y\to-\sqrt{\frac{\upsilon}{b}} \text{ and } u\to 0,\ T\to 0 \text{ as } y\to\sqrt{\frac{\upsilon}{b}} \end{aligned} \tag{4}$$

where b is stretching constant.

Using similarity transformations

$$u=bxe^{-\lambda^2 t}f'(\eta), \tag{5}$$

$$v=-\sqrt{b\upsilon}\,f(\eta)e^{-\lambda^2 t} \tag{6}$$

$$\theta(\eta)T_0=T,\ \eta=\sqrt{\frac{b}{\upsilon}}y \tag{7}$$

The equations (1) and (3) are respectively transformed to ordinary differential form,

$$f'''+(\frac{\lambda^2}{c}-M^2)f'+G_r\theta-h=0 \tag{8}$$

$$\frac{Kb}{\rho\upsilon c_p}\theta''+(\frac{Q'}{\rho c_p}+\lambda^2)\theta=0 \tag{9}$$

$$\theta''+\frac{1}{b}(\frac{Q'\upsilon}{K}+\lambda^2\frac{\rho\upsilon c_p}{K})\theta=0 \tag{10}$$

Where M is magnetic parameter, G_r is the Grashof number and $h = \frac{dP}{dx} / b^2 x\rho e^{-\lambda^2 t}$ is pressure gradient. The equation (10) is simplified as:

$$\theta'' + \frac{1}{b} R\theta = 0 \tag{11}$$

Where, $R = N + P_r\lambda^2\upsilon$ and $N = \frac{Q'\upsilon}{K}$ is heat source parameter, $P_r = \rho c_p / K$ is the Prandtl number. The boundary conditions in equation (4) take the form as:

$$f'(\eta) = 1,\ f(\eta) = 0, \theta(\eta) = 1 \text{ as } \eta \to -1. \tag{12}$$

$$f'(\eta) = 0, \text{ and } \theta(\eta) = 0 \text{ as } \eta \to 1 \tag{13}$$

The second order, homogeneous ordinary differential equation (11) subject to the above boundary conditions is readily solved for $\theta(\eta)$.

$$\theta(\eta) = \frac{1}{2\cos\sqrt{\frac{R}{b}}} \cos\left(\sqrt{\frac{R}{b}}\,\eta\right) - \frac{1}{2\sin\sqrt{\frac{R}{b}}} \sin\left(\sqrt{\frac{R}{b}}\,\eta\right) \tag{14}$$

Temperature distribution T is

$$T = \left[\frac{1}{2\cos\sqrt{\frac{R}{b}}} \cos\left(\sqrt{\frac{R}{b}}\,\eta\right) - \frac{1}{2\sin\sqrt{\frac{R}{b}}} \sin\left(\sqrt{\frac{R}{b}}\,\eta\right)\right] e^{-\lambda^2 t} \tag{15}$$

The complimentary solution of equation (8) is

$$f = Ae^{\sqrt{Q}\eta} + Be^{-\sqrt{Q}\eta} + C, \text{ where } Q = M^2 - \frac{\lambda^2}{b} \tag{16}$$

$$f = Ae^{\sqrt{Q}\eta} + Be^{-\sqrt{Q}\eta} + C \tag{17}$$

Where as the particular solution of equation (8) is as follows:

$$f_p = \frac{1}{D(D^2 - Q)} \left[h - Gr \left[\begin{array}{l} \frac{1}{2\cos\sqrt{\frac{R}{b}}} \cos\left(\sqrt{\frac{R}{b}}\,\eta\right) \\ -\frac{1}{2\sin\sqrt{\frac{R}{b}}} \sin\left(\sqrt{\frac{R}{b}}\,\eta\right) \end{array} \right] \right] \tag{18}$$

$$\begin{aligned} f_p = & -\frac{h}{Q}\eta + G_r \frac{1}{2\cos\sqrt{\frac{R}{b}}} \frac{1}{(Q + \frac{R}{b})\sqrt{\frac{R}{b}}} \sin\left(\sqrt{\frac{R}{b}}\,\eta\right) \\ & + G_r \frac{1}{2\sin\sqrt{\frac{R}{b}}} \frac{1}{(Q + \frac{R}{b})\sqrt{\frac{R}{b}}} \cos\left(\sqrt{\frac{R}{b}}\,\eta\right), \end{aligned} \tag{19}$$

then the general solution of equation (8):

$$f=\begin{bmatrix} Ae^{\sqrt{Q}\eta}+Be^{-\sqrt{Q}\eta}+C-\frac{h}{Q}\eta \\ +G_r\,\frac{1}{2\cos\sqrt{\frac{R}{c}}}\,\frac{1}{(Q+\frac{R}{b})\sqrt{\frac{R}{b}}}\sin\left(\sqrt{\frac{R}{b}}\,\eta\right)+ \\ G_r\,\frac{1}{2\sin\sqrt{\frac{R}{b}}}\,\frac{1}{(Q+\frac{R}{b})\sqrt{\frac{R}{b}}}\cos\left(\sqrt{\frac{R}{b}}\,\eta\right) \end{bmatrix}. \tag{20}$$

Using boundary conditions (12) and (13), the values of unknown constant A, B and C are found and the normal velocity and horizontal velocity components are thus obtained as below:

$$v=\begin{bmatrix} Ae^{\sqrt{Q}\eta}+Be^{-\sqrt{Q}\eta}+C-\frac{h}{Q}\eta \\ +G_r\,\frac{1}{2\cos\sqrt{\frac{R}{b}}}\,\frac{1}{(Q+\frac{R}{b})\sqrt{\frac{R}{b}}}\sin\left(\sqrt{\frac{R}{b}}\,\eta\right)+ \\ G_r\,\frac{1}{2\sin\sqrt{\frac{R}{b}}}\,\frac{1}{(Q+\frac{R}{b})\sqrt{\frac{R}{b}}}\cos\left(\sqrt{\frac{R}{b}}\,\eta\right) \end{bmatrix}e^{-\lambda^2 t} \tag{21}$$

$$u=\begin{bmatrix} \left((1-\frac{G_r}{R+Q})+\frac{h}{Q}(1-e^{2\sqrt{Q}}\ )\right)\frac{e^{\sqrt{Q}(1+\eta)}}{1-e^{4\sqrt{Q}}} \\ -\left((1-\frac{G_r}{R+Q})+\frac{h}{Q}(1-e^{-2\sqrt{Q}}\ )\right)\frac{e^{\sqrt{Q}(3-\eta)}}{1-e^{4\sqrt{Q}}}-\frac{h}{Q}+ \\ G_r\,\frac{1}{2(Q+\frac{R}{b})}\left(\frac{1}{\cos\sqrt{\frac{R}{b}}}\cos\left(\sqrt{\frac{R}{b}}\,\eta\right)+\frac{1}{\sin\sqrt{\frac{R}{b}}}\sin\left(\sqrt{\frac{R}{b}}\,\eta\right)\right) \end{bmatrix}e^{-\lambda^2 t} \tag{22}$$

III RESULTS AND DISCUSSION

We examined the effects of physical parameters involved in this problem for velocity and temperature distributions. Results have been computed for several values of the parameters namely heat source parameter , Prandtl number, decay parameter, Magnetic parameter and the Grashof number. The values of other parameters are fixed arbitrarily for computational purpose, generally as t = 1, λ = 0.5, $\upsilon=0.5,$ $h=0.5,$ $P_r=1,$ $G_r=4.5,$ $N=1$ and $M^2=1$. Three cases have been considered for stretching velocity parameter b namely $b<1,$ $b=1$ and $b>1.$ Results are presented in graphical form. Figure 1 to figure 3, respectively demonstrate temperature distribution for three cases of b to observe the effect of heat source parameter N . It is found that temperature field increases with increase in the values of N and it attains the maximum value at $\eta=0$ and stronger effect of the physical parameters is observed for lesser values of $b.$ Figure 4 and figure 5 presents the distribution of the temperature for different values of λ . The temperature distribution decreases against the decay parameter λ . Figure 6 to figure 8 show respectively that the effect of Grashof number G_r, Prandtl number P_r and heat source parameter $N,$ on the axial velocity . It is noticed that velocity increases with increase in values of these parameters. The decay parameter λ has decreasing effect on the velocity as shown in figure 9.

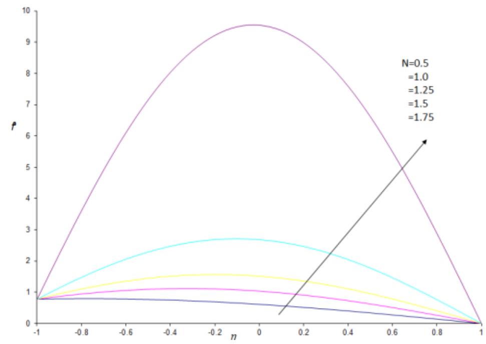

Fig. 1 Graph of temperature distribution for different values of N, when b = 0.8.

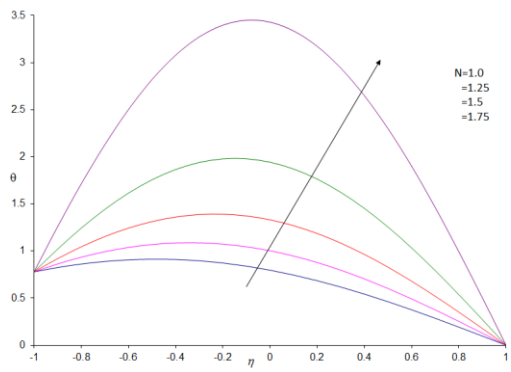

Fig. 2: Graph of temperature distribution for different values of N, when b = 1.

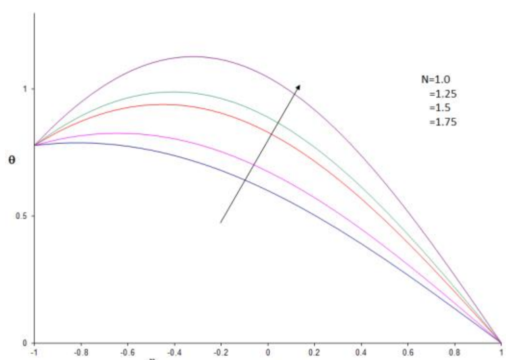

Fig. 3: Graph of temperature distribution for different values of N when b = 1.5.

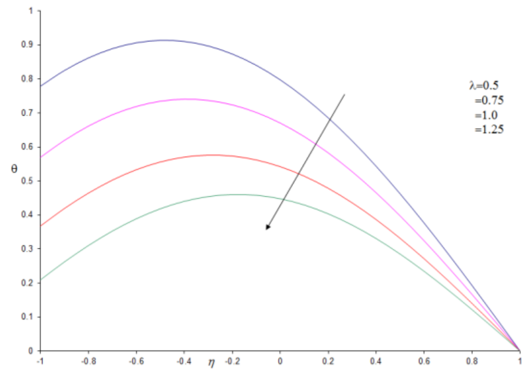

Fig. 4: Graph of temperature distribution for different values of λ when b = 1

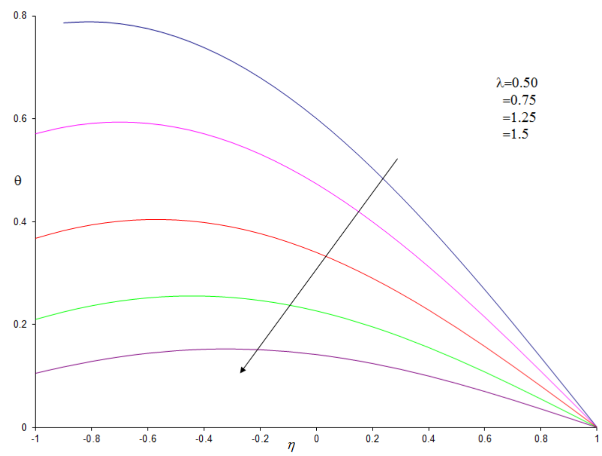

Fig. 5: Graph of temperature distribution for different values of λ when b = 1.5

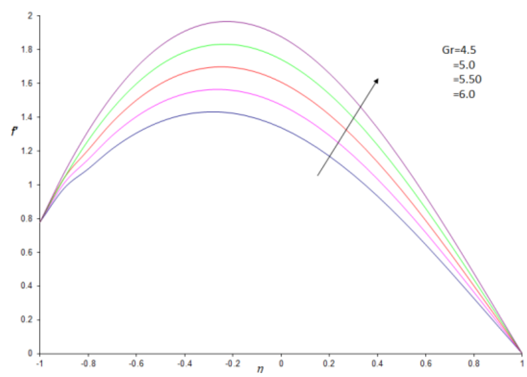

Fig.6:Graph of axial velocity for different values of G_r when $b = 1.5$

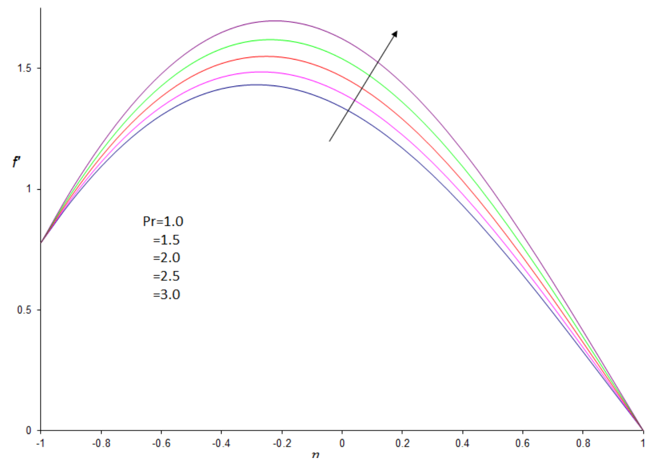

Fig.7: Graph of axial velocity for different values of $\Pr$ when $b=1$.

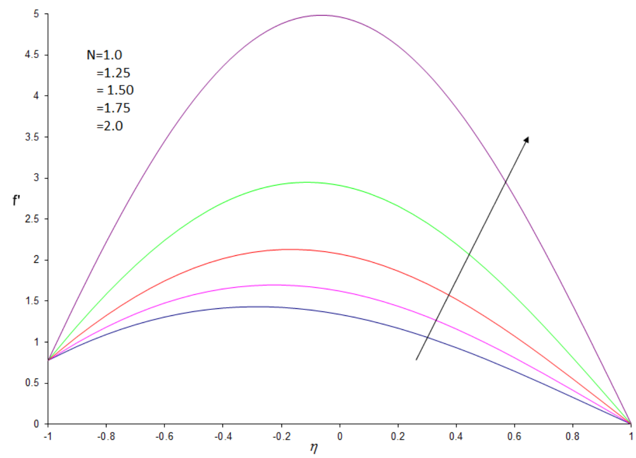

Fig.8: Graph of axial velocity for different values of N when $b=1.0$.

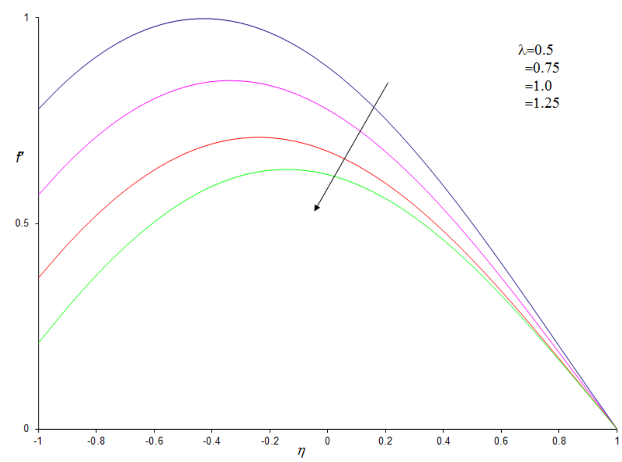

Fig.9:Graph of axial velocity for different values of λ when $b=1.0$.

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