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Gravity Driven Flow of an Unsteady Second Order Fluid Between two Parallel and Vertical Oscillating Plates.

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ABSTRACT

The flow of magnetohydrodynamic unsteady second grade fluid problem is examined between two vertical and oscillating parallel plates. The parallel plates are oscillating and the fluid drain down due to gravity. A uniform magnetic field is applied perpendicularly to the plates in the presence of temperature field. The model differential equations are solved analytically by using Optimal Homotopy Asymptotic Method (OHAM). The effect of various physical parameters are studied and discussed.

KEYWORDS: Unsteady Second Grade Fluid, Parallel Vertical Plates, MHD, Temperature Field, Optimal Homotopy Asymptotic Method (OHAM).

INTRODUCTION

The study of non-Newtonian fluids has received considerable attention in the past due to its application and a lot of importance in over daily life. The usage of these fluids can be seen in several areas including plastic manufacture, performance of lubricants, processing of food, movement of biological fluids, wire and fiber coating, paper production, transpiration cooling, gaseous diffusion, drilling muds' and heat pipes. Considerable efforts have been made to study non-Newtonian fluids through analytical and numerical treatment. Taza Gul et al [1, 2] have discussed the unsteady thin film flow of a second grade fluid over a vertical oscillating belt. Van Rossum et al [3] has study has study a thin film flow for lifting and drainage problem. Raghu Raman et al [4] discussed non-Newtonian fluids passing over vertical flat plates. Hoyt et al [5] have shown experimentally that the fluids containing minute polymeric additives can reduce skin friction 25-30%. This reduction was explained with the theory of micro polar fluids. Zueco et al [6] has investigated an unsteady free convection flow of a micro polar fluid between two parallel porous vertical walls. S zeriet et al [7] have examine the flow of a third grade fluid between heated parallel plates caused by external pressure gradient and obtained similarity solutions of the energy equation, numerically. Siddiqui et al [8] has discussed a thin film flow of a third grade fluid down an inclined plane. Fetecau et al [9, 11] investigated the second grade fluid between the oscillating plates for unsteady unidirectional flow. M. Hussain, et al [12] as examine oscillatory flows of second grade fluid in a porous space. R.A. Shah et al [13] observed OHAM solution of unsteady second grade fluid in wire coating analysis. Application of optimal homotopy asymptotic method for solving non-linear equations arising in heat transfer was discussed V. Marinca et al in [14]. Aamer Khan et al [15] have observed OHAM Solution of Thin Film Non-Newtonian Fluid on a Porous and Lubricating Vertical Belt. S. Nadeem et al [16] has discussed a thin film flow of a second grade fluid over a stretching/shrinking sheet with variable temperature-dependent viscosity. M. Qasim, et al [17] has examined heat transfer and mass diffusion in nano fluids with convective boundary conditions. The flow through a channel due to transversely oscillating walls was also found in [18, 20]. For the solution of nonlinear differential equations different method have been used like Adomian decomposition method (ADM) and Optimal Homotopy Asymptotic Method (OHAM) [21-32].

The purpose of the present attempt is to analyze the flow between two oscillating and parallel plates. We have used the Optimal Homotopy Asymptotic Method (OHAM) for the solution of differential equations.

Basic equations

The MHD equations which governs the unsteady incompressible flow of second grade fluid are

$$\nabla.\mathbf{v} = 0, \tag{1}$$

$$\rho\frac{D\mathbf{v}}{Dt} = \nabla.\mathbf{T} - \rho g + \mathbf{J}\times\mathbf{B}, \tag{2}$$

$$\rho c_p\frac{D\Theta}{Dt} = k\nabla^2\Theta + tr(T.L), \tag{3}$$

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Where $\rho$is the fluid density,$\mathbf{v}$is the velocity vector of the fluid,gis gravity,$\mathbf{J} = \sigma(\mathbf{E} + \mathbf{v} \times \mathbf{B})$ is the current density, σ is the electrical conductivity,$\mathbf{B} = (0, \mathrm{B}_0, 0)$is uniform magnatic field, Θ is temprature, $k$is the thermal conductivity, $c_p$is specific heat, $\mathbf{T}$is the caushy stress tensor and the material time derivative$\frac{D}{Dt}$ define as

$$\frac{D(*)}{Dt} = \frac{\partial(*)}{\partial t} + (\mathbf{v}.\boldsymbol{\nabla}) *, \tag{4}$$

$$\mathbf{J} \times \mathbf{B} = \left[0, -\sigma \mathrm{B}_0{}^2 \mathbf{v}, 0\right], \tag{5}$$

The cauchy stress tensor $\mathbf{T}$ for second grade fluid is given by

$$\mathbf{T} = -p\mathbf{I} + \mu \mathbf{A}_1 + \alpha_1 \mathbf{A_2} + \alpha_2 \mathbf{A}_1^2, \tag{6}$$

Here α_1 and α_2 are the material constants,$\mathbf{A}_1 and$ $\mathbf{A}_2$are the Rivlin-Ericksen tensor given by

$$\mathbf{A}_1 = \mathbf{I}, \quad \mathbf{A}_1 = \boldsymbol{L} + \boldsymbol{L}^{\boldsymbol{T}}, \quad \boldsymbol{L} = grad\mathbf{v}, \tag{7}$$

$$\mathbf{A}_n = \frac{D\mathbf{A}_{n-1}}{Dt} + \mathbf{A}_{n-1}\boldsymbol{L} + \boldsymbol{L}^{\boldsymbol{T}}\mathbf{A}_{n-1}, \quad n = 0,1,2, \ldots, \tag{8}$$

Formulation of problem

Consider two vertical parallel and oscillating plates. The fluid is flowing in between these two plates due to gravity. A uniform Magnetic field is applied perpendicularly to the plates. $2h$is the total thickness of fluid between two plates. The temperature field is also applied to the flow field. The coordinate system is chosen as in which the x-axis is taken perpendicular and y-axis parallel to the plates.

The velocity and temperature fields are

$$\mathbf{V} = (0, v(x,t), 0) \text{and} \Theta = \Theta(x) \tag{9}$$

Boundary conditions are:

$$\mathbf{v}(h,t) = V cos\omega t \ , \qquad \mathbf{v}(-h,t) = V cos\omega t \tag{10}$$

$$\Theta(h,t) = \Theta_0 \ , \quad \Theta(-h,t) = \Theta_1, \tag{11}$$

$\omega$is used as frequency of the oscillating belt.

The continuity equation (1) is satisfied identically, the momentum and energy equations reduces to the form

$$\rho \frac{\partial \mathbf{v}}{\partial t} = \frac{\partial}{\partial x} T_{xy} - \rho \mathrm{g} - \sigma B_0{}^2 \mathbf{v}, \tag{12}$$

The energy equation

$$\rho c_p \left(\frac{\partial \Theta}{\partial t}\right) = k \left(\frac{\partial^2 \Theta}{\partial x^2}\right) + \mu \left(\frac{\partial \mathbf{v}}{\partial x}\right)^2 + \alpha_1 \frac{\partial \mathbf{v}}{\partial x}\frac{\partial}{\partial t}\left(\frac{\partial \mathbf{v}}{\partial x}\right), \tag{13}$$

The components of the Cauchy stress tensor $\mathbf{T}$ as

$$T_{xx} = -p + 2\alpha_1 \left(\frac{\partial \mathbf{v}}{\partial x}\right)^2, \tag{14}$$

$$T_{xy} = \mu \frac{\partial \mathbf{v}}{\partial x} + \alpha_1 \frac{\partial}{\partial t}\left(\frac{\partial \mathbf{v}}{\partial x}\right) = T_{yx}, \tag{15}$$

Putting equation (14) in equation (12) and (13)we get

$$\rho \frac{\partial \mathbf{v}}{\partial t} = \mu \frac{\partial^2 \mathbf{v}}{\partial x^2} + \alpha_1 \frac{\partial}{\partial t}\left(\frac{\partial^2 \mathbf{v}}{\partial x^2}\right) - \rho \mathrm{g} - \sigma B_0 \mathbf{v}, \tag{16}$$

The energy equation is

$$\rho c_p \left(\frac{\partial \Theta}{\partial t}\right) = k \left(\frac{\partial^2 \Theta}{\partial x^2}\right) + \mu \left(\frac{\partial \mathbf{v}}{\partial x}\right)^2 + \alpha_1 \frac{\partial \mathbf{v}}{\partial x}\frac{\partial}{\partial t}\left(\frac{\partial \mathbf{v}}{\partial x}\right), \tag{17}$$

Introducing the following non-dimensional physical quantities

$$\bar{v} = \frac{\mathbf{v}}{V}, \bar{x} = \frac{x}{h}, \bar{t} = \frac{t\mu}{\delta^2 \rho}, \alpha = \frac{\alpha_1}{\rho \delta^2}, M = \frac{\delta^2 \sigma \mathrm{B}_0}{\mu V}, B_r = \frac{V^2 \mu}{k(\Theta_1 - \Theta_0)}, P_r = \frac{c_p \mu}{k}, \ \bar{\Theta} = \frac{\Theta - \Theta_0}{\Theta_1 - \Theta_0}, \tag{18}$$

Where M is the magnetic parameter, α is the non-dimensional variable, B_r is the Brinkman number and $P_r$is the Prandtl number.

Using the above dimensionless variables in equation (18) and dropping bars we obtain

$$\frac{\partial v}{\partial t} = \frac{\partial^2 v}{\partial x^2} + \alpha \frac{\partial}{\partial t}\left(\frac{\partial^2 v}{\partial x^2}\right) - S_t - Mv, \tag{19}$$

$$P_r \left(\frac{\partial \Theta}{\partial t}\right) = \frac{\partial^2 \Theta}{\partial x^2} + B_r \left(\frac{\partial v}{\partial x}\right)^2 + \alpha \left(\frac{\partial v}{\partial x}\right)\left(\frac{\partial^2 v}{\partial t \partial x}\right), \tag{20}$$

And the boundary conditions are

$$v(1,t) = cos\omega t \ , \qquad v(-1,t) = cos\omega t \ , \tag{21}$$

$$\Theta(0,t) = 0 \ , \quad \Theta(1,t) = 1, \tag{22}$$

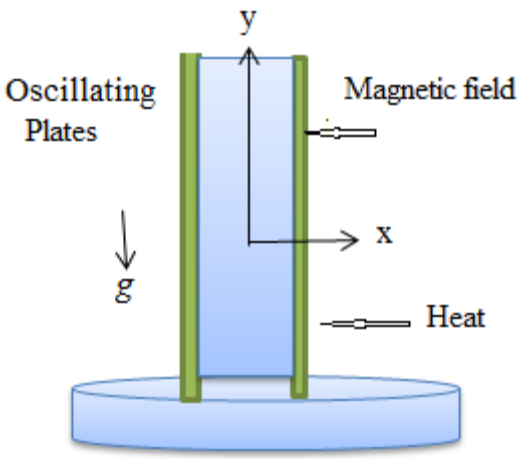

Figure: 1. Geometry of the problem

Optimal Homotopy Asymptotic Method (OHAM)
Basic Idea of OHAM

Here we discuss the concept of OHAM; we consider the boundary value problem as consider:

$$L(u(x))+N(u(x))+g(x)=0, \qquad B\left(u,\frac{\partial u}{\partial x}\right)=0, \tag{23}$$

Where L is a linear operator in the differential equation, N is a non-linear term, $x \in R$ is an independent variable, B is a boundary operator and g is a source term. We construct a set of equation for OHAM.

$$[1-p][L\varphi(x,p)+g(x)]=H(p)[L\varphi(x,p)+g(x)+N\varphi(x,p)], \quad B\left(\varphi(x,p),\frac{\partial\varphi(x,p)}{\partial x}\right)=0 \tag{24}$$

$p \in [0,1]$ is an embedding parameter, H(p), is a non-zero auxiliary function for $p \neq 0$ and $H(0)=0$. $\varphi(x,p)$ isan unknown function. Obviously, when $p=0 \quad and \quad p=1$, it holds that:

$$\varphi(x,0)=u_0(x), \quad \varphi(x,1)=u(x), \tag{25}$$

When p varies from $0 \quad to \quad 1$ then $\varphi(x,p)$ also varies from $u_0(x) \quad to \quad u(x)$. Where the zero component solution $u_0(x)$ is obtained from equation (25) when $p=0$,

$$L(u_0(x))+g(x)=0, \qquad B\left(u_0(x),\frac{\partial u_0(x)}{\partial x}\right)=0, \tag{26}$$

Auxiliary function $H(p)$is choosing as

$$H(p)=pc_1+p^2c_2+\cdots, \tag{27}$$

c_1, c_2 are auxiliary constants.

Marinca uses a special procedure to expand $\varphi(x,p)$ with respect to p by using Taylor Series.

$$\varphi(x,p,c_i)=u_0(x)+\sum_{k\geq 1}u_k(x,c_i)p^k, \qquad i=1,2,\cdots \tag{28}$$

Inserting Eq. (28) into Eq. (24), collecting the same powers of p and equating each coefficient of p. The zero order problems given in equation (26) and the first order and second order given in equations (29, 30).

$$L(u_1(x))+g(x)=c_1N_0(u_0(x)), \qquad B\left(u_1(x),\frac{\partial u_1(x)}{\partial x}\right)=0, \tag{29}$$

$$L(u_2(x))-L(u_1(x))=c_2N_0(u_0(x))+c_1[L(u_1(x))+N_1(u_0(x),u_1(x))], \quad B\left(u_2(x),\frac{\partial u_2(x)}{\partial x}\right)=0 \tag{30}$$

The general governing equations for $u_k(x)$are given by

$$L(u_k(x)) - L(u_{k-1}(x)) = c_k N_0(u_0(x)) + \sum_{i=1}^{k-1} c_i \left[L(u_{k-i}(x)) + N_{k-1}(u_0(x), u_1(x) \cdots u_{k-i}(x)) \right]$$

$$, k = 2,3,\cdots, \qquad B\left(u_k(x), \frac{\partial u_k(x)}{\partial x} \right) = 0, \tag{31}$$

Here $N_m(u_0(x), u_1(x) \cdots u_{m-1}(x))$ is the coefficient of p^m, in the expansion of $N\varphi(x, p)$.

$$N(\varphi(x, p, c_i)) = N_0(u_0(x)) + \sum_{m=1}^{\infty} N_m(u_0(x), u_1(x) \cdots u_m(x)) p^m \tag{32}$$

The convergence of the Series in equation (28) depend upon the auxiliary constants $c_1, c_2, \cdots$

If it converges at $p = 1$, then the mth order approximation u is

$$u(x, c_1, c_2, \cdots c_m) = u_0(x) + \sum_{i=1}^{m} u(x, c_1, c_2, \cdots c_i). \tag{33}$$

Inserting Eq. (33) into Eq. (23), the residual is obtained as:

$$R(x, c_i) = L(u(x, c_i)) + g(x) + N(u(x, c_i)), \quad i = 1,2 \cdots m \tag{34}$$

Numerous methods like Ritz Method, Method of Least Squares, Galerkin's Method and Collocation Method are used to find the optimal values of $c_{i,}$ $i = 1,2,3,4,\cdots$ We apply the Method of Least Squares in our problem as given below:

$$J(c_1, c_2, \cdots c_n) = \int_a^b (x, c_1, c_2, \cdots c_m) dx, \tag{35}$$

Where a and b are the constant values taking from domain of the problem.

Auxiliary constants $(c_1, c_2, \cdots c_n)$ can be identified from:

$$\frac{\partial J}{\partial c_1} = \frac{\partial J}{\partial c_2} = \cdots = 0. \tag{36}$$

The OHAM Solution

In this section we apply OHAM method on equation (19, 20) with boundary condition in equation (21, 22) and study zero, first and second component problems

Zero and first component problem for velocity and temperature profiles are

$$p^0: \qquad \frac{d^2 v_0}{dx^2} = S_t, \tag{37}$$

$$\frac{d^2 \Theta_0}{dx^2} = 0, \tag{38}$$

$$p^1: \quad \frac{d^2 v_1}{dx^2} = S_t + c_1 S_t - M c_1 v_0 - c_1 \frac{dv_0}{dt} + \frac{d^2 v_0}{dx^2} + c_1 \frac{d^2 v_0}{dx^2} + \alpha c_1 \frac{d^2}{dx^2}\left(\frac{dv_0}{dt}\right), \tag{39}$$

$$\frac{d^2 \Theta_1}{dx^2} = -P_r c_3 \frac{d\Theta_0}{dt} + B_r c_3 (\frac{dv_0}{dx})^2 + \alpha B_r c_3 \frac{dv_0}{dx} \frac{d}{dx}\left(\frac{dv_0}{dt}\right) + \frac{d^2 \Theta_0}{dx^2} + c_3 \frac{d^2 \Theta_0}{dx^2}, \tag{40}$$

$$p^2: \quad \frac{d^2 v_2}{dx^2} = c_2 S_t - M c_2 v_0 - c_2 \frac{dv_0}{dt} - M c_1 v_1 - c_1 \frac{dv_1}{dt} + c_2 \frac{d^2 v_0}{dx^2} + \alpha c_2 \frac{d^2}{dx^2}\left(\frac{dv_0}{dt}\right) + \frac{d^2 v_1}{dx^2} + c_1 \frac{d^2 v_1}{dx^2} + \alpha c_1 \frac{d^2}{dx^2}\left(\frac{dv_1}{dt}\right), \tag{41}$$

$$\frac{d^2 \Theta_2}{dx^2} = -P_r c_4 \frac{d\Theta_0}{dt} - P_r c_3 \frac{d\Theta_1}{dt} + B_r c_4 (\frac{dv_0}{dx})^2 + \alpha B_r c_4 \frac{dv_0}{dx} \frac{d}{dx}\left(\frac{dv_0}{dt}\right) + 2 B_r c_3 \frac{dv_0}{dx} \frac{dv_1}{dx} + \alpha B_r c_3 \frac{d}{dx}\left(\frac{dv_0}{dt}\right) \frac{dv_1}{dx} + \alpha B_r c_3 \frac{dv_0}{dx} \frac{d}{dx}\left(\frac{dv_1}{dt}\right) + c_4 \frac{d^2 \Theta_0}{dx^2} + \frac{d^2 \Theta_1}{dx^2} + c_3 \frac{d^2 \Theta_1}{dx^2}, \tag{42}$$

Solutions of zero, first and second components problem using boundary conditions from equation (21-22) in equations (37-42) are given by:

$$v_0[x, t] = \mathrm{Cos}[t\omega] - \frac{1}{2}(-1 + x^2) S_t \tag{43}$$

$$\Theta_0[x, t] = \frac{1+x}{2} \tag{44}$$

$$v_1[x, t] = \frac{1}{24}(12 M \mathrm{Cos}[t\omega] c_1 - 12 M x^2 \mathrm{Cos}[t\omega] c_1 - 12\omega \mathrm{Sin}[t\omega] c_1 + 12 x^2 \omega \mathrm{Sin}[t\omega] c_1 + 5 M c_1 S_t - 6 M x^2 c_1 S_t + M x^4 c_1 S_t), \tag{45}$$

$$\Theta_1[x, t] = \frac{S_t^2 B_r}{12}(-c_3 + x^4 c_3), \tag{46}$$

$$v_2[x, t] = v_1^* + v_2^* + v_3^* + v_4^* + v_5^*, \tag{47}$$

where

$$v_1^* = \frac{M c_1}{720}(180 + 60x - 180x^2 - 60x^3 - 150\Omega + 180x^2\Omega - 30x^4\Omega + 360\mathrm{Cos}[t\omega] - 360x^2 \mathrm{Cos}[t\omega]),$$

$$v_2^* = \frac{c_1}{720}(360\omega\mathrm{Sin}[t\omega](x^2-1) + Mc_1(180 + 75 + 60x + 7\mathrm{M}x - 180x^2 - 90\mathrm{M}x^2 - 60Mx^3c_1^2 - 10\mathrm{M}x^3 + 15\mathrm{M}x^4 + 3\mathrm{M}x^5)),$$

$$v_3^* = \frac{Mc_1^2}{720}(\mathrm{V}x^6\Omega - 150\Omega - 61\mathrm{M}\Omega + 180x^2\Omega + 75\mathrm{M}x^2\Omega - 30x^4\Omega - 15\mathrm{M}x^4\Omega + 360\mathrm{Cos}[t\omega] + 150\mathrm{M}\mathrm{Cos}[t\omega] - 360Mx^2\mathrm{Cos}[t\omega] - 180\mathrm{M}x^2\mathrm{Cos}[t\omega]),$$

$$v_4^* = \frac{c_1^2}{720}(30M^2x^4\mathrm{Cos}[t\omega] - 150\omega^2\mathrm{Cos}[t\omega] + 180x^2\omega^2\mathrm{Cos}[t\omega] - 30x^4\omega^2\mathrm{Cos}[t\omega] - 120x\alpha\omega^2\mathrm{Cos}[t\omega] + 120x^3\alpha\omega^2\mathrm{Cos}[t\omega] - 360\omega\mathrm{Sin}[t\omega] - 300M\omega\mathrm{Sin}[t\omega] + 360x^2\omega\mathrm{Sin}[t\omega] + 360Mx^2\omega\mathrm{Sin}[t\omega] - 60Mx^4\omega\mathrm{Sin}[t\omega] - 120Mx\alpha\omega\mathrm{Sin}[t\omega]),$$

$$v_5^* = \frac{1}{720}120Mx^3\alpha\omega\mathrm{Sin}[t\omega]c_1^2 + 180Mc_2 + 60Mxc_2 - 180Mx^2c_2 - 60Mx^3c_2 - 720\Omega c_2 - 150M\Omega c_2 + 720x^2\Omega c_2 + 180Mx^2\Omega c_2 - 30Mx^4\Omega c_2 + 360M\mathrm{Cos}[t\omega]c_2 - 360Mx^2\mathrm{Cos}[t\omega]c_2 - 360\omega\mathrm{Sin}[t\omega]c_2 + 360x^2\omega\mathrm{Sin}[t\omega]c_2).$$

$$\Theta_2[x,t] = \frac{\mathrm{S}_t B_r}{180}(-30M\mathrm{Cos}[t\omega]c_1c_3 + 30Mx^4\mathrm{Cos}[t\omega]c_1c_3 + 15\alpha\omega^2\mathrm{Cos}[t\omega]c_1c_3 - 15x^4\alpha\omega^2\mathrm{Cos}[t\omega]c_1c_3 + 30\omega\mathrm{Sin}[t\omega]c_1c_3 - 30x^4\omega\mathrm{Sin}[t\omega]c_1c_3 + 15M\alpha\omega\mathrm{Sin}[t\omega]c_1c_3 - 15Mx^4\alpha\omega\mathrm{Sin}[t\omega]c_1c_3 - 15c_3\mathrm{S}_t + 15x^4c_3\mathrm{S}_t - 13Mc_1c_3\mathrm{S}_t + 15Mx^4c_1c_3\mathrm{S}_t - 2Mx^6c_1c_3\mathrm{S}_t - 15c_3^2\mathrm{S}_t + 15x^4c_3^2\mathrm{S}_t - 15c_4\mathrm{S}_t + 15x^4c_4\mathrm{S}_t \quad . \qquad (48)$$

Now the series solution of velocity distribution up to second component is:

$$v(x,t) = v_0(x,t) + v_1(x,t) + v_2(x,t), \qquad (49)$$

Putting equations (43, 45,47) in equation (49), we obtain the series solution.

Now the series solution of temperature distribution up to second term is;

$$\Theta[x,t] = \Theta_0[x,t] + \Theta_1[x,t] + \Theta_2[x,t] \qquad (50)$$

Putting equations (44, 46,48) in equation (50), we have

$$\Theta[x,t] = \frac{1+x}{2} + \frac{S_t^2B_r}{12}(-c_3 + x^4c_3) + \frac{\mathrm{S}_tB_r}{180}(-30M\mathrm{Cos}[t\omega]c_1c_3 + 30Mx^4\mathrm{Cos}[t\omega]c_1c_3 + 15\alpha\omega^2\mathrm{Cos}[t\omega]c_1c_3 - 15x^4\alpha\omega^2\mathrm{Cos}[t\omega]c_1c_3 + 30\omega\mathrm{Sin}[t\omega]c_1c_3 - 30x^4\omega\mathrm{Sin}[t\omega]c_1c_3 + 15M\alpha\omega\mathrm{Sin}[t\omega]c_1c_3 - 15Mx^4\alpha\omega\mathrm{Sin}[t\omega]c_1c_3 - 15c_3\mathrm{S}_t + 15x^4c_3\mathrm{S}_t - 13Mc_1c_3\mathrm{S}_t + 15Mx^4c_1c_3\mathrm{S}_t - 2Mx^6c_1c_3\mathrm{S}_t - 15c_3^2\mathrm{S}_t + 15x^4c_3^2\mathrm{S}_t - 15c_4\mathrm{S}_t + 15x^4c_4\mathrm{S}_t), \qquad (51)$$

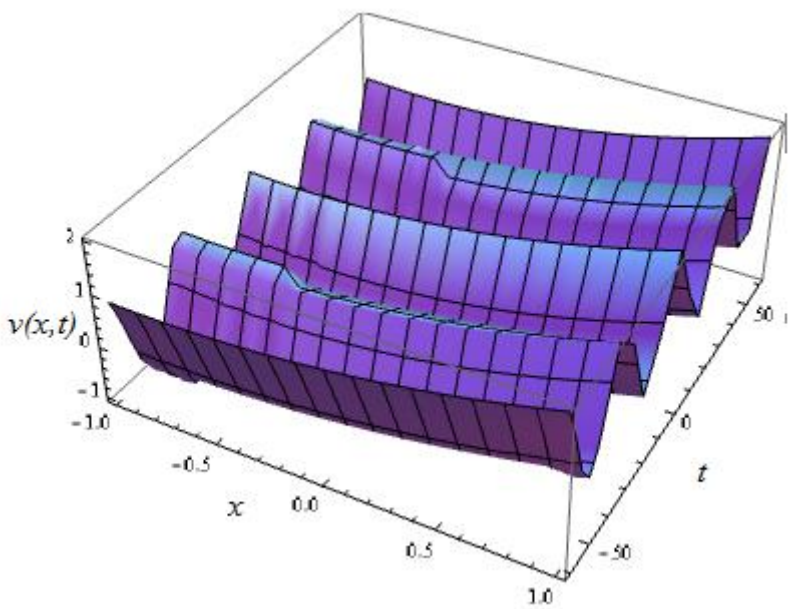

Figure 2: The fluid flow during different time level. when $\alpha = 0.1, t = 0.3, \omega = 0.2, = 0.4$.

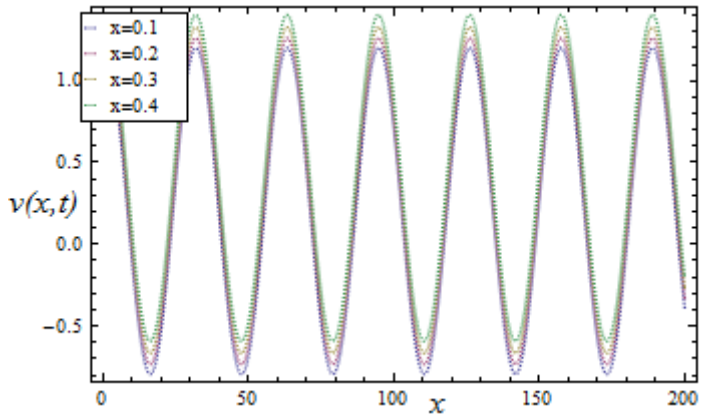

Figure 3: The fluid flow during different time period. when $\alpha = 0.1, t = 0.3, \omega = 0.2, = 0.4$.

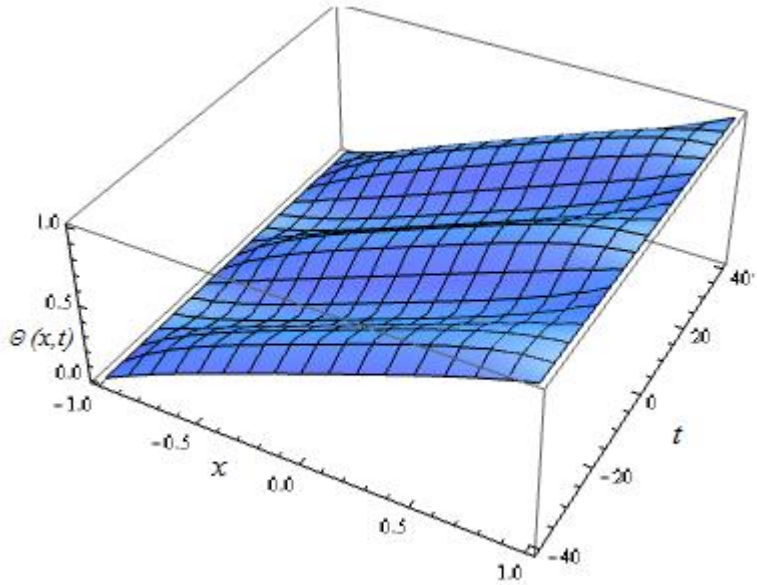

Figure 4: The temperature distribution of flow during different time period. When $\alpha = 0.1, \omega = 0.2, = 0.4$.

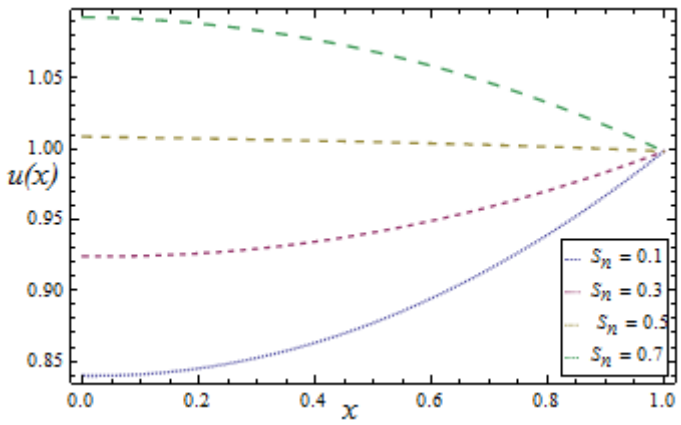

Figure5 : Effect of Stock number $S_t$for velocity distribution when $\alpha = 0.3, t = 0.2, \omega = 0.3, M = 0.4$.

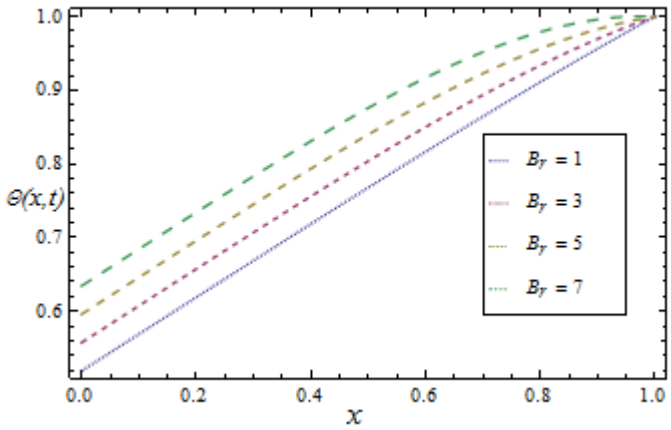

Fig 6: Effect of Brankman number$B_r$temperature distribution when $\alpha = 0.07, t = 0.01, M = 0.3, S_t = 0.9, \omega = 0.1.$

RESULTS AND DISCUSSION

In the present study we discussed the two vertical parallel and oscillating plates. OHAM method has been used to obtain the solution of the partial differential equations. The effects of model parameters on velocity and temperature distribution have been discussed graphically.Figure1 represent geometry of the problem. Figures 2 and 3 show the fluid flow motion between two oscillating vertical plate at the same period. The fluid near the flats oscillating in the same phase with the plates and the fluid flow oscillation reduces when we increase the distance between the plates. The effect of temperature is shown in figure 4. The fluid flow increases near the heated plate and decreases gradually away from the plates. Because the cohesive forces reduce near the plates and are strong away from the plates. Figure 5 shows the stock number for velocity profile as when we increase the value of parameter $S_t$the velocity of fluid increases. The reason is that the force of friction decrease with increase in stock number. Figure 6 shows the effect of Brinkman number in temperature distribution. Increase in Brinkman number increase the velocity profile.

Conclusion

The MHD flow of an unsteady second grade fluid problem is examined between two vertical and oscillating parallel plates in the presence of temperature distribution. We have shown the effect of different time level on the fluid flow motion between two oscillating vertical plate at the same period. The fluid near the flats oscillating in the same phase with the plates and the fluid flow oscillation reduces when we increase the distance between the plates. The fluid flow increases near the heated plate and decreases gradually away from the plates. Because the cohesive forces near the heated plates reduces and are stronger away from the plates.

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