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Unsteady Third Order Fluid Flow with Heat Transfer Between Two Vertical Oscillating Plates

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ABSTRACT

In this research article we study an unsteady flow of a third grade fluid with heat. The flow phenomenon of Non-Newtonian fluid between two oscillating vertical plates.The problem is modeled in terms of non-linear partial differential equation. The model differential equations are solved analytically by using Adomain Decomposition Method (ADM). This method is frequently used for solving non-linear differential equations arise in various applied sciences and is found quite useful.The physical influence of various parameters on velocity is studied graphically and discussed.

KEYWORDS: Unsteady, Temperature Field, Non-Newtonian, Adomain Decomposition Method (ADM).

INTRODUCTION

Non-Newtonian fluids has got attractive importance in research field, especially in mathematics, industry and engineering problems. Examples of non-Newtonian fluids include plastic manufacturing, food processing, movement of biological fluids, wire and fiber coating, paper production, gaseous diffusion, transpiration cooling, heat pipes etc. Several complex fluids such as polymer melts, paint, shampoo, mud, ketchup, blood, certain oils and greases, and many emulsions are involved in the class of nonNewtonian fluids. These fluids are described by a non-linear relationship between stress and the rate of deformation tensors and therefore several models have been proposed. Over the past few decades, there has been growing recognition of the fact that many fluids of industrial significance do not obey the Newtonian postulate of linear relationship between the shear stress and shear rate. Therefore, these fluids are known as non-Newtonian fluids. Common examples of such fluids are slurries, pastes, gels, molten plastics and lubricants containing polymer additives.Akyildiz et al [1] discussed a note on the flow of a non-Newtonian fluid film.Sankar et al [2] observed a non-Newtonian fluid flow model for blood flow through a catheterized artery Steady flow.Erdogan et al [3] study an unsteady flows of a non-Newtonian fluids. Moulic et al [4,5] dicussed Non-Newtonian Natural Convection flow.The unsteady flow of Non-Newtonian fluid was discussed in [6,7]. Qayyum Shah et al in [8] have investigated gravity driven flow of an unsteady second order fluid between two parallel and vertical oscillating plates.Taza Gul et al [9] observed a heat transfer analysis of thin film flow of an unsteady second grade fluid past a vertical oscillating belt. Sidra Abid et al [10] discussed an unsteady second grade fluid between two vertical plates with oscillating boundary conditions. A thin film flow of unsteady third grade fluid between two vertical plates when one of the plates oscillating and the other is stationary was discussed in [11,12]. Taza Gul et al [13] have observed a temperature dependent viscosity ofthin film flow of third grade fluid on a vertical belt. Ali et al [14] has investigated a Laplace transform method for unsteady thin film flow of a second grade fluid through a porous medium.Unsteady Couette Flow between Two Infinite Parallel Porous Plates in an Inclined Magnetic Field with Heat Transfer was discussed in [15, 16].Hameed et al [17]an unsteady flow of a non-Newtonian fluid on a porous plate.

For the solution of nonlinear differential equation,Mabood et al [18] used Optimal Homotopy Asymptotic Method for the solution of differential equation various method are used. Aamer Khan et al [19] have observed OHAM Solution for the non-linear differential equation. Wazwaz et al [20] discussed a new modification of the Adomian Decomposition Method for linear and nonlinear operators.. Adomian decomposition method for solving boundary value problems for higher order nonlinear differential equations was used in [21, 23].

In the present article we will use Adomian decomposition method (ADM) for the solution of nonlinear differential equations of gravity driven flow of an unsteady third order fluid between two parallel and vertical oscillating plates.

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Basic equations

The MHD equations which govern the unsteady incompressible flow of third grade fluid are

$$\mathbf{V} = (0, v(x,t), 0), \quad (1)$$

$$\Theta = \Theta(x,t). \quad (2)$$

Subject to the following boundary conditions as

$$\mathbf{v}(h,t) = V cos\omega t \ , \qquad \mathbf{v}(-h,t) = V cos\omega t \quad (3)$$

$$\Theta(h,t) = \Theta_0 \ , \quad \Theta(-h,t) = \Theta_1, \quad (4)$$

Here$v(x,t)$,$\Theta(x,t)$, are the velocity and temperature components in x-direction where$\omega$is the frequency of the oscillating belt.

TheCauchy stress tensor$\mathbf{T}$,for the third grade fluid model is

$$\mathbf{T} = -p\mathbf{I} + \mu\mathbf{A}_1 + \alpha_1\mathbf{A}_2 + \alpha_2\mathbf{A}_1^2 + \beta_1\mathbf{A}_3 + \beta_2(\mathbf{A}_1\mathbf{A}_2 + \mathbf{A}_2\mathbf{A}_1) + \beta_3(tr\mathbf{A}_1^2)\mathbf{A}_1, \quad (5)$$

Where$\mathbf{I}$ is identity tensor, p is fluid pressure,μ, is the coefficient of viscosity, $\alpha_i(i = 1,2), \beta_j(j = 1,2,3)$, are the material constants second grade and third grade fluids.The first four Rivlin-Ericksentensors $\mathbf{A}_m(m = 1,2,3,4)$ are defined by the following relations

$$\mathbf{A}_0 = \mathrm{I}, \quad (6)$$

$$\mathbf{A}_1 = \boldsymbol{L} + \boldsymbol{L}^T, \ \boldsymbol{L} = grad(\mathbf{v}), \quad (7)$$

$$\mathbf{A}_n = \frac{D\mathbf{A}_{n-1}}{Dt} + \mathbf{A}_{n-1}(\mathrm{L}) + (\mathrm{L})^T\mathbf{A}_{n-1}, \ n = 2,3,4. \quad (8)$$

For the electrically conducting unsteady incompressible third grade fluid flow with body force the continuity and momentum equations are

$$\mathrm{div}\mathbf{V} = 0 \ , \quad (9)$$

$$\rho\frac{D\mathbf{v}}{Dt} = \mathrm{div}\mathbf{T} + \rho \mathrm{g}, \quad (10)$$

$$\rho c_p\frac{D\Theta}{Dt} = k\nabla^2\Theta + tr(\boldsymbol{T}.\boldsymbol{L}), \quad (11)$$

Whereρ is the fluid density,$\frac{D}{Dt}$,is the material time derivative, g is the external body force, $k$is the thermal conductivity and c_p, is specific heat.

Using Eq. (1), the continuity Eq. (9), is identically satisfied and Eqs. (10, 11)reduced to the form

$$\rho\frac{\partial \mathrm{v}}{\partial t} = \frac{\partial}{\partial x}T_{yx} - \rho g. \quad (12)$$

$$\rho c_p\frac{\partial\Theta}{\partial t} = k\left(\frac{\partial^2\Theta}{\partial x^2}\right) + T_{yx}\left(\frac{\partial \mathrm{v}}{\partial x}\right), \quad (13)$$

The component of Cauchy stressT_{xy},obtained

$$T_{xy} = \mu\frac{\partial \mathrm{v}}{\partial x} + \alpha_1\frac{\partial}{\partial t}\left(\frac{\partial \mathrm{v}}{\partial x}\right) + \beta_1\frac{\partial^2}{\partial t^2}\left(\frac{\partial \mathrm{v}}{\partial x}\right) + 2(\beta_2 + \beta_3)\left(\frac{\partial \mathrm{v}}{\partial x}\right)^3, \quad (14)$$

Substituting Eq.(14) in momentum Eq.(12),we get

$$\rho\frac{\partial \mathrm{v}}{\partial t} = \mu\frac{\partial^2 \mathrm{v}}{\partial x^2} + \alpha_1\frac{\partial}{\partial t}\left(\frac{\partial^2 \mathrm{v}}{\partial x^2}\right) + 6(\beta_2 + \beta_3)\left(\frac{\partial \mathrm{v}}{\partial x}\right)^2\left(\frac{\partial^2 \mathrm{v}}{\partial x^2}\right) - \rho g. \quad (15)$$

Heat equation become

$$\rho c_p\frac{\partial\Theta}{\partial t} = k\left(\frac{\partial^2\Theta}{\partial x^2}\right) + \mu\left(\frac{\partial \mathrm{v}}{\partial x}\right)^2 + \alpha_1\left(\frac{\partial^2 \mathrm{v}}{\partial t\partial x}\right)\left(\frac{\partial \mathrm{v}}{\partial x}\right) + \beta_1\left(\frac{\partial^3 \mathrm{v}}{\partial^2 t\partial x}\right)\left(\frac{\partial \mathrm{v}}{\partial x}\right) + 2(\beta_2 + \beta_3)\left(\frac{\partial \mathrm{v}}{\partial x}\right)^4. \quad (16)$$

We are introducing somenon-dimensionalquantitiesas fallows

$$\widetilde{v} = \frac{\mathbf{v}}{V}, \quad \widetilde{x} = \frac{x}{\delta}, \quad \widetilde{t} = \frac{t\mu}{\delta^2\rho}, \quad \widetilde{\alpha} = \frac{\alpha_1}{\rho\delta^2}, \quad \widetilde{\beta_1} = \frac{\beta_1\mu}{\delta^2}, \quad \widetilde{\beta} = \frac{(\beta_2 + \beta_3)V^2}{\mu\delta^2},$$

$$, S_t = \frac{\rho\delta^2 g}{\mu V}, P_r = \frac{c_p\mu}{k}, \overline{\Theta} = \frac{\Theta - \Theta_0}{\Theta_1 - \Theta_0}, E_c = \frac{V^2}{c_p(\Theta_1 - \Theta_0)}. \quad (17)$$

In above expressionE_c, is Eckert number, P_r, is Prandtl number, $\widetilde{t}$,is a dimensionless time parameter.

Substitute the above non-dimensional variablesinto Eqs. (15,16), we get

$$\frac{\partial v}{\partial t} = -S_t + \frac{\partial^2 v}{\partial x^2} + \alpha\frac{\partial}{\partial t}\left(\frac{\partial^2 v}{\partial x^2}\right) + \beta_1\frac{\partial^2}{\partial t^2}\left(\frac{\partial^2 v}{\partial x^2}\right) + 6\beta\left(\frac{\partial v}{\partial x}\right)^2\left(\frac{\partial^2 v}{\partial x^2}\right), \quad (18)$$

$$P_r\frac{\partial\Theta}{\partial t} = \frac{\partial^2\Theta}{\partial x^2} + P_r E_c\left\{\left(\frac{\partial v}{\partial x}\right)^2 + \alpha\left(\frac{\partial^2 v}{\partial t\partial x}\right)\left(\frac{\partial v}{\partial x}\right) + \beta_1\left(\frac{\partial^3 v}{\partial^2 t\partial x}\right)\left(\frac{\partial v}{\partial x}\right) + 2\beta\left(\frac{\partial v}{\partial x}\right)^4\right\}, \quad (19)$$

Non-dimensional boundary conditions are

$$\mathbf{v}(1,t) = cos\omega t \ , \qquad \mathbf{v}(-1,t) = cos\omega t \ , \quad (20)$$

$$\Theta(1,t) = 1 \ , \quad \Theta(-1,t) = 0, (21)$$

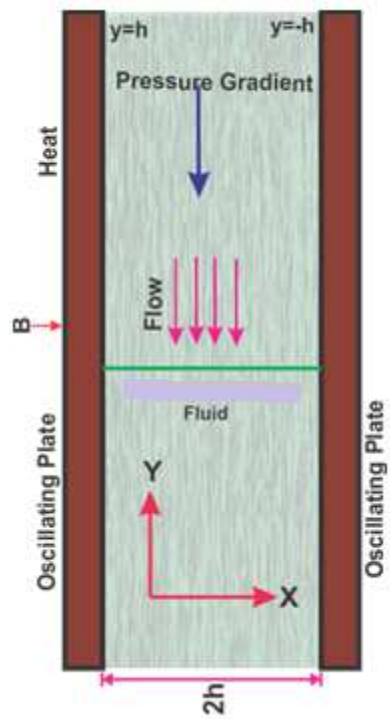

Figure 1. Geometry of Problem

2.2 Basic concepts of Adomain Decomposition Method (ADM)

We consider a nonlinear partial differential equation in an operator form as

$$\Lambda_t v(x,t) + \Lambda_x v(x,t) + Sv(x,t) + Nv(x,t) = g(x,t), \quad (22)$$

$$\Lambda_x v(x,t) = g(x,t) - \Lambda_t v(x,t) - Sv(x,t) - Nv(x,t). \quad (23)$$

Where $\Lambda_x = \frac{\partial^2}{\partial x^2}$ and $\Lambda_t = \frac{\partial}{\partial t}$, are linear operators in the partial differential equation and are easily invertible, $g(x,t)$, is a source term, $Rv(x,t)$, is a remaining linear term and $Nv(x,t)$ is non-linear analytical term expandable in the adomian polynomials A_n.

Taking the inverse operator ${\Lambda_x}^{-1}$ to both sides of Eq. (23), we write

$${\Lambda_x}^{-1}\Lambda_x v(x,t) = {\Lambda_x}^{-1} g(x,t) - {\Lambda_x}^{-1}\Lambda_t v(x,t) - {\Lambda_x}^{-1} Sv(x,t) - {\Lambda_x}^{-1} Nv(x,t), \quad (24)$$

$$v(x,t) = \psi(x,t) - {\Lambda_x}^{-1}\Lambda_t v(x,t) - {\Lambda_x}^{-1} Sv(x,t) - {\Lambda_x}^{-1} Nv(x,t). \quad (25)$$

Here the function $f(x,t)$ represents the terms arising from $\Lambda^{-1} g(x,t)$, after using the given conditions. ${\Lambda_x}^{-1} = \iint(.)dxdx$ is used as inverse operator for the second order partial differential equation.

In this method, the series solution $v(x,t)$ is defined as:

$$v(x,t) = \sum_{k=0}^{\infty} v_n(x,t), \quad (26)$$

$$\sum_{n=0}^{\infty} v_n(x,t) = \psi(x,t) - {L_x}^{-1}\Lambda_t \sum_{n=0}^{\infty} v_n(x,t) - {\Lambda_x}^{-1} S \sum_{n=0}^{\infty} v_n(x,t) - {\Lambda_x}^{-1} N \sum_{n=0}^{\infty} v_n(x,t) \quad (27)$$

The non-linear term is expanded in adomian polynomials as:

$$N\sum_{n=0}^{\infty} v_n(x,t) = \sum_{n=0}^{\infty} A_n. \quad (28)$$

Where the components $v_0(x,t), v_1(x,t), v_2(x,t) \ldots \ldots$ are periodically derived as:

$$v_0(x,t)+v_1(x,t)+v_2(x,t) \ldots = \psi(x,t) - {\Lambda_x}^{-1}\Lambda_t(v_0(x,t)+v_1(x,t)+v_2(x,t) \ldots) - {\Lambda_x}^{-1} S(v_0(x,t)+v_1(x,t)+v_2(x,t) \ldots) - {\Lambda_x}^{-1}(A_0+A_1+A_2 \ldots). \quad (29)$$

To determine the components of the series $v_0(x,t)+v_1(x,t)+v_2(x,t) \ldots \ldots$ it is important to note that ADM suggests that the zeroth component $v_0(\mathrm{x},\mathrm{t})$ is usually defined by the function $\psi(\mathrm{x},\mathrm{t})$ described above.

The formal recursive relation is defined as:

$$v_0(x,t) = \psi(x,t),$$
$$v_1(x,t) = -\Lambda_x{}^{-1}\Lambda_t[v_0(x,t)] - \Lambda_x{}^{-1}S[v_0(x,t)] - \Lambda_x{}^{-1}[A_0],$$
$$v_2(x,t) = -\Lambda_x{}^{-1}\Lambda_t[v_1(x,t)] - \Lambda_x{}^{-1}S[v_1(x,t)] - \Lambda_x{}^{-1}[A_1],$$
$v_3(x,t) = -\Lambda_x{}^{-1}\Lambda_t[v_2(x,t)] - \Lambda_x{}^{-1}S[v_2(x,t)] - \Lambda_x{}^{-1}[A_2]$, and so on. (30)

2.4The ADM solution of lift velocity and heat problem

Apply Adomian Decomposition Method to themodel equations (18,19), we get

$$L_x v(x,t) = S_t + \frac{\partial v}{\partial t} - \alpha\frac{\partial}{\partial t}\left(\frac{\partial^2 v}{\partial x^2}\right) - \beta_1\frac{\partial^2}{\partial t^2}\left(\frac{\partial^2 v}{\partial x^2}\right) - 6\beta\left(\frac{\partial v}{\partial x}\right)^2\left(\frac{\partial^2 v}{\partial x^2}\right) \quad (31)$$

$$L_x\Theta(x,t) = P_r\frac{\partial\Theta}{\partial t} - P_rE_c\left[\left(\frac{\partial v}{\partial x}\right)^2 + \alpha\left(\frac{\partial^2 v}{\partial t\partial x}\right)\left(\frac{\partial v}{\partial x}\right) + \beta_1\left(\frac{\partial^3 v}{\partial^2 t\partial x}\right)\left(\frac{\partial v}{\partial x}\right) + 2\beta\left(\frac{\partial v}{\partial x}\right)^4\right]. \quad (32)$$

Using the inverse operator, $L_x{}^{-1}$on Eq. (31, 32), we get

$$L_x{}^{-1}L_x v(x,t) = L_x{}^{-1}\left(S_t + \frac{\partial v}{\partial t}\right) - \alpha L_x{}^{-1}\left[\frac{\partial}{\partial t}\left(\frac{\partial^2 v}{\partial x^2}\right)\right] - \beta_1 L_x{}^{-1}\left[\frac{\partial^2}{\partial t^2}\left(\frac{\partial^2 v}{\partial x^2}\right)\right] -$$
$$6\beta L_x{}^{-1}\left[\left(\frac{\partial v}{\partial x}\right)^2\left(\frac{\partial^2 v}{\partial x^2}\right)\right], \quad (33)$$

$$L_x{}^{-1}L_x\Theta(x,t) = P_rL_x{}^{-1}\left[\frac{\partial\Theta}{\partial t}\right] - P_rE_cL_x{}^{-1}\left[\left(\frac{\partial v}{\partial x}\right)^2 + \alpha\left(\frac{\partial^2 v}{\partial t\partial x}\right)\left(\frac{\partial v}{\partial x}\right) + \beta_1\left(\frac{\partial^3 v}{\partial^2 t\partial x}\right)\left(\frac{\partial v}{\partial x}\right) + 2\beta\left(\frac{\partial v}{\partial x}\right)^4\right], \quad (34)$$

$$\sum_{n=0}^{\infty} v_n(x,t) = b_0 + b_1x + S_t\frac{x^2}{2} - \alpha L_x{}^{-1}\left[\sum_{n=0}^{\infty} A_n\right] - \beta_1 L_x{}^{-1}\left[\sum_{n=0}^{\infty} B_n\right] -$$
$$6\beta L_x{}^{-1}[\textstyle\sum_{n=0}^{\infty} C_n], \quad (35)$$

$$\textstyle\sum_{n=0}^{\infty}\Theta_n(x,t) = f_0 + f_1x - P_rE_cL_x{}^{-1}\left[[\sum_{n=0}^{\infty} P_n] + \alpha[\sum_{n=0}^{\infty} Q_n] + \beta_1[\sum_{n=0}^{\infty} R_n] + 2\beta[\sum_{n=0}^{\infty} S_n]\right], \quad (36)$$

where$A_n, B_n, C_n, D_n,\ E_n, P_n, Q_n, R_n, S_n, T_n$ *and* $U_n$are adomian polynomials and define as

$\frac{\partial}{\partial t}\left(\frac{\partial^2 v}{\partial x^2}\right) = \sum_{n=0}^{\infty} A_n$, $\frac{\partial^2}{\partial t^2}\left(\frac{\partial^2 v}{\partial x^2}\right) = \sum_{n=0}^{\infty} B_n$, $\left(\frac{\partial v}{\partial x}\right)^2\left(\frac{\partial^2 v}{\partial x^2}\right) = \sum_{n=0}^{\infty} C_n$, $\frac{\partial^3}{\partial t^3}\left(\frac{\partial^2 \mathbf{v}}{\partial x^2}\right) = \sum_{n=0}^{\infty} D_n$, $\frac{\partial}{\partial x}\left[\left(\frac{\partial \mathbf{v}}{\partial x}\right)^2\frac{\partial}{\partial t}\left(\frac{\partial^2 \mathbf{v}}{\partial x^2}\right)\right] = \sum_{n=0}^{\infty} E_n$, $\left(\frac{\partial v}{\partial x}\right)^2 = \sum_{n=0}^{\infty} P_n$, $\left(\frac{\partial^2 v}{\partial t\partial x}\right)\left(\frac{\partial v}{\partial x}\right) = \sum_{n=0}^{\infty} Q_n$, $\left(\frac{\partial^3 v}{\partial^2 t\partial x}\right)\left(\frac{\partial v}{\partial x}\right) = \sum_{n=0}^{\infty} R_n$, $\left(\frac{\partial v}{\partial x}\right)^4 = \sum_{n=0}^{\infty} S_n$, $\left(\frac{\partial^4 v}{\partial^3 t\partial x}\right)\left(\frac{\partial v}{\partial x}\right) = \sum_{n=0}^{\infty} T_n$, $\left(\frac{\partial v}{\partial x}\right)\frac{\partial}{\partial t}\left(\frac{\partial v}{\partial x}\right)^3 = \sum_{n=0}^{\infty} U_n$, (37)

$v_0(x,t) + v_1(x,t) + \cdots = b_0 + b_1x + S_t\frac{x^2}{2} - \alpha L_x{}^{-1}[A_0 + A_1 + \cdots] - \beta_1 L_x{}^{-1}[B_0 + B_1 + \cdots] - 6\beta L_x{}^{-1}[C_0 + C_1 + \cdots]$,(38)

$\Theta_0(x,t) + \Theta_1(x,t) + \cdots = f_0 + f_1x - P_rE_cL_x{}^{-1}\big[[P_0 + P_1 + \cdots] + \alpha[Q_0 + Q_1 + \cdots] + \beta_1[R_0 + R_1 + \cdots] + 2\beta[S_0 + S_1 + \cdots]\big]$.(39)

The components of velocity and temperature profile obtained by comparing both side of Eq. (38, 39)

Solution of Zeroth and first component problem:

According to[15, 16] the zero first and second component solutions of velocity and temperature fields are.

$$\frac{\partial^2 v_0}{\partial x^2} = -S_t, \quad (40)$$
$$\frac{\partial^2\Theta_0}{\partial x^2} = 0, \quad (41)$$

$\frac{\partial^2 v_1}{\partial x^2} = \left(\frac{\partial v_0}{\partial t}\right) - 6\beta\left(\frac{\partial v_0}{\partial x}\right)^2\frac{\partial^2 v_0}{\partial x^2} - \alpha\left(\frac{\partial^3 v_0}{\partial t\partial x^2}\right)$,(42)

$$\frac{\partial\Theta_1}{\partial t} = P_r\frac{\partial\Theta_0}{\partial t} - P_rE_c\left[\left(\frac{\partial v_0}{\partial x}\right)^2 - 2\beta\left(\frac{\partial v_0}{\partial x}\right)^4 - \alpha\frac{\partial^2 v_0}{\partial t\partial x} - \beta_1\frac{\partial^3 v_0}{\partial x\partial t^2}\right], \quad (43)$$

$$\frac{\partial^2 v_2}{\partial x^2} = \left[\frac{\partial v_1}{\partial t} - 12\beta\frac{\partial v_0}{\partial x}\frac{\partial v_1}{\partial x}\frac{\partial^2 v_0}{\partial x^2} - 6\beta\left(\frac{\partial v_0}{\partial x}\right)^2\frac{\partial^2 v_1}{\partial x^2} + \alpha\frac{\partial^2}{\partial x^2}\left(\frac{\partial v_1}{\partial t}\right)\right], \quad (44)$$

$$\frac{\partial\Theta_2}{\partial t} = P_r\frac{\partial\Theta_1}{\partial t} - P_rE_c\left[2\frac{\partial v_0}{\partial x}\frac{\partial v_1}{\partial x} + 8\beta E_c\left(\frac{\partial v_0}{\partial x}\right)^3\frac{\partial v_1}{\partial x} + \alpha\frac{\partial^2 v_1}{\partial t^2} + \beta_1\frac{\partial^2 v_1}{\partial t^2}\right] \quad (45)$$

Solutions of zeroth, first and the second component problem of velocity and temperature distribution using boundary conditions given Eq. (20, 21) are

$v_0[x,t] = \mathrm{Cos}[t\omega] - \frac{1}{2}(-1 + x^2)S_t$,(46)

$$\Theta_0[x,t] = \frac{1+x}{2}, \quad (47)$$
$$v_1[x,t] = \frac{1}{24}[-12\omega\mathrm{Sin}[t\omega](x^2 - 1) - 12\beta S_t^3(x^4 - 1)], \quad (48)$$

$$\Theta_1[x,t] = \frac{1}{60}[5P_rE_cS_t^2(1-x^4) + 4\beta P_rE_cS_t^4(1-x^6)], \quad (49)$$

$$v_2[x,t] = \left[\left(-\frac{5\omega^2}{24} - \frac{\alpha\omega^2}{2}\right)\mathrm{Cos}[t\omega] - \left(\frac{3\beta\omega S_t^2}{2}\right)\mathrm{Sin}[t\omega] + 2\beta^2S_t^5\right] - \left[\left(\frac{\omega^2}{4} + \frac{\alpha\omega^2}{2}\right)\mathrm{Cos}[t\omega]\right]x^2 + \left[\left(\frac{-\omega^2}{24}\right)\mathrm{Cos}[t\omega] + \left(\frac{3\beta\omega S_t^2}{2}\right)\mathrm{Sin}[t\omega]\right]x^4 - [2\beta^2S_t^5]x^6, \quad (50)$$

$$\Theta_2[x,t] = \varphi_{11} + x\varphi_{12} + x^2\varphi_{13} + x^3\varphi_{14} + x^4\varphi_{15} + x^6\varphi_{16} + x^8\varphi_{17}, \quad (51)$$

Where $\varphi_{11} \ldots\ldots\ldots. \varphi_{17}$, are defined as

$$\varphi_{11} = P_rE_c\left[\left(\frac{4}{15}\beta\omega S_t^3 - \frac{5}{24}\beta_1\omega^3 + \frac{1}{6}\omega S_t\right)\mathrm{Sin}[t\omega] - \frac{2}{15}\beta S_t^4 - \frac{2}{7}\beta^2S_t^6\right],$$

$\varphi_{12} = -\frac{\alpha P_rE_c}{6}(\omega^2\mathrm{Cos}[t\omega])$,

$\varphi_{13} = \frac{\beta_1P_rE_c}{4}(\omega^3\mathrm{Sin}[t\omega])$,

$\varphi_{14} = \frac{\alpha P_rE_c}{6}(\alpha\omega^2\mathrm{Cos}[t\omega])$,

$\varphi_{15} = \frac{P_rE_c}{24}(-\beta_1\omega^3\mathrm{Sin}[t\omega] - 4\omega\mathrm{Sin}[t\omega]S_t)$,

$\varphi_{16} = \frac{P_rE_c}{90}(-24\beta\omega\mathrm{Sin}[t\omega]S_t^3 + 12S_t^4)$,

$\varphi_{17} = \frac{P_rE_c}{42}\left(\frac{2}{7}\beta^2S_n^6\right)$,

Inserting equations (46),(48) and (50)into equation (52) we get the series solution for fluid motion.

$v[x,t] = v_0[x,t] + v_1[x,t] + v_2[x,t]$.(52)

Similarly the series solution for temperature distribution can be obtained by using equations (47),(49) and (51)into equation (54).

$\Theta[x,t] = \Theta_0[x,t] + \Theta_1[x,t] + \Theta_2[x,t]$ (53)

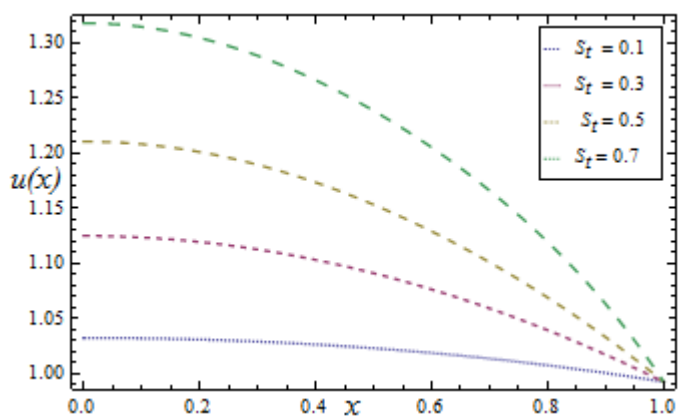

Figure2: Velocity profile for stock numberS_t, when other parameter is constant $\omega = 0.3, \alpha = 0.2, \beta = 0.5, t = 0.4$,

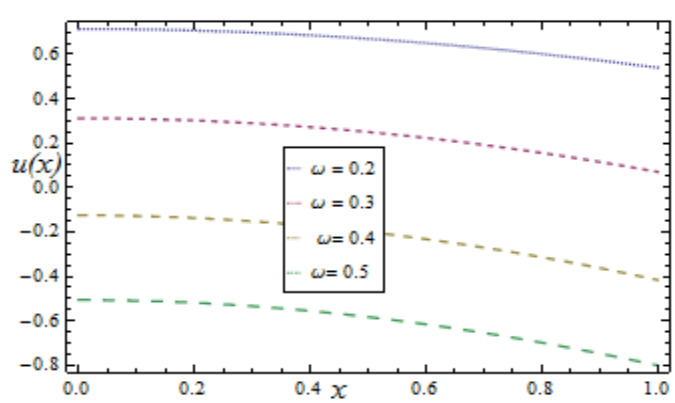

Figure 3: Velocity profile for pressure gradient ω, when other parameter is $S_t = 0.2; \alpha = 0.1; \beta = 0.3; t = 5$

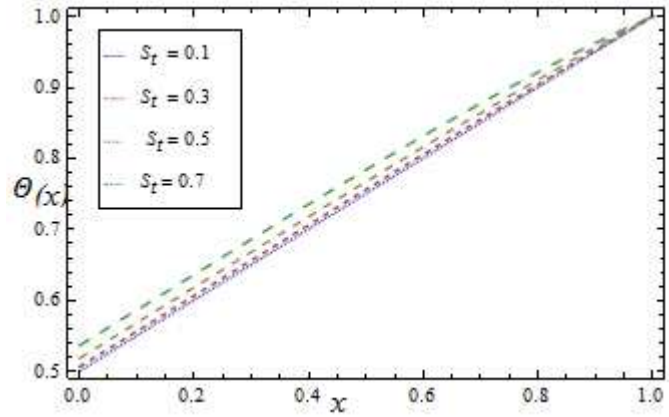

Figure 4: The influence of stock number S_t, for temperature profile when other parameter is $\omega = 0.01, \alpha = 0.07, E_c = 1, P_r = 0.9, \beta_1 = 0.5, \beta = 0.01, t = 0.1$.

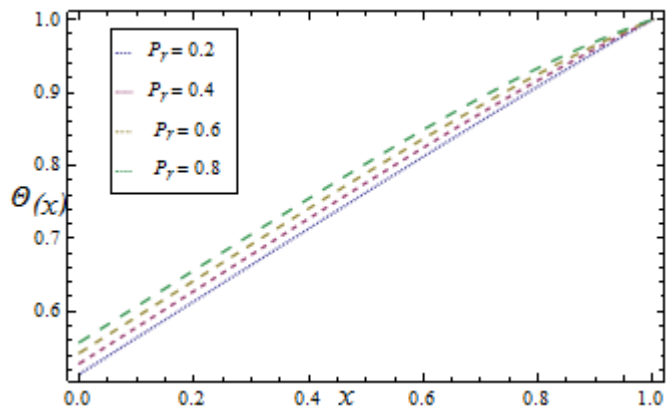

Figure 5: The profile of temperature distribution of Prandtl numberP_r,when $\omega = 0.02, \alpha = 0.07, E_c = 1, S_t = 0.9, \beta_1 = 0.5, \beta = 0.01, t = 0.1$.

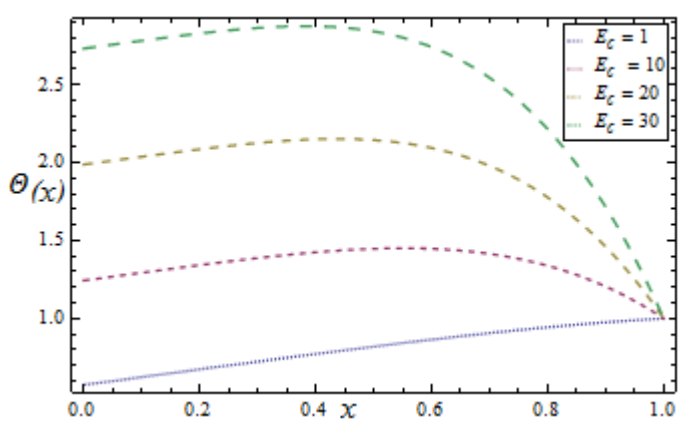

Figure 6: The figure shows the Eckertnumber onthe temperature profile by keeping other fixed$\omega = 0.01, \alpha = 0.07, S_t = 1, P_r = 0.9, \beta_1 = 0.5, \beta = 0.01, t = 0.1$.

RESULTS AND DISCUSSION

In the present study we discussed the velocity field and temperature of unsteady flow problems of magneto-hydrodynamics third grade fluid. ADM method has been used to obtain the solution of non-linear partial differential equations of temperature and velocity distribution. The effects of model parameters on velocity and temperature distribution have been discussed graphically. Figure 1 shows the geometry of the problem. From the Figure 2 we observed that velocity increases when the value of stock number S_t, increases.Figure 3 shows the velocity profile for pressure gradient ω. We see that there is decrease in the velocity as we increase the value of pressure gradient ω. Figure 4 to 6 are plotted to examine the effect of $E_c, P_r and\ S_t$ on temperature distribution. Figure 4 shows the effect of S_t on temperature distribution, there is an increase in the temperature if we increase the value of stock numberS_t. Figure 5 shows the effect of P_r, on temperature field. From this figure we observed that temperature increases when the value of P_r increases. Figure 6 are plotted to examine the effect of Eckert number E_c on temperature distribution. We observe that temperature is increasing by increasing Eckert number E_c .

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