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Unsteady MHD Blood Flow with Micropolar Fluid Characteristics and Heat Source through Parallel Plate Channel

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ABSTRACT

Unsteady blood flow having micropolar fluid properties with heat source through parallel plates channel is studied under the influence of a uniform transverse magnetic field. Suitable similarity transformations have been used to transform the governing highly nonlinear partial differential equations into ordinary differential form. The effect is strong form nondimensional material constants C_1, C_2 and C_3 which show micropolar behavior. A comparison of Newtonian fluids is made with micropolar fluids. The results have been computed numerically for several values of the parameters namely magnetic parameter *M*, heat source parameter *N*, Prandtl number P_r and presented in graphical form.
KEYWORDS: Micropolar fluids, Blood flow, Similarity Transformations, Hartmann number, Prandtl number, Decay parameter.

1. INTRODUCTION

The analysis of blood flow through parallel plate channel can be employed in biomedical research to examine the effects of fluid shear stress on the structure and function of endothelial cells. It is widely believed as Isenberg et al. [1] considered that vascular endothelial cells bear the earliest responses to blood shear stress and thus undergo reorganization. Khalil et al. [2] and Krizanac-Bengez et al. [3] studied the effects of ischemia/reperfusion on inflammatory gene expression of endothelial cells. Li et al. [4] remarked that the blood shear stress causes cardiovascular diseases by regulating inflammatory reactions in the vascular endothelium. A parallel plate channel is constituted by two plates in parallel with a thin rectangular flow of blood. The concept has been attributed to wide spread usage because of its simplicity and its various designs have been developed for different applications of biomedical research. Among many others, Albuquerque et al. [5] studied platelet adhesion, Gray et al. [6] examined cell morphology, Miyagi et al.[7] investigated gene and protein expression, Yee et al. [8] and Chen et al. [9] analyzed cell responses under special conditions. Hussain et al. [10] obtained numerical solution for a similar flow between two disks in the presence of a magnetic field. Ahmad et al. [11] also obtained numerical solution for hydromagnatic fluid flow between two horizontal plates, both the plates being stretching sheets.

Since blood is an electrically conducting fluid as reported in [12].The magneto hydrodynamic principles are applied in medicine and physiological flow problems. The MHD principles may be used to impede the flow of blood in arterial system and it is useful in the treatment of certain blood flow disorders and in the diseases like hypertension and hemorrhages [13]. Ali et al. [14] obtained analytical solution of unsteady MHD blood flow and heat transfer through parallel plates when lower plate stretches exponentially.

Several mathematical models have been developed for blood flow and many authors like [15-18] assumed blood as Newtonian fluid. No doubt, this assumption provides enough simplicity but blood is more likely non-Newtonian fluid as it consists of plasma, blood cells and suspended particles. Micropolar fluid theory introduced by Eringen [19] provides an important branch of non-Newtonian fluid dynamics where micro-rotation effects as well as micro inertia are exhibited. Chaturani and Mahajan [20] reexamined Poiseuille flow of a micropolar fluid from the point of view of its applications to blood flow. The results obtained have been compared with experimental values for blood flow.
Hussain et al. [21-22] discussed effects of heat source/sink on MHD flow of micropolar fluids over a shrinking/stretching sheet with mass suction.

Olajuwon et al. [23] investigated unsteady free convection heat and mass transfer in an MHD micropolar fluid in the presence of thermo diffusion and thermal radiation. A mathematical study of non-Newtonian micropolar fluid in arterial blood flow through composite stenosis is undertaken by Ellahi et al. [24].

The main advantage of using micropolar fluid to study the blood flow in comparison with other classes of non-Newtonian fluids is that it takes care of the rotation of the fluid particles by means of an independent kinematic vector

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called micro-rotation vector. However, a great deal of research work warrants the attention of researchers to use micropolar fluid model for rheology.

Ali et al. [25-26] numerical solution of MHD flow of fluid and heat transfer over porous stretching sheet. Hussain et al. [27] investigated unsteady MHD flow and heat transfer for Newtonian fluids over an exponentially stretching sheet.

Motivated by these facts, the present work has been undertaken to extend the work of Eldesoky [17]. Physical problem is first modelled and then simplified by using the nondimensional variables. The numerical solution of the simplified equations are found and the results for axial velocity, microrotation and temperature functions are discussed through graphs for various physical parameters of the problem. After the introduction in Section 1, the outlines of this paper are as follows. Section 2 contains analysis of the problem. Discussion and results are given in Sections 3. Finally Section 4 summaries the concluding remarks.

2. ANALYSIS OF THE PROBLEM

The animal blood sample is assumed to be micropolar fluid. The fluid flow is incompressible, two dimensional and unsteady. The velocity vector is $V = v(u,v)$ and micro rotation vector is $W = w(0,0,w_3)$. A magnetic field of strength B_0 is applied perpendicular to the fluid flow. The governing equations of the motion are:

$$\partial u^* / \partial x^* + \partial v^* / \partial y^* = 0 \tag{1}$$

$$(\mu + \kappa)\partial^2 u^* / \partial y^{*2} + \kappa \partial w_3^* / \partial t^* - \sigma B_0^2 u^* + g\beta(T - T_0) = \rho \partial u^* / \partial t^* + dP / dx^* \tag{2}$$

$$\gamma\, \partial^2 w_3^* / \partial y^{*2} - 2\kappa w_3^* + \kappa\left(\partial u^* / \partial x^* - \partial v^* / \partial y^*\right) = \rho j \partial w_3^* / \partial t^* \tag{3}$$

$$\partial T / \partial t^* = \frac{K^/}{\rho\, C_p} \partial^2 T / \partial y^{*2} + \frac{Q}{\rho\, C_p}(T - T_0) \tag{4}$$

where B_0 is strength of magnetic field, P is the pressure, g is acceleration due to gravity and β is the volumetric coefficient of thermal expansion, ρ is fluid density, μ is the coefficient of viscosity, σ is electrical charge density, T is the temperature inside the boundary layer, k is vortex viscosity, γ is spin gradient viscosoyi, $K^/$ the thermal conductivity and C_p is the specific heat at constant pressure.

The following non- dimensional variables are introduced to simplify the above equations:

$$x^* = \frac{x}{b},\quad y^* = \frac{y}{b},\quad u^* = \frac{u}{(m/2\rho b)},\quad v^* = \frac{v}{(m/2\rho b)}$$

$$t^* = \frac{t}{(\rho b^2/\mu)},\quad h^*(x,t) = \frac{dp/dx}{(\mu m/2\rho^2 b^3)},\quad \theta^* = \frac{\theta}{\left(\mu m/2\rho^2 b^3\right)},\quad w_3^* = \frac{w_3}{\left(m/2\rho b^2\right)}$$

The equations (1) to (4) respectively become:

$$\partial u / \partial x + \partial v / \partial y = 0 \tag{5}$$

$$(\mu + \kappa)\partial^2 u / \partial y^2 + \kappa \partial w_3 / \partial t - \sigma B_0^2 b^2 u + g\beta\theta = \partial u / \partial t + h \tag{6}$$

$$\gamma \partial^2 w_3 / \partial y^2 - 2b^2 \kappa w_3 + \kappa b^2\left(\partial v / \partial x - \partial u / \partial y\right) = j\mu \partial w_3 / \partial t \tag{7}$$

$$\partial\theta / \partial t = \frac{1}{\upsilon\rho_r}\partial^2\theta / \partial y^2 + \frac{1}{\upsilon\rho_r}\theta \tag{8}$$

The associated boundary conditions are:

$$u = e^{-\lambda^2 t},\ w_3 = 0,\ \theta = e^{-\lambda^2 t} \text{ at } y = -1 \text{ and } u = 0,\ w_3 = 0,\ \theta = 0 \text{ at } y = 1 \tag{9}$$

Let us take the transformations as:

$$u = f(y)e^{-\lambda^2 t},\ v = g(y)e^{-\lambda^2 t},\ \theta = H(y)e^{-\lambda^2 t},\ w_3 = L(y)e^{-\lambda^2 t}$$

The equations (5) is readily satisfied and (6) to (8) are respectively transformed to ordinary differential equations,

$$(1+c_1)f'' + c_1L' + (\lambda^2 - H_a^2)f - h = 0 \tag{10}$$

$$L'' + c_1c_2(f' - 2L) + c_3\lambda^2 L = 0 \tag{11}$$

$$H'' + (w + \lambda^2 P_r \upsilon)H = 0 \tag{12}$$

$$f = 1,\ L = 0,\ H = 1 \text{ at } y = -1,\ f = 0,\ L = 0,\ H = 0 \text{ at } y = 1. \tag{13}$$

Where $M = B_0 b\sqrt{\frac{\sigma}{\mu}}$ is Magnetic Parameter, $N = \frac{Qb^2}{K'}$ is heat source parameter, $P_r = \frac{\rho C_p}{K'}$ is Prandtl number, h is pressure gradient and υ is kinematic viscosity. Also, $C_1 = \frac{\mu}{k}$, $C_2 = \frac{b^2\mu}{\gamma}$, $C_3 = \frac{j\mu}{\gamma}$ are nondimensional material constants associated with micropolar behavior of fluids.

3. RESULTS AND DISCUSSION

The ordinary differential equations (10) to (12) are solved along with the boundary conditions (13) using Mathematica-6. The validity of the present results has been checked through comparison with the previous results where it is possible. Main focus of the study is to examine effects of nondimensional material constants namely C_1 (vortex viscosity parameter), C_2 (spin gradient viscosity parameter) and C_3 (micro inertia density parameter) on fluid velocity, microrotation and temperature distribution.

In order to develop better understanding of micropolar fluid model for blood, four different sets of these parameters have been chosen arbitrarily as given below:

Case	I	II	III	IV
C_1	0.0	0.2	0.5	1.8
C_2	0.0	1.4	1.5	2.0
C_3	0.0	1.8	2.5	3.5

For case I, the blood flow resembles with that Newtonian fluid flow model as described in [14] and other three cases stand for micropolar fluid flow model. The effects of other physical parameters of the problems namely magnetic parameter M, heat source parameter N, Prandtl number P_r have also been examined for blood flow characteristics and presented graphically.

The horizontal velocity component increases from the lower boundary at $y = -1$, to maximum value at the center of the region at $y = 0$ and decreases to other boundary at $y = 1$. The magnetic field has decreasing effect on this velocity component as shown in Fig-1. Prandtl number and heat source parameter N both show increasing effect on the velocity as depicted in Fig-2 and Fig-3 respectively. The effect of heat source parameter is strong.

Fig-4 demonstrates the effect of material constants C_1, C_2 and C_3 on flow velocity. The increasing values of these parameters decrease the blood velocity. Similar effect of these parameters is observed on the angular velocity as presented in Fig-5. The above mentioned two findings especially convince that micropolar fluid model provides better description of the blood flow than that of Newtonian fluid flow model of blood.

Fig-6 and Fig-7 respectively show that angular velocity of microrotation of blood particles increases with the increase in heat source and decreases with the increase in magnetic field.

Fig-8 and Fig-9 respectively show that temperature distribution increases with increase in heat source and Prandtl number.

4. CONCLUSION

The magnetic field parameter M, has decreasing effect where as heat source parameter and Prandtl number have increasing effect on blood flow velocity and angular velocity of microrotation of blood particles. The material constants show decreasing effect on blood flow.

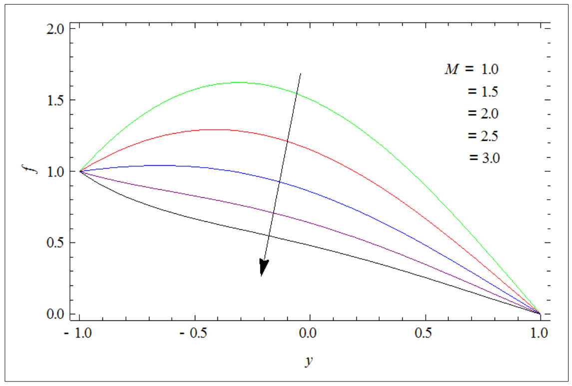

Fig.1 Graph of horizontal velocity for different values of Magnetic field M from top to bottom

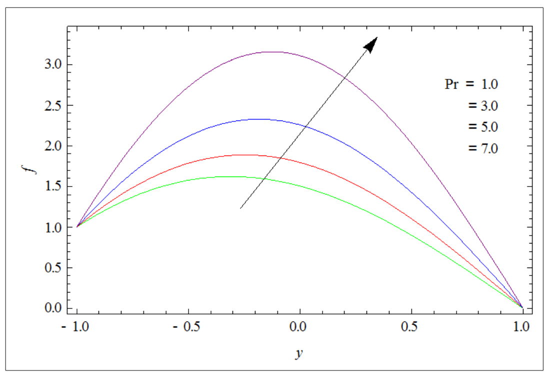

Fig.2 Graph of horizontal velocity f for different values of Prandtl Number Pr from bottom to top

Fig.3 Graph of Horizontal velocity f for different values of Heat Source Parameter N from bottom to top

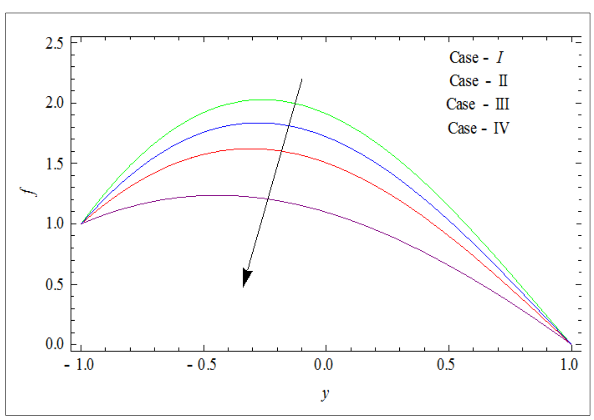

Fig .4 Graph of horizontal velocity f for comparison of Newtonian fluids and micropolar fluids

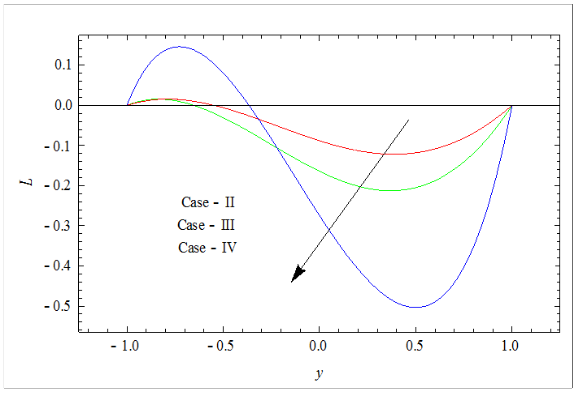

Fig.5 Graph of microrotation L for different values of M

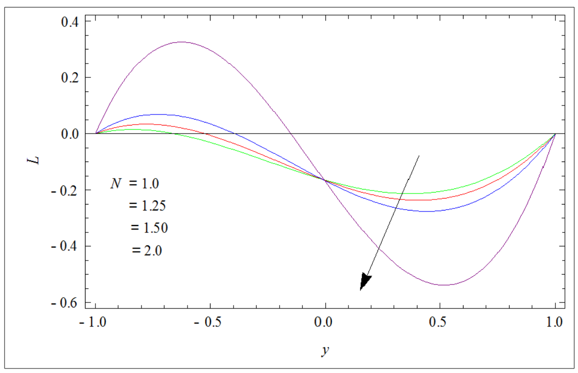

Fig.6 Graph of microrotation L for different values 0f N

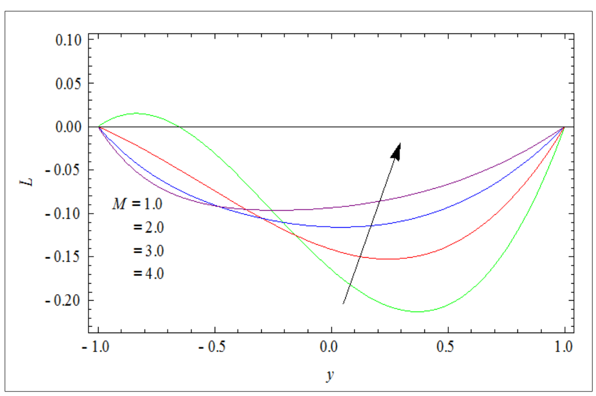

Fig.7 Graph of microrotation L for three different cases

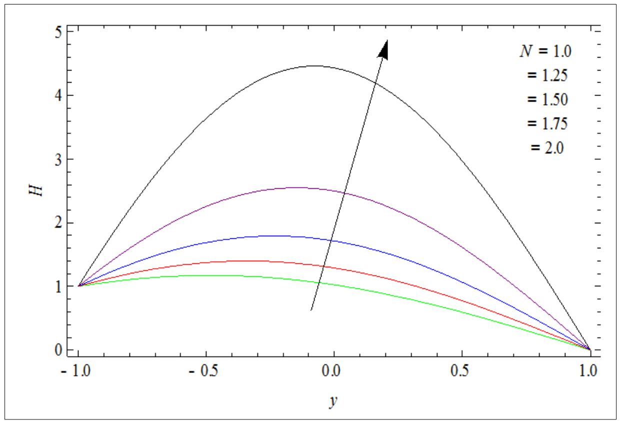

Fig.8 Graph of Temperature function H for different values of Heat Source Parameter N from bottom to top

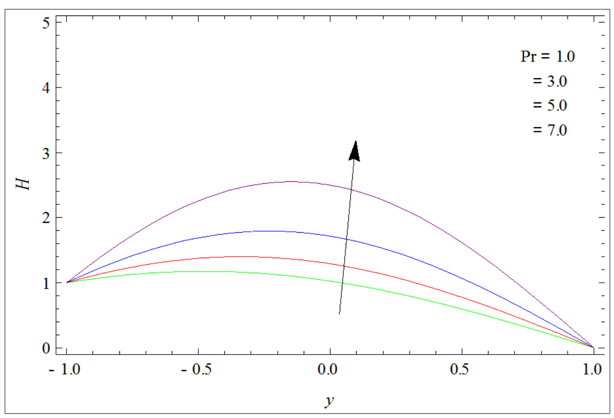

Fig.9 Graph of Temperature function H for different values of Parndtl Number Pr from bottom to top

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