Ideal $\mu$-Weak Structure Space with Some Applications

M. M. Khalaf, F. Ahmad, S. Hussain

1, 2, 3 Mathematics Department, Majmaah University, College of Science, Alzulfi, KSA

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ABSTRACT

A. Császár [13] introduced generalized structures on a nonempty set $X$ called a generalized topology. Also, A. Császár [2] introduced and studied more generalized topology and generalized continuity between generalized spaces. The purpose of this paper is to define $i_{\mu}^*$ and $c_{\mu}^*$ under more general conditions and to show that the important properties of these operations remain valid under these conditions. Also we define and study weaker form of $\tau_{\mu}$-open sets, $\tau_{\mu}^*$-continuity and an ideal $\ast$-$\mu$-weak structure space and $\mu$-weak local function with some applications on a set $X$ are defined and their properties are discussed.

KEYWORDS: ideal $\ast$-$\mu$- weak structure space, $\mu$-weak local function.

1. INTRODUCTION

A. Császár [3] has introduced a new notion of structures called weak structure. Every generalized topology is a weak structure. In [3], A. Császár, defined some structures and operators under more general conditions. Let $X$ be a nonempty set and $\tau \in P(X)$ where $P(X)$ is the power set of $X$. Then $\tau$ is called a weak structure $\tau \in P(X)$ and $\tau$ is called simply a space $(X, \tau)$. A nonempty set $X$ with a weak structure $\tau$ is denoted by the pair $(X, \tau)$ and is called simply a space $(X, \tau)$. The elements of $\tau$ are called $\tau$-open sets and the complements of $\tau$-open sets are called $\tau$-closed sets [14]. For a weak structure $\tau$ on $X$, the intersection of all $\tau$-closed sets containing a subset $A \subseteq X$ is denoted by $c_{\tau}(A)$. The union of all $\tau$-open sets contains in $A$ is denoted by $i_{\tau}(A)$. A subfamily $\mathcal{F} \subseteq P(X)$ is called a generalized topology $\mathcal{F}$ if $\phi \in \mathcal{F}$ and $\tau$ is closed under arbitrary union. A generalized topology on $X$ is said to be a quasi-topology [14] if $\tau$ is closed under finite intersection. The concept of ideals in topological spaces is treated in the classic text by Kuratowski [5] and Vaidyanathaswamy [11]. Jankovic and Ha-Mlett [4] investigated further properties of ideal spaces. An ideal $I$ on a topological space $(X, \tau)$ is a non-empty collection of subsets of $X$ which satisfies the following properties: (1) $A \in I$ and $B \subseteq A$ implies $B \in I$; (2) $A \in I$ and $B \in I$ implies $A \cup B \in I$.

An ideal topological space (or an ideal space) is a topological space $(X, \tau)$ with an ideal $I$ on $X$ and is denoted by $(X, \tau, I)$. For a subset $A \subseteq X$, $A^*(I, \tau) = \{x \in X : x \in A \cap \cup \in \tau \text{ for every } \tau \in \tau \}$ is called the local function of $A$ with respect to $I$ and $\tau$ (see [5]). We simply write $A^*$ in case there is no chance for confusion. A Kuratowski closure operator $\mathcal{C}^*(\cdot)$ or a topology $\tau^*(I, \tau)$ called the $I$-topology, finer than $\tau$, is defined by $\mathcal{C}^*(A) = A \cup A^*$ (see [11]).

Definition 1.1. let $(X, \tau)$ be a weak structure of $X$, $\lambda \subseteq X$ is called

$$
1. \eta_{\tau_{\mu}-set} \text{ if } \lambda = U \cup B \text{ where } U \in \tau_{\mu} \text{ and } B \in \tau_{\mu}-\text{ss}(\lambda)
$$

a Corresponding Author: Farooq Ahmad, Punjab Higher Education Department, Government College Bhakkar, Pakistan. +923336482936 Presently at Department of Mathematics, College of Science, Majmaah University, Alzulfi, KSA, +966597626606 Email: farooqgujjar@gmail.com & f.ishaq@mu.edu.sa
(2) $\beta_{t_\mu}$-set if $\lambda = U \cup B$ where $U \in t_\mu$ and $B$ is $t_\mu$-sc-b set

We denote the family of all $\eta_{t_\mu}$-set ( resp. $\beta_{t_\mu}$-set ) by $\eta_{t_\mu}(X)$ ( resp. $\beta_{t_\mu}(X)$ )

**Remark 1.1.** (1) Since $X$ is $t_\mu$-closed, as well as $t_\mu$-semiclosed and so every $t_\mu$-open set is $\eta_{t_\mu}$-sets, as well as $\beta_{t_\mu}$-set

(2) If $X \in t_\mu$ every $t_\mu$-ssc-set is an $\eta_{t_\mu}$-set. And every $t_\mu$-sc-set is $\beta_{t_\mu}$-set

(3) $\eta_{t_\mu}(X) \subseteq \beta_{t_\mu}(X)$

**Example 1.1.** Let $X = \{a, b, c, d\}$, $\tau = \{\phi, \{a\}, \{b\}, \{c\}\}$ and $\mu = \{a, b, c\}$. then $t_\mu = \{\phi, \{a\}, \{b\}, \{c\}\}$. If $\lambda = \{a\}$, then $i_{t_\mu}(\lambda) = \{a\}$, then $i_{t_\mu}(c_{t_\mu}(\lambda) = \{a\} \subseteq \lambda$, and so $\lambda$ is $t_\mu$-sc. Since $\lambda = \lambda \cap \{a\}, \{a\} \in t_\mu$, then $\lambda \in \eta_{t_\mu}(X)$.

**Theorem 1.1.** Let $(X, \tau)$ be a weak structure of $X$ and $A, \mu \in X$. then the following stamens are equivalent:

(1) $\lambda$ is an $\eta_{t_\mu}$-set

(2) $\lambda = U \cap c_{t_\mu}(t_\mu - sso(\lambda))$ for some $t_\mu$-open set $U$ and $t_\mu sso(X)$ is generalized topology on $X$

**Proof**

(1) $\Rightarrow$ (2). Suppose $\lambda$ is an $\eta_{t_\mu}$-set. Then $\lambda = U \cap F$ where $U \in t_\mu$ and $F$ is $t_\mu$-ssc-set. Now $\lambda \subseteq F$ implies that $c_{t_\mu}(t_\mu - sso(\lambda)) \subseteq F$, since $t_\mu - sso$ is a generalized space by Lemma 1.1 and so $\lambda \subseteq U \cap c_{t_\mu}(t_\mu - sso(\lambda)) \subseteq U \cap F = \lambda$ which implies that $\lambda = U \cap c_{t_\mu}(t_\mu - sso(\lambda))$.

(2) $\Rightarrow$ (3). By Lemma 1.1, since $t_\mu - sso(\lambda)$ is a generalized space and so $c_{t_\mu}(\lambda)$ is $t_\mu$-ssc. Hence $\lambda \in \eta_{t_\mu}(X)$

**Theorem 2.1.** Let $(X, \tau)$ be a weak structure of $X$, $\lambda, \mu \in X$, then the following stamens are equivalent:

(a) $\lambda$ is an $\eta_{t_\mu}$-set

(b) $c_{t_\mu}(t_\mu - sso(\lambda))$ is $t_\mu$-ssc

(c) $\lambda \cup (X - (t_\mu - sso(\lambda)))$ is $t_\mu$-ssc-set

(d) $\lambda \subseteq i_{t_\mu}(\lambda \cup (X - c_{t_\mu}(t_\mu - sso(\lambda)))$

Then (a) $\Rightarrow$ (b) $\Rightarrow$ (c) $\Rightarrow$ (d)

**Proof**

(a) $\Rightarrow$ (b). If $\lambda$ is an $\eta_{t_\mu}$-set, by Theorem 1.1, $\lambda = U \cap c_{t_\mu}(t_\mu - sso(\lambda))$, for some $t_\mu$-open set $U$. Now $c_{t_\mu}(t_\mu - sso(\lambda)) = \lambda = c_{t_\mu}(t_\mu - sso(\lambda)) - (U \cap c_{t_\mu}(t_\mu - sso(\lambda))) = c_{t_\mu}(t_\mu - sso(\lambda)) \cap ((X - U) \cup (X - c_{t_\mu}(t_\mu - sso(\lambda)))$

(b) $\Rightarrow$ (c) $c_{t_\mu}(t_\mu - sso(\lambda)) - \lambda$ is $t_\mu$-ssdc, implies that $X - (c_{t_\mu}(t_\mu - sso(\lambda)) - \lambda$ is $t_\mu$-ssc, and so $\lambda \cup (X - c_{t_\mu}(t_\mu - sso(\lambda)))$ is $t_\mu$-ssc.

(c) $\Rightarrow$ (d). Since $\lambda \subseteq \lambda \cup (X - c_{t_\mu}(t_\mu - sso(\lambda)))$, by (c), $\lambda \subseteq i_{t_\mu}(\lambda \cup (X - c_{t_\mu}(t_\mu - sso(\lambda)))$

**Example 1.2** Let $X = \{a, b, c, d\}$, $\tau = \{\phi, \{a\}, \{b\}, \{c\}\}$, $\lambda = \mu = \{a, b, c\}$, and $t_\mu = \{\phi, \{a\}, \{b\}, \{c\}\}$. If $\lambda = \{a, b, c\}$, then $c_{t_\mu}(t_\mu - sso(\lambda)) = \{d\}$ is $t_\mu$-ssc, and $\lambda \cup (X - c_{t_\mu}(t_\mu - sso(\lambda))) = \{a, b, c\} \cup (X - X) = \{a, b, c\} \cup \emptyset = \{a, b, c\} = \lambda$ is $t_\mu$-ssc-set but $\lambda$ is not an $\eta_{t_\mu}$-set
Corollary 1.1. Following statements.

Remark 1.2. Since $X$ is $\tau_\mu$-closed, every $\tau_\mu$-open set is $\tau_\mu$-locally closed. The following theorem is given some properties of $\tau_\mu$-locally closed sets.

Theorem 1.3. Let $(X, \tau)$ be a weak structure space and $\lambda, \mu \subseteq X$. Then the following hold:

(a) If $\lambda$ is $\tau_\mu$-locally closed, then $\lambda = U \cap c_{\tau_\mu}(\lambda)$ for some $\tau_\mu$-open set $U$.

(b) If $\tau$ is a generalized topology and $\lambda = U \cap c_{\tau_\mu}(\lambda)$ for some $\tau_\mu$-open set $U$, then $\lambda$ is $\tau_\mu$-locally closed.

Proof. (a) $\Rightarrow$ (b). Suppose $\lambda$ is $\tau_\mu$-locally closed set. Then $\lambda = U \cap F$ where $U \in \tau_\mu$ and $F$ is $\tau_\mu$-closed. Now $\lambda \subseteq F$ implies that $c_{\tau_\mu}(\lambda) \subseteq F$, by Lemma and so $\lambda \subseteq U \cap c_{\tau_\mu}(\lambda) \subseteq U \cap F = \lambda$ which implies that $\lambda = U \cap c_{\tau_\mu}(\lambda)$.

(b) $\Rightarrow$ (a). If $\tau$ is a generalized topology, then $c_{\tau_\mu}(\lambda)$ is $\tau_\mu$-closed and so $\lambda$ is $\tau_\mu$-locally closed.

Corollary 1.1 ([12], Theorem 2.8). Let $(X, \tau)$ be a generalized space and $\lambda, \mu \subseteq X$. Then the following are equivalent.

(a) $\lambda$ is $\tau_\mu$-locally closed.

(b) $\lambda = U \cap c_{\tau_\mu}(\lambda)$ for some $\tau_\mu$-open set $U$.

Theorem 1.4. Let $(X, \tau)$ be a WS space and $\lambda, \mu \subseteq X$. If $\lambda$ is $\tau_\mu$-open, then $\lambda$ is a $\tau_\mu$-sso-set and an $\eta_{\tau_\mu}$-set.

Proof. If $\lambda$ is $\tau_\mu$-open, then clearly, $\lambda$ is $\tau_\mu$-sso-set and an $\eta_{\tau_\mu}$-set.

Theorem 1.5. Let $(X, \tau)$ be a generalized space and $\lambda, \mu \subseteq X$. Consider the following statements.

(a) $\lambda$ is $\tau_\mu$-open.

(b) $\lambda$ is $\tau_\mu$-sso-set and an $\eta_{\tau_\mu}$-set.

(c) $\lambda$ is $\tau_\mu$-open and $\tau_\mu$-locally closed.

(d) $\lambda$ is $\tau_\mu$-preopen and $\tau_\mu$-locally closed.

(e) $\lambda$ is $\tau_\mu$-preopen and an $\eta_{\tau_\mu}$-set.

(f) $\lambda$ is $\tau_\mu$-preopen and a $\tau_\mu$-spo-set.

Then (a) $\Rightarrow$ (b) $\Rightarrow$ (c) $\Rightarrow$ (d) $\Rightarrow$ (e) $\Rightarrow$ (f). If $\tau_\mu$ is a quasi-topology, then (f) $\Rightarrow$ (a).

Proof. (a) $\Rightarrow$ (b). The proof follows from Theorem 1.3.

(b) $\Rightarrow$ (c). Since $\lambda$ is an $\eta_{\tau_\mu}$-set, $\lambda = U \cap c_{\tau_\mu}(\tau_\mu - sso(X)(\lambda))$ for some $\tau_\mu$-open set $U$. Since $\lambda$ is $\tau_\mu$-spo set, $c_{\tau_\mu}(c_{\tau_\mu}(\tau_\mu - sso(X)(\lambda))) = c_{\tau_\mu}(\lambda \cup c_{\tau_\mu}(i_{\tau_\mu}(c_{\tau_\mu}(\lambda))))$ by Lemma 2.2 of [3] and so $c_{\tau_\mu}(c_{\tau_\mu}(\tau_\mu - sso(X)(\lambda))) = c_{\tau_\mu}(i_{\tau_\mu}(c_{\tau_\mu}(\lambda)))) = c_{\tau_\mu}(\tau_\mu - sso(X)(\lambda))$ which implies that $c_{\tau_\mu}(\tau_\mu - sso(X)(\lambda))$ is $\tau_\mu$-closed. Hence $\lambda$ is $\tau_\mu$-locally closed.

The proofs of (c) $\Rightarrow$ (d), (d) $\Rightarrow$ (e) and (e) $\Rightarrow$ (f) are clear.

(f) $\Rightarrow$ (a). Conversely, suppose $\lambda$ is both a $\tau_\mu$-preopen set and a $\tau_\mu$-sso- set. Then $\lambda = U \cap F$ where $U$ is $\tau_\mu$-open and $i_{\tau_\mu}(c_{\tau_\mu}(F)) = i_{\tau_\mu}(F)$. Since $\lambda$ is $\tau_\mu$-preopen, $\lambda \subseteq i_{\tau_\mu}(c_{\tau_\mu}(\lambda)) = i_{\tau_\mu}(c_{\tau_\mu}(U \cap F)) \subseteq i_{\tau_\mu}(c_{\tau_\mu}(U)) \cap i_{\tau_\mu}(c_{\tau_\mu}(F)) = i_{\tau_\mu}(c_{\tau_\mu}(U)) \cap i_{\tau_\mu}(F)$. Hence $\lambda = \lambda \cap U \subseteq i_{\tau_\mu}(c_{\tau_\mu}(U)) \cap i_{\tau_\mu}(F) \cap U = U \cap i_{\tau_\mu}(F) \subseteq U \cap F = \lambda$. Therefore, $\lambda = U \cap i_{\tau_\mu}(F)$. Since $\tau_\mu$ is a quasi-topology, $\lambda$ is $\tau_\mu$-open.
Definition 1.3. Let \((X, \tau)\) and \((Y, \gamma)\) be a weak structures, \(\mu \subset X, \mu' \subset Y\). A function \(f: X \rightarrow Y\) is said to be \((\tau_\mu\)-continuous \[2\]) (resp., \((\eta_\tau _\mu\)-continuous, \(\beta_\tau _\mu\)-Continuous, \(\tau_\mu s\)-continuous, \(\tau_\mu ss\)-continuous, \(\tau_\mu p\)-continuous, \(\tau_\mu lc\)-continuous) if \(f^{-1}(V) \cap \mu \in \tau_\mu (\text{resp., } \eta_\tau _\mu \cap \mu \in \eta_\tau _\mu , \beta_\tau _\mu , f^{-1}(V) \cap \mu \in \tau_\mu , f^{-1}(V) \cap \mu \in \tau_\mu - so(X) , f^{-1}(V) \cap \mu \in \tau_\mu - sso(X), f^{-1}(V) \cap \mu \in \tau_\mu - po(X) , f^{-1}(V) \cap \mu \in \tau_\mu - lc(X))\) for every \(V \in \gamma_f(\mu)\).

The following theorem gives decompositions of \((\tau_\mu, \gamma_f(\mu))\)-continuous functions, the proof of which follows from Theorem 1.5.

Theorem 1.6. Let \((X, \tau)\) and \((Y, \gamma)\) be a weak structure spaces and \(\mu \subset X, \mu' \subset Y\) where \((X, \tau)\) is a quasi topological space, and let \(f: X \rightarrow Y\) be a function. Then the following are equivalent:

(a) \(f\) is \(\tau_\mu\)-continuous.
(b) \(f\) is \(\eta_\tau _\mu \)-continuous and \(\tau_\mu s\)-continuous.
(c) \(f\) is \(\tau_\mu lc\)-continuous and \(\tau_\mu ss\)-continuous.
(d) \(f\) is \(\tau_\mu lc\)-continuous and \(\tau_\mu p\)-continuous.
(e) \(f\) is \(\eta_\tau _\mu \)-continuous and \(\tau_\mu p\)-continuous.
(f) \(f\) is \(\beta_\tau _\mu\)-continuous and \(\tau_\mu p\)-continuous.

Theorem 1.7. Let \((X, \tau)\) and \((Y, \gamma)\) be \(\tau S', s, \mu \subset X\), and \(f: X \rightarrow Y\). Then \(f\) is \(\tau_\mu^*\)-continuous if for each \(v \in \gamma_f(\mu)\), we have \(\mu \cap f^{-1}(v) \in \tau_\mu^*\).

Definition 1.4. Let \(f: (X, \tau) \rightarrow (Y, \gamma)\) be a mapping from a weak structure \((X, \tau)\) to another \((Y, \gamma)\), and \(\mu \subset X\). Then \(f\) is called:

(i) an \(\tau_\mu\)-semicontinuous (briefly, \(\tau_\mu sc\)) mapping if for each \(v \in \gamma_f(\mu)\), we have \(\mu \cap f^{-1}(v) \in \tau_\mu - sp\);
(ii) an \(\tau_\mu\)-precontinuous (briefly, \(\tau_\mu pc\)) mapping if for each \(v \in \gamma_f(\mu)\), we have \(\mu \cap f^{-1}(v) \in \tau_\mu - po\);
(iii) an \(\tau_\mu\)-strongly semi continuous (briefly, \(\tau_\mu ssc\)) mapping if for each \(v \in \gamma_f(\mu)\), we have \(\mu \cap f^{-1}(v) \in \tau_\mu - sso\);
(iv) an \(\tau_\mu\)-semi precontinuous (briefly, \(\tau_\mu spc\)) mapping if for each \(v \in \gamma_f(\mu)\), we have \(\mu \cap f^{-1}(v) \in \tau_\mu - spo\);

Remark 1.3. The implications between these different concepts are given by the following diagram:

\[
\begin{array}{c}
\tau - c \\
\downarrow \\
\tau_\mu - c \\
\downarrow \\
\tau_\mu - ssc \\
\downarrow \\
\tau_\mu - pc \\
\downarrow \\
\tau_\mu - sc \\
\downarrow \\
\tau_\mu - ssc \\
\downarrow \\
\tau_\mu - spc \\
\end{array}
\]
Examples 1.3. Let \( X = \{a, b, c\} \), \( \tau = \{\emptyset, \{a\}, \{b\}\} \), \( \mu = \{b, c\} \), \( \tau_\mu = \{\emptyset, \{b\}\} \), \( Y = \{x, y\} \), \( \gamma = \{\phi, \{x\}\} \), \( f : X \to Y \),  and  \( f(a) = f(b) = x \), \( f(c) = y \), then \( f(\mu) = \{x, y\} \), then \( \gamma f(\mu) = \emptyset, \{x, y\} \), since: \( f^{-1}(\emptyset) = \emptyset \cap \mu = \emptyset \in \delta_\mu \), then \( f^{-1}(x) = \{a, b\} \cap \mu = \{b\} \in \delta_\mu \), then \( \tau \) is continuous. and \( f^{-1}(a) = \emptyset, f^{-1}(x) = \{a, b\} \notin \tau \), then \( f \) is not \( \tau \)-continuous mapping.

2. IDEAL \( \Rightarrow \mu \)-WEAK STRUCTURE SPACE AND \( \mu \)-WEAK LOCAL FUNCTION

For a subset \( \mu \subseteq X \). Let \( \tau_\mu \) and \( I \) a \( \mu \)-weak structure space and an ideal \( \mu \)-weak structure space (denoted by \( (\tau_\mu)_I \) on a set \( X \). Let \( \tau_\mu \) is a \( \mu \)-weak structure space and \( \mathfrak{A}_\tau(x) = \{U : x \in U, U \in \tau_\mu\} \) be the family of \( \tau_\mu \)-open sets which contain a point \( x \in X \).

**Definition 2.1.** An ideal \( I \) on a \( \mu \)-weak structure space \( (X, \tau_\mu) \) is a non-empty collection of subsets of \( X \) which satisfies the following properties: (1) \( A \subseteq I \) and \( B \subseteq A \) implies \( B \in I \); (2) \( A \in I \) and \( B \in I \) implies \( A \cup B \in I \).

**Definition 2.2.** Let \( (X, (\tau_\mu)_I) \) be an ideal \( \mu \)-weak structure space. For a subset \( A, \mu \subseteq X \), \( A^{*}_\mu (I, \tau_\mu) = \{x \in X : U \cap A \notin I \text{ for every } U \in \mathfrak{A}_\tau(x)\} \) is called the \( \mu \)-weak local function of \( A \) with respect to \( I \) and \( \tau_\mu \).

We will simply write \( A^{*}_\mu \) for \( A^{*}_\mu (\tau_\mu, I) \).

**Theorem 2.1.** Let \( (\tau_\mu) \) be a \( \mu \)-weak structure on a set \( X, I, J \) ideals on \( X \) and \( A, B, \mu \) be subsets of \( X \). The following properties hold:

1. If \( A \subseteq B \), then \( A^{*}_\mu \subseteq B^{*}_\mu \).
2. If \( I \subseteq J \), then \( A^{*}_\mu (J) \subseteq A^{*}_\mu (I) \).
3. \( A^{*}_\mu = c_{\tau_\mu} (A^{*}_\mu) \subseteq c_{\tau_\mu} (A) \).
4. \( A^{*}_\mu \cup B^{*}_\mu \subseteq (A \cup B)^{*}_\mu \).
5. \( (A^{*}_\mu)^{*}_\mu \subseteq A^{*}_\mu \).
6. If \( A \subseteq I \), then \( A^{*}_\mu = \emptyset \).

**Proof.** (1) Let \( A \subseteq B \), then \( x \notin B^{*}_\mu \). Implies that \( U \cap B \in I \) for some \( U \in \mathfrak{A}_\tau(x) \). Since \( U \cap A \subseteq U \cap B \) and \( U \cap B \in I \). Then \( U \cap A \in I \) from the definition of ideals. Thus, we have \( x \notin A^{*}_\mu \). Hence we have \( A^{*}_\mu \subseteq B^{*}_\mu \).

(2) Let \( I \subseteq J \) and \( x \in A^{*}_\mu (J) \). Then \( U \cap A \notin J \) for every \( U \in \mathfrak{A}_\tau(x) \). By hypothesis, \( U \cap A \notin I \). So \( x \in A^{*}_\mu (I) \).

(3) Since \( A^{*}_\mu \subseteq c_{\tau_\mu} (A^{*}_\mu) \). Let \( x \in c_{\tau_\mu} (A^{*}_\mu) \). Then \( A^{*}_\mu \cap U \neq \emptyset \) for every \( U \in \mathfrak{A}_\tau(x) \). Therefore, there exists some \( y \in A^{*}_\mu \cap U \) and \( y \in \mathfrak{A}_\tau(y) \). Since \( y \in A^{*}_\mu \), \( A \subseteq U \notin I \) and hence \( x \in A^{*}_\mu \). Hence \( c_{\tau_\mu} (A^{*}_\mu) \subseteq A^{*}_\mu \) and \( c_{\tau_\mu} (A^{*}_\mu) = A^{*}_\mu \). Again, let \( x \in c_{\tau_\mu} (A^{*}_\mu) = A^{*}_\mu \). Then \( A \cap U \notin I \) for every \( U \in \mathfrak{A}_\tau(x) \). This implies \( A \cap U \neq \emptyset \) for every \( U \in \mathfrak{A}_\tau(x) \). Therefore, \( x \in c_{\tau_\mu} (A) \).

(4) This follows from (1).

(5) Let \( x \in \left( A^{*}_\mu \right)^{*}_\mu \). Then, for every \( U \in \mathfrak{A}_\tau(x) \), \( U \cap A^{*}_\mu \notin I \) and hence \( U \cap A^{*}_\mu \neq \emptyset \). Let \( y \in U \cap A^{*}_\mu \). Then \( U \in \mathfrak{A}_\tau(y) \) and \( y \in A^{*}_\mu \). Hence we have \( U \cap A \notin I \) and \( x \in A^{*}_\mu \). This shows that \( A^{*}_\mu \subseteq A^{*}_\mu \).

(6) Suppose that \( x \in A^{*}_\mu \). Then for any \( U \in \mathfrak{A}_\tau(x) \), \( U \cap A \notin I \). But, since \( A \subseteq I \), \( U \cap A \subseteq I \). This is a contradiction. Hence \( A^{*}_\mu = \emptyset \).

The converses of Theorem 2.1 need not be true as seen in the following examples.

**Example 2.1.** Let \( X = \{a, b, c, d\} \), \( \mu = \{a, b, c\} \) and \( \tau_\mu = \tau = \{\emptyset, \{a\}, \{b\}, \{a, b, c\}, \{b, c\}, \{a, c\}\} \) be a \( \mu \)-weak structure on the set \( X \) with \( I = \{\emptyset, \{a\}\} \). For \( A = \{a, c\} \) and \( B = \{b, c\} \), we have \( A^{*}_\mu = \{c, d\} \subseteq B^{*}_\mu = \{b, c, d\} \) but \( A \not\subseteq B \).
Example 2.2. Let $X = \{a, b, c, d\}, \mu = \{a, b, c\}$ and $\tau_\mu = \tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, c\}\}$ be a $\mu$-weak structure on the set $X$ with $I = \{\emptyset, \{b\}\}$ and $J = \{\emptyset, \{a\}\}$. It is easily seen that $I \not\subseteq J$ but for $A = \{a, c\}$ we have $A^*_\mu(I) = \{c, d\} \subseteq A^*_\mu(I) = \{a, c, d\}$.

Example 2.3. Let $X = \{a, b, c\}, \mu = \{a, c\}$ and $\tau_\mu = \tau = \{\emptyset, \{a\}, \{c\}\}$ be a $\mu$-weak structure on the set $X$ with $I = \{\emptyset, \{a\}\}$. For $A = \{a, c\}$, we have $c_\mu(A) = X \neq A^*_\mu(I) = \{b, c\} = c_\mu(A^*_\mu(I))$.

Example 2.4. Let $X = \{a, b, c\}, \mu = \{a, b, c\}$ and $\tau_\mu = \tau = \{\emptyset, \{a, b\}, \{b, c\}\}$ be a $\mu$-weak structure on the set $X$ with $I = \{\emptyset, \{b\}\}$. For $A = \{a\}$ and $B = \{c\}$ we have $A^*_\mu(I) = \{a\}, B^*_\mu(I) = \{c\}$ and $(A \cup B)^*_\mu(I) = \emptyset$. Hence, we have $(A \cup B)^*_\mu = A^*_\mu \cup B^*_\mu$.

Definition 2.3. Let $(X, (\tau_\mu))$ be an ideal $\mu$-weak structure space. The set operator $c^*_\mu$ is called a $\mu$-weak structure $*$-closure and is defined as follows: $c^*_\mu(A) = A \cup A^*_\mu$ for $A, \mu \subseteq X$. We will denote by $\tau^*_\mu(\tau_\mu)$ the $\mu$-weak structure determined by $c^*_\mu$, that is, $\tau^*_\mu(\tau_\mu) = \{U \subseteq X : c^*_\mu(\mu - U) = \mu - U\} = \{U \subseteq X : i^*_\mu(U) = U\}$. $\tau^*_\mu(\tau_\mu)$ is called an $\ast$-$\mu$-weak structure which is finer than $\tau_\mu$. The elements of $\tau^*_\mu(\tau_\mu)$ are said to be $\tau^*_\mu$-open and the complement of a $\tau^*_\mu$-open set is said to be $\tau^*_\mu$-closed.

Throughout the paper we simply write $\tau^*_\mu(I)$ for $\tau^*_\mu(\tau_\mu)$. If $I$ is an ideal on $X$, then $(X, \tau^*_\mu)$ is called an ideal $\ast$-$\mu$-weak structure space.

Proposition 2.1. The set operator $c^*_\mu$ satisfies the following conditions:
1. $A \subseteq c^*_\mu(A)$
2. $c^*_\mu(\emptyset) = \emptyset$ and $c^*_\mu(X) = X$
3. If $A \subseteq B$, then $c^*_\mu(A) \subseteq c^*_\mu(B)$
4. $c^*_\mu(A) \cup c^*_\mu(B) \subseteq c^*_\mu(A \cup B)$
5. $c^*_\mu(A \cap B) \subseteq c^*_\mu(A) \cap c^*_\mu(B)$

Proof. The proofs are clear from Theorem 2.1 and the definition of $c^*_\mu$.

3. IDEAL $\mu$-WEAK STRUCTURE SPACE

In this section let a $\mu$-weak structure $\tau_\mu$ have the property any finite intersection of $\tau_\mu$-open sets is $\tau_\mu$-open and $(X, \tau_\mu)$ is called an ideal $\mu$-weak structure space with this property.

Proposition 3.1. Let $(X, \tau_\mu)$ be an ideal $\mu$-weak structure space and $A, \mu \subseteq X$, then $U \cap A^*_\mu = U \cap (U \cap A)^*_\mu \subseteq (U \cap A)^*_\mu$ for every $U \in \tau_\mu$.

Proof. Suppose that $U \in \tau_\mu$ and $x \in U \cap A^*_\mu$. Then $x \in U$ and $x \in A^*_\mu$. Let $V \in \mathfrak{V}(\tau_\mu)(x)$. Then $V \subseteq U \cap A^*_\mu$. Moreover, $U \cap A^*_\mu \subseteq U \cap (U \cap A)^*_\mu$ and by Theorem 2.3 (1) $(U \cap A)^*_\mu \subseteq A^*_\mu$ and $U \cap (U \cap A)^*_\mu \subseteq U \cap A^*_\mu$. Therefore, $U \cap A^*_\mu \subseteq U \cap (U \cap A)^*_\mu$.

Theorem 3.2. Let $(X, \tau_\mu)$ be an ideal $\mu$-weak structure space $A, B, \mu \subseteq X$. Then $A^*_\mu \cup B^*_\mu = (A \cup B)^*_\mu$.

Proof. It follows from Theorem 2.1 that $A^*_\mu \cup B^*_\mu \subseteq (A \cup B)^*_\mu$. To prove the reverse inclusion, let $x \notin A^*_\mu \cup B^*_\mu$. Then $x$ belongs neither to $A^*_\mu$ nor to $B^*_\mu$. Therefore there exist $U, V \in \mathfrak{V}(\tau_\mu)(x)$ such that $U \cap A \in I$ and $V \cap B \in I$. Since $I$ is additive, $(U \cap A) \cup (V \cap B) \in I$. Moreover, since $I$ is hereditary and

\[
(U \cap A) \cup (V \cap B) = [(U \cap A) \cup V] \cap [(U \cap A) \cup B] = (U \cup V) \cap (A \cup B) \cap (U \cup B) \\
\supseteq (U \cap V) \cap (A \cup B) \cap (U \cup B),
\]

Thus, $(U \cap V) \cap (A \cup B) \cap (U \cup B) \in I$. Since $(U \cap V) \in \mathfrak{V}(\tau_\mu)(x)$, $x \notin (A \cup B)^*_\mu$. Hence $(A \cup B)^*_\mu \subseteq A^*_\mu \cup B^*_\mu$. Hence we obtain $A^*_\mu \cup B^*_\mu = (A \cup B)^*_\mu$.

Theorem 3.3. Let $(X, \tau_\mu)$ be an ideal $\mu$-weak structure space $A, B, \mu \subseteq X$. Then the following properties hold:
1. $c^*_\mu(A \cup B) = c^*_\mu(A) \cup c^*_\mu(B)$
2. $c^*_\mu(A) = c^*_\mu(c^*_\mu(A))$

Proof. By Theorem 3.2, we obtain
(1) \( c_{\tau_\mu}(A \cup B) = (A \cup B)_{\tau_\mu} \cup (A \cup B) = (A_{\tau_\mu} \cup B_{\tau_\mu}) \cup (A \cup B) = c_{\tau_\mu}(A) \cup c_{\tau_\mu}(B) \)

(2) \( c_{\tau_\mu}(c_{\tau_\mu}(A)) = c_{\tau_\mu}(A_{\tau_\mu} \cup A) = (A_{\tau_\mu} \cup A)_{\tau_\mu} \cup (A_{\tau_\mu} \cup A) = ((A_{\tau_\mu})_{\tau_\mu} \cup A_{\tau_\mu}) \cup (A_{\tau_\mu} \cup A) = A_{\tau_\mu} \cup A = c_{\tau_\mu}(A) \)

**Corollary 3.1.** Let \((X, \tau)\) be an ideal \(\mu\)-weak structure space, \(A, \mu \subseteq X\) and \(c_{\tau_\mu}(A) = A \cup A_{\tau_\mu}\). Then \(\gamma_{\tau_\mu} = \{U \subseteq X : c_{\tau_\mu}(\mu - U) = \mu - U\}\) is a topology for \(X\)

**Proof.** By Proposition 2.1 and Theorem 3.3, \(c_{\tau_\mu}(A) = A \cup A_{\tau_\mu}\) is a Kuratowski closure operator. Therefore, \(\gamma_{\tau_\mu}\) a topology for \(X\).

**Lemma 3.1.** Let \((X, \tau)\) be an ideal \(\mu\)-weak structure space and \(A, B, \mu \subseteq X\). Then \(A_{\tau_\mu} - B_{\tau_\mu} = (A - B)_{\tau_\mu} - B_{\tau_\mu}\)

**Proof.** By Theorem 3.2, \(A_{\tau_\mu} = [(A - B) \cap (A \cap B)]_{\tau_\mu} = (A - B)_{\tau_\mu} \cup (A \cap B)_{\tau_\mu} \subseteq (A - B)_{\tau_\mu} \cup B_{\tau_\mu}\). Thus \(A_{\tau_\mu} - B_{\tau_\mu} \subseteq (A - B)_{\tau_\mu} - B_{\tau_\mu}\). By Theorem 2.3, \((A - B)_{\tau_\mu} \subseteq A_{\tau_\mu}\) and hence \((A - B)_{\tau_\mu} - B_{\tau_\mu} \subseteq A_{\tau_\mu} - B_{\tau_\mu}\). Hence \(A_{\tau_\mu} - B_{\tau_\mu} = (A - B)_{\tau_\mu} - B_{\tau_\mu}\)

**Corollary 3.2.** Let \((X, \tau)\) be an ideal \(\mu\)-weak structure space and \(A, B, \mu \subseteq X\) with \(B \in I\). Then \((A \cup B)_{\tau_\mu} = A_{\tau_\mu} = (A - B)_{\tau_\mu}\)

**Proof.** Since \(B \in I\), by Theorem 2.1, \(B_{\tau_\mu} = \emptyset\). By Lemma 3.1, \(A_{\tau_\mu} = (A - B)_{\tau_\mu}\) and by Theorem 3.2 \((A \cup B)_{\tau_\mu} = A_{\tau_\mu} = (A - B)_{\tau_\mu}\)

**Theorem 3.7.** Let \((X, \tau)\) be an ideal \(\mu\)-weak structure space. Then \(\beta(\tau_\mu) = \{V - l : V \in \tau_\mu, \ l \in I\}\) is a basis for \((\gamma)_{\tau_\mu}\).

**Proof.** Let \((X, \tau)\) be an ideal \(\mu\)-weak structure space, \(\mu \subseteq X\). It is obvious that \(A\) is \((\gamma)_{\tau_\mu}\)-closed if and only if \(A_{\tau_\mu} \subseteq A\). Now we have \(U \in (\gamma)_{\tau_\mu}\) if and only if \((\mu - U)_{\tau_\mu} \subseteq \mu - U\) if and only if \(U \subseteq \mu - \mu_{\tau_\mu}\). Therefore \(x \in U \in (\gamma)_{\tau_\mu}\) implies that \(x \in \mu_{\tau_\mu}\). This implies that there exists \(V \in \forall_{\tau_\mu}(x)\) such that \(V \cap (\mu - U) \in I\). Now let \(l = V \cap (\mu - U)\) and we have \(x \in V - l \subseteq U\), where \(V \in \forall_{\tau_\mu}(x)\) and \(l \in I\). Now we need only show that \(\beta\) is \(\mu\)-closed under finite intersection. Let \(A, B \in \beta\), then \(A = H - l\) and \(B = K - j\), where \(H, K \in \tau_{\mu}\) and \(l, j \in I\). Now we have

\[
(H - l) \cap (K - j) = [H \cap (X - l)] \cap [K \cap (\mu - j)]
\]

\[
= [H \cap K] \cap [\mu - l \cup (\mu - j)]
\]

\[
= [H \cap K] \cap [l \cup (\mu - j)]
\]

\[
= [H \cap K] - (l \cup j).
\]

Since \((l \cup j) \in I\) and \([H \cap K] \in \tau_{\mu}, A \cap B \in \beta\). Therefore \(\beta\) is \(\mu\)-closed under finite intersection. Thus \(\beta = \{V - l : V \in \tau_\mu, \ l \in I\}\) is a basis for \((\gamma)_{\tau_\mu}\).

### 4. APPLICATIONS

**Definition 4.1.** Let \((X, \tau_{\mu})\) be \(\mu\)-weak structure, \(A, \mu \subseteq X\). A point \(x \in X\) is called accumulation points of a subset \(A\) iff for every \(\tau_{\mu}\)-open sets \(\lambda\) containing \(x\), \((X - \lambda) \cap A \neq \emptyset\).

**Remark 4.1.** The set of all \(\tau_{\mu}\)-accumulation points of a subset \(A\) of a \(\mu\)-weak structure \((X, \tau_{\mu})\) is \(A_{\tau_{\mu}} = \{x \in X : U \cap A\text{ is ininite for every } U \in \mathcal{N}(x)\}\) where \(\mathcal{N}\) is the ideal of all nowhere dense sets.

**Definition 4.2.** Let \((X, \tau_{\mu})\) be \(\mu\)-weak structure, \(A, \mu \subseteq X\). A point \(x \in X\) is called condensation points of \(A\) iff for every \(U \in \mathcal{N}(x)\), \(U \cap A\) is uncountable. The set of all condensation points of \(A\) is \(A^* = \{x \in X : U \cap A\text{ is uncountable for every } U \in \mathcal{N}(x)\}\). It is interesting to note that \(A_{\tau_{\mu}}(l)\) is a generalization of closure points, \(\tau_{\mu}\)-accumulation points and condensation points.
We call a class \(\Omega \subseteq P(X)\) a generalized topology \cite{2} (briefly, \(GT\)) if \(\phi \in \Omega\) and the arbitrary union of elements of \(\Omega\) belongs to \(\Omega\).

A set \(X\) with a \(GT\ \Omega\) on it is called a generalized topological space (briefly, \(GT\ S\)) and is denoted by \((X, \Omega)\).

The proofs of the following theorem is clear.

**Theorem 4.1.** For a \(\mu\)-weak structure space \((X, \tau_\mu), \mu \subseteq X\), the following properties are equivalent:
1. \(\tau_\mu = \Omega\) i.e. \(\tau\) is a generalized topology in the sense of Császár.
2. \(i_{\tau_\mu}(A)\) is \(\tau_\mu\)-open for every subset \(A\) of \(X\).
3. \(c_{\tau_\mu}(A)\) is \(\tau_\mu\)-closed for every subset \(A\) of \(X\).

**Remark 4.2.** For a \(\mu\)-weak structure space \((X, \tau_\mu), \mu \subseteq X\), and \(\tau_\mu^* = \{A \subseteq X : A = i_{\tau_\mu}(A)\}\). Then:
1. \(\tau_\mu^*\) is a \(GT\) containing \(\tau_\mu\).
2. \(\tau_\mu \subseteq \tau_\mu^* \subseteq \tau_\mu^*(I)\).

**Theorem 4.2.** Let \((X, \tau_\mu)\) be an ideal \(\mu\)-weak structure space. Then, \(\tau_\mu^*(I)\) is a \(GT\) containing \(\tau_\mu^*\).

**Proof.** If \(A \in \tau_\mu^*, A = i_{\tau_\mu}(A) \subseteq i_{\tau_\mu}^*(A)\) and hence \(A \in \tau_\mu^*(I)\). Therefore, \(\tau_\mu^*(I)\) contains \(\tau_\mu^*\). Let \(A_\alpha \in \tau_\mu^*(I)\) for each \(\alpha \in \Delta\). Then \(A_\alpha = i_{\tau_\mu}^*(A_\alpha) \subseteq i_{\tau_\mu}^*(\cup A\alpha)\) for each \(\alpha \in \Delta\). Hence \(\cup A\alpha \subseteq i_{\tau_\mu}^*(\cup A\alpha)\) and \(\cup A\alpha = i_{\tau_\mu}^*(\cup A\alpha)\). Therefore, \(\cup A\alpha \in \tau_\mu^*(I)\). And \(\tau_\mu^*(I)\) is a \(GT\).

**Definition 4.3.** Let \((X, \tau_\mu)\) be an ideal \(\mu\)-weak structure space and \(A, \mu \subseteq X\). Then
1. \(A \in \tau_\mu - \text{sso}(\tau_\mu)\) if \(A \subseteq i_{\tau_\mu}(c_{\tau_\mu}(i_{\tau_\mu}(A)))\)
2. \(A \in \tau_\mu - \text{so}(\tau_\mu)\) if \(A \subseteq c_{\tau_\mu}(i_{\tau_\mu}(A))\)
3. \(A \in \tau_\mu - \text{po}(\tau_\mu)\) if \(A \subseteq i_{\tau_\mu}(c_{\tau_\mu}(i_{\tau_\mu}(A)))\)
4. \(A \in \tau_\mu - \text{s-po}(\tau_\mu)\) if \(A \subseteq c_{\tau_\mu}(i_{\tau_\mu}(c_{\tau_\mu}(i_{\tau_\mu}(A))))\)

**Lemma 4.5.** Let \((X, \tau_\mu)\) be an ideal \(\mu\)-weak structure space, we have the following:
1. \(\tau_\mu \subseteq \tau_\mu - \text{sso}(\tau_\mu) \subseteq \tau_\mu - \text{so}(\tau_\mu) \subseteq \tau_\mu - \text{s-po}(\tau_\mu)\)
2. \(\tau_\mu \subseteq \tau_\mu - \text{sso}(\tau_\mu) \subseteq \tau_\mu - \text{s-po}(\tau_\mu) \subseteq \tau_\mu - \text{po}(\tau_\mu)\)

**Definition 4.4** Let \((X, \tau_\mu)\) be an ideal \(\mu\)-weak structure space. The ideal \(\mu\)-weak structure space is said to be \(\tau_\mu\)-externally disconnected if \(c_{\tau_\mu}(A) \in \tau_\mu\) for \(A, \mu \subseteq X\) and \(A \in \tau_\mu\).

**Theorem 4.3.** Let \((X, \tau_\mu)\) be an ideal \(\mu\)-weak structure space. Then the implications \((1) \Rightarrow (2), (3) \Rightarrow (4)\) and \((5) \Rightarrow (6) \Rightarrow (7)\) hold. If \(\tau_\mu = \tau_\mu^*\) then the following statements are equivalent:
1. \((X, \tau_\mu)\) is \(\tau_\mu\)-externally disconnected.
2. \(i_{\tau_\mu}(A)\) is \(\tau_\mu\)-closed for each \(\tau_\mu\)-closed set \(A, \mu \subseteq X\)
3. \(c_{\tau_\mu}(i_{\tau_\mu}(A)) \subseteq i_{\tau_\mu}(c_{\tau_\mu}(i_{\tau_\mu}(A)))\) for each \(A, \mu \subseteq X\)
4. \(A \in \tau_\mu - \text{po}(\tau_\mu)\) for each \(A \in \tau_\mu - \text{so}(\tau_\mu)\)
5. \(c_{\tau_\mu}(A) \in \tau_\mu\) for each \(A \in \tau_\mu - \text{s-po}(\tau_\mu)\)
6. \(A \in \tau_\mu - \text{po}(\tau_\mu)\) for each \(A \in \tau_\mu - \text{po}(\tau_\mu)\)
7. \(A \in \tau_\mu - \text{so}(\tau_\mu)\) if and only if \(A \in \tau_\mu - \text{so}(\tau_\mu)\)

**Proof.** (1) \(\Rightarrow\) (2). Let \(A\) be a \(\tau_\mu\)-closed set. Then \(\mu - A\) is \(\tau_\mu\)-open. By using (1), \(c_{\tau_\mu}(\mu - A) = \mu - i_{\tau_\mu}(A) \subseteq \tau_\mu\). Thus \(i_{\tau_\mu}(A)\) is \(\tau_\mu\)-closed

(2) \(\Rightarrow\) (3). Let \(\mu, A \subseteq X\). Then \(\mu - i_{\tau_\mu}(A)\) is \(\tau_\mu\)-closed and by (2) \(i_{\tau_\mu}(\mu - i_{\tau_\mu}(A))\) is \(\tau_\mu\)-closed. Therefore, \(c_{\tau_\mu}(i_{\tau_\mu}(A))\) is \(\tau_\mu\)-open and hence \(c_{\tau_\mu}(i_{\tau_\mu}(A)) \subseteq i_{\tau_\mu}(c_{\tau_\mu}(i_{\tau_\mu}(A)))\)

(3) \(\Rightarrow\) (4). Let \(A \in \tau_\mu - \text{so}(\tau_\mu)\). By (3), we have \(A \subseteq c_{\tau_\mu}(i_{\tau_\mu}(A)) \subseteq i_{\tau_\mu}(c_{\tau_\mu}(i_{\tau_\mu}(A)))\). Thus, \(A \in \tau_\mu - \text{po}(\tau_\mu)\)
(4) ⇒ (5). Let $A \in \tau_{\mu}-spo(\tau_I)$. Then $c^*_{\mu}(A) = i_{\mu}(c^*_{\mu}(A))$ and $c^*_{\mu}(A) \in \tau_{\mu-so}(\tau_I)$. By (4), $c^*_{\mu}(A) \in \tau_{\mu-po}(\tau_I)$. Thus $c^*_{\mu}(A) \subseteq i_{\mu}(c^*_{\mu}(A))$ and hence $c^*_{\mu}(A)$ is $\tau_{\mu}$-open.

(5) ⇒ (6). Let $A \in \tau_{\mu}-spo(\tau_I)$. By (5), $c^*_{\mu}(A) = i_{\mu}(c^*_{\mu}(A))$. Thus, $A \in c^*_{\mu}(A) \subseteq i_{\mu}(c^*_{\mu}(A))$ and hence $A \in \tau_{\mu-po}(\tau_I)$.

(6) ⇒ (7). Let $A \in \tau_{\mu}-so(\tau_I)$ then $A \in \tau_{\mu}-spo(\tau_I)$. Then by (6), $A \in \tau_{\mu-po}(\tau_I)$. Since $A \in \tau_{\mu-so}(\tau_I)$ and $A \in \tau_{\mu-po}(\tau_I)$, then $A \in \tau_{\mu-ssso}(\tau_I)$.

(7) ⇒ (1). Let $A$ be a $\tau_{\mu}$-open set. Then $c^*_{\mu}(A) \in \tau_{\mu-so}(\tau_I)$ and by using (7), $c^*_{\mu}(A) \in \tau_{\mu-ssso}(\tau_I)$. Therefore, $c^*_{\mu}(A) \subseteq i_{\mu}(c^*_{\mu}((i_{\mu}(c^*_{\mu}(A))))) = i_{\mu}(c^*_{\mu}(A))$ and hence $c^*_{\mu}(A) = i_{\mu}(c^*_{\mu}(A))$. Hence $c^*_{\mu}(A)$ is $\tau_{\mu}$-open and $(X, \tau_{\mu})$ is $\tau_{\mu}$-externally disconnected.

5. RESULTS AND DISCUSSION

The purpose of this paper is to define $i^*_{\mu}$ and $c^*_{\mu}$ under more general conditions and to show that the important properties of these operations remain valid under these conditions. Also we define and study weaker form of $\tau_{\mu}$-open sets, $\tau_{\mu}$-continuity and an ideal $\ast-\mu$-weak structure space and $\mu$-weak local function with some applications on a set $X$ are defined and their properties are discussed.

6. CONCLUSION

We can add many applications in ideal $\mu$-weak structure space like if $(X, \tau_I)$ be an ideal $\mu$-weak structure space. If $\tau_{\mu} = \tau_{\mu}^*(I)$. Then if $(X, \tau_I)$ is $\tau_{\mu}$-externally disconnected then $c^*_{\mu}(A) \cap c^*_{\mu}(B) \subseteq c^*_{\mu}(A \cap B)$ for each $A \in \tau_{\mu}$, $B \in \tau_{\mu}^*$ and $c^*_{\mu}(A) \cap c^*_{\mu}(B) = \emptyset$ for each $A \in \tau_{\mu}$, $B \in \tau_{\mu}^*$ with $A \cap B = \emptyset$.

REFERENCES

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