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MHD Boundary Layer Flow and Heat Transfer for Micropolar Fluids over a Shrinking Sheet

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ABSTRACT

The MHD boundary layer flow and heat transfer for micropolar fluids over a shrinking sheet is investigated numerically. The governing partial differential equations of motion have been transformed to ordinary differential form by using suitable similarity functions. The resulting equations have been solved by using Runge-Kutta method with shooting technique. Numerical results have been obtained for several values of the parameters involved in the study namely, magnetic parameter M, suction parameter S, micropolar parameter D, Prandtl number Pr and Ekert number E_c. The effects of these parameters have been observed on velocity, microrotation and temperature functions. The results have been presented in graphical form.

KEYWORDS: Micropolar Fluids, Shrinking Sheet, Boundary Layer, Heat Transfer, Prandtl number, Ekert number

1. INTRODUCTION

Over the past few decades, fluid dynamics at micro and nano scales has been expanding research field. The effect of molecular spin is not considered in classical continuum, when the channel size is comparable to the molecular size. Such motions can be described with micro continuum theory developed by Eringen [1–4]. Micropolar theory offered by Eringen [2] is a higher-order theory for fluid dynamics. Each material point is a finite size particle, which contains six degrees of freedom (Three translation and Three rotation). In Continuum Mechanics, each material point only has three DOF. The extra three rotational DOF can be used to describe the gyration not the vorticity. Hence, for solving gyration, the balance law of angular momentum is given to take into account the effect of molecular spin. Physically, micropolar fluids can be seen in ferrofluids, blood flows, bubbly liquids, liquid crystals, and so on, all of them containing intrinsic polarities. Excellent reviews about the applications of micropolar fluids have been presented by Airman et al. [5, 6]. A comprehensive review of the subject and applications of micropolar fluid mechanics was given by Khonsari and Brewe [7], Chamkha et al. [8], Bachok et al. [9] and Kim and Lee [10]. Moreover, Lukaszewicz [11] provided extensive surveys of literature of the theory of micro polar fluids.

The flow induced by stretching boundaries is important for metal industries and extrusion processes in plastic. The stretching problem steady flow has been studied extremely in various topics; such as porous medium, MHD flows, heat transfer analysis and Non-Newtonian fluids. But the flow due to a shrinking surface is different from forward stretching flow, as first observed by Wang [12]. Goldstein [13] reported that the shrinking flow is essentially a backward flow. After few years, Miklavcic and Wang [14] established the existence and uniqueness of the similarity solution of the equation for the steady flow due to a shrinking sheet and they also reported that an adequate suction is necessary to maintain the steady flow. If the physical background of the flow is examined then it can be observed that the vorticity generated due to the shrinking of sheet is not confined within the boundary layer, and the steady flow exists only when adequate suction on the boundary is imposed. Later on, Hayat et al. [15-16] obtained analytic solutions of magnetohydrodynamic (MHD) rotating and non-rotating flows of a second grade fluid over a shrinking sheet using homotopy analysis method (HAM). Fang [17] reported an analytic solution of the boundary layer flow over a shrinking sheet with a power law surface velocity and wall mass transfer. Also, Fang and Zhang [18] found a closed-form analytic solution for two dimensional MHD flow over a porous shrinking sheet subjected to wall mass transfer. Further, Fang and his co-authors [19–20] discussed some other important aspects of shrinking flow. Bhattacharyya [21] studied the flow over an exponentially shrinking sheet. An analytic solution of steady two dimensional MHD rotating flow of a second grade over a porous shrinking surface was reported by Faraz and Khan [22] using homotopy perturbation method. Yacob and Ishak [23] and Bhattacharyya et al. [24] discussed

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the micropolar fluid flow over a shrinking sheet with out and with thermal radiation, respectively. Sajjad et al.[25] considered MHD boundary layer flow of micropolar fluids over a permeable shrinking sheet. Mishra and Jena [27] presented numerical solution of boundary layer MHD flow with viscous dissipation.

This work examines MHD boundary layer flow and heat transfer for micropolar fluids over a shrinking sheet. The effects of the physical parameters of the study have been observed on fluid velocity, microrotation and temperature distribution. Comparison of the results for Newtonian fluids and micropolar fluids is presented. The effect of micropolar parameter D is particularly taken in to account.

2. MATHEMATICAL ANALYSIS

Consider viscous and electrically conducting fluid flow over a permeable shrinking sheet which coincides with the plane y=0and the flow is confined in the region$y$>0. The fluid flow is incompressible, steady and two dimensional. The x- and y-axes are taken along and perpendicular to the sheet, respectively. Two equal and opposite forces are applied along the$x$-axis so that the sheet is stretched keeping the origin fixed. A uniform magnetic field of strength B_0 is assumed to be applied normal to the sheet. The magnetic Reynolds number is taken to be small and therefore the induced magnetic field is neglected. All the fluid properties are assumed to be constant. The body couple is neglected.

The governing equations of motion are given below:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$(\mu+\kappa)(\frac{\partial^2 u}{\partial y^2})+\kappa(\frac{\partial w_3}{\partial y})-\sigma B_0^2 u=\rho(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}) \tag{2}$$

$$\gamma(\frac{\partial^2 w_3}{\partial x^2}+\frac{\partial^2 w_3}{\partial^2 y})+\kappa(-\frac{\partial u}{\partial y})-2\kappa w_3=\rho j(u\frac{\partial w_3}{\partial x}+v\frac{\partial w_3}{\partial y}) \tag{3}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\kappa_0}{\rho c_p}\frac{\partial^2 T}{\partial y^2}+\frac{Q_0}{\rho c_p}(T-T_\infty)+\frac{\mu}{\rho c_p}(\frac{\partial u}{\partial y})^2+\frac{\sigma B_0^2}{\rho c_p}u^2 \tag{4}$$

where u, v are velocity components along x and y directions and w_3 is microrotation component. ρ is the density, μ is the coefficient of viscosity, κ is the vortex density, γ is the spin gradient viscosity coefficient, σ is the electrical conductivity of the fluid, T is the temperature, T_∞ is the free stream temperature, κ_0 is the thermal conductivity of the fluid, c_p is the specific heat at constant pressure, and Q_0 is the volumetric rate of heat generation or absorption.

The boundary conditions are

$$u=-cx,\quad v=v_w,\quad T=T_w,\ w_3=0 \qquad \text{at y=0}$$

$$u=0,\quad T\longrightarrow T_\infty,\ w_3\to 0 \qquad \text{as}\quad y\longrightarrow\infty \tag{5}$$

where c>0, (0<c<1) is the shrinking constant,

T_w is temperature of the sheet, and v_w > 0) is a prescribed distribution of wall mass suction through the porous sheet.

The stream function $\psi=\psi(x, y)$ is such that

$$u=\frac{\partial\psi}{\partial y},\quad v=-\frac{\partial\psi}{\partial x} \tag{6}$$

Using similarity transformations:

$$\psi = \sqrt{c\upsilon}\, x f(\eta)\,,\ \ \eta = y\sqrt{\frac{c}{\upsilon}} \tag{7}$$

$$u = cxf'(\eta),\quad v = -\sqrt{c\upsilon}\, f(\eta)\ ,\ w_3 = c\sqrt{\frac{c}{\upsilon}}\, x g(\eta) \tag{8}$$

Where υ is kinematic viscosity coefficient. The equation of continuity is identically satisfied and the equations (2) to (4) respectively yield:

$$(1+D)f''' + ff'' - f'^2 - M\ \ f' + Dg' = 0 \tag{9}$$

$$C_2 g'' - DC_1 f'' - 2DC_1 g = (f'g - f'g') \tag{10}$$

$$\theta'' + \Pr(f\theta' + \lambda\theta) + \Pr Ecf''' + M \Pr Ecf'^2 = 0 \tag{11}$$

where D, C_1, C_2 are non dimensional material constants. $M = \dfrac{\sigma B_0^2}{\rho c}$ is magnetic parameter, $\Pr = \dfrac{\mu c_p}{\kappa}$ is Prandtl number $\lambda = \dfrac{Q_0}{c\rho c_p}$ is heat source parameter (λ <0) and heat sink parameter(λ >0), $S = \dfrac{v_w}{\sqrt{c\upsilon}}$ and $E_c = \dfrac{Q_0}{c\rho c_p}$

$$\left.\begin{array}{l} \eta = 0 : f = S, f' = -1, g = 0, \theta = 0 \\ \eta \to 0 : f' \to 0, g \to 0, \theta \to 0 \end{array}\right\} \tag{12}$$

3. RESULTS & DISCUSSION

The ordinary differential equations (9) to (11) are solved subject to the boundary conditions (12). The numerical results have been computed for different values of the parameters namely M , S, λ , *D*, *Pr* and *Ec* .The effects of these parameters have been studied on velocity, microrotation, and heat transfer distributions. It is to know that the problem reduces to Newtonian fluids flow [27] when *D* and w_3 vanish. The comparison of the results for Newtonian fluids and micropolar fluids is shown in table 1 for skin friction coefficient $-f''(0)$. It is noted that the magnitude of $-f''(0)$ is lesser for micropolar fluids than for Newtonian fluids. Table 2 presents the effect of micropolar parameter *D* on skin friction coefficient and on couple stress $-g'(0)$. The magnitude of skin friction coefficient decreases while that of couple stress increases with increase in the values of *D*.

The effect of magnetic parameter M on the horizontal velocity f' is presented in Fig.1. The magnetic field increases the horizontal flow of the fluid. The boundary layer thickness decreases with increase in the values of magnetic field. Fig.2 illustrates the curves of f' due to suction parameter *S*. This component of velocity increases with increase in the values of *S*. The boundary layer thickness also decreases in this case. The magnetic field has small decreasing effect on the non dimensional temperature function θ as shown in Fig.3. The suction parameter *S* has decreasing effect on θ , it can be observed in the Fig. 4 that the effect of *S* is significant.

Fig.5 demonstrates the effect of magnetic field on the micro rotation *g*. The curve pattern shows that the microrotation decreases near the boundary and then increases. The effect of the suction parameter *S*, on microrotation is depicted in Fig.6. It is seen that the microrotation component curves have same pattern under *S* as in case of Magnetic parameter *M*.

Fig.7 illustrates the temperature distribution curves under the effect of Prandtl number. The temperature function increases with increase in the values of Prandtl number. Fig.8 shows that temperature function θ increases with increase in the magnitude of λ. The Eckert number shows increasing effect on θ as depicted in Fig.9. The effect of micropolar parameter *D* is observed on fluid velocity, microrotation and temperature functions. Fig.10 indicates that the velocity component decreases with increase in the values of D. Fig.11 shows that microrotation component *g* increases with increase in the values of *D*. The temperature function increases with *D*, as demonstrated in fig.12.

Table 1: Results for $-f''(0)$, when M=2

S	Results for micropolar fluids	Present results for Newtonian fluids	Mishra and Jena [27]
2	1.72054	2.41358	2.414476
3	2.29087	3.301724	3.302813
4	2.89661	4.235123	4.236068
5	3.52243	5.145362	

Table 2: Results for $-f''(0)$, and $-g'(0)$ for various values of micropolar parameter *D*.

D	$-f''(0)$	$-g'(0)$
0.5	1.72054	0.66256
1.5	1.0001	1.52318
3.0	0.79692	3.78869
5.0	0.59003	5.10607

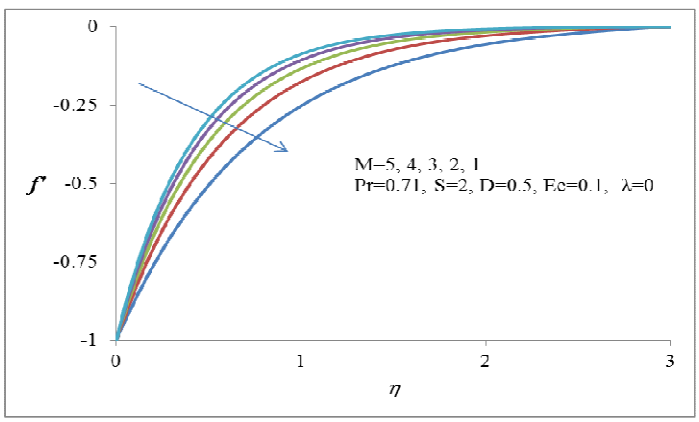

Fig.1: graph of f' for different values of *M*.

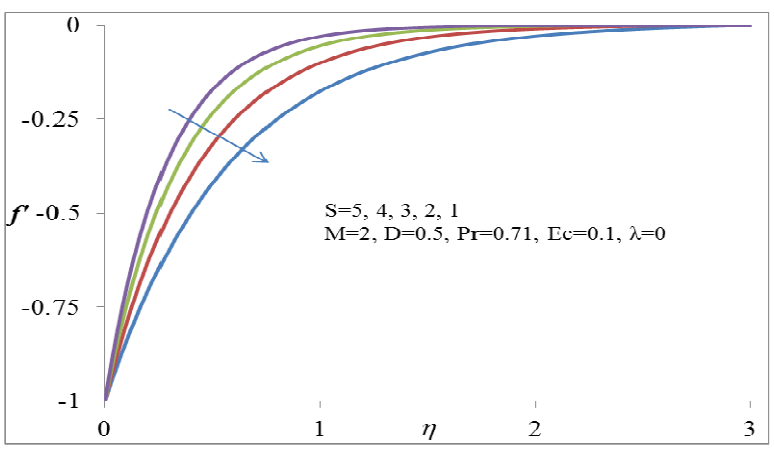

Fig.2: Graph of f' for different values of *S*.

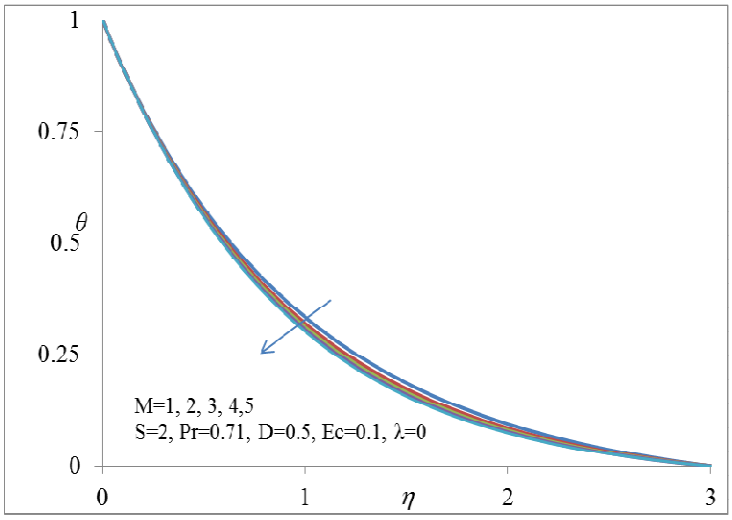

Fig.3: Graph of θ for different values of *M*.

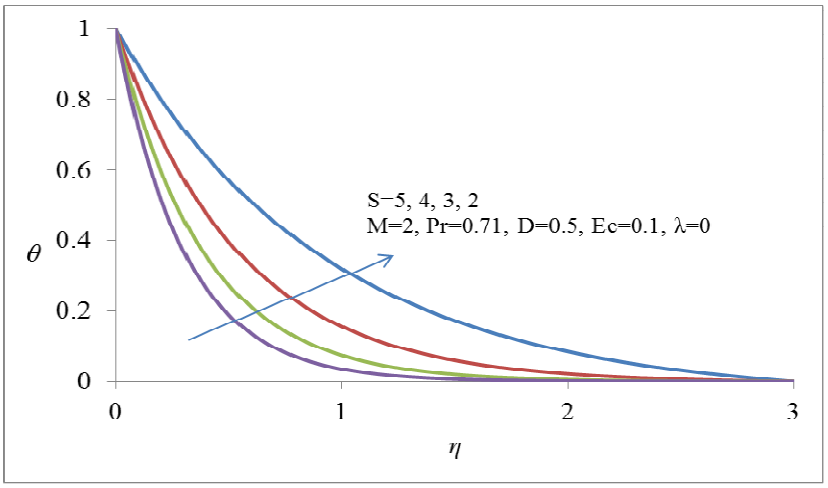

Fig.4: Graph of θ for different values of *S*.

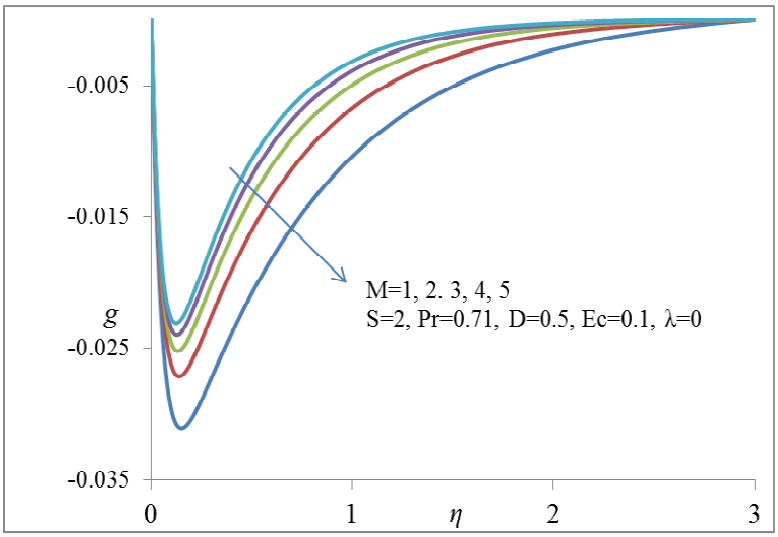

Fig.5: Graph of *g* for different values of *M*.

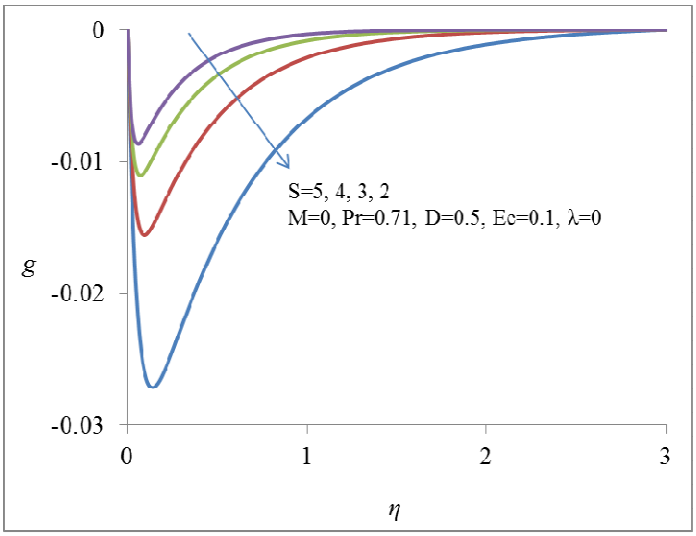

Fig.6: Graph of g for different values of S.

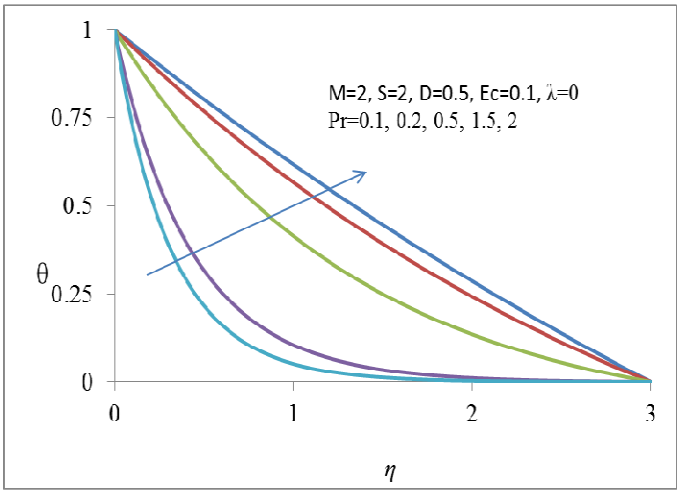

Fig.7: Graph of θ for different values of Pr.

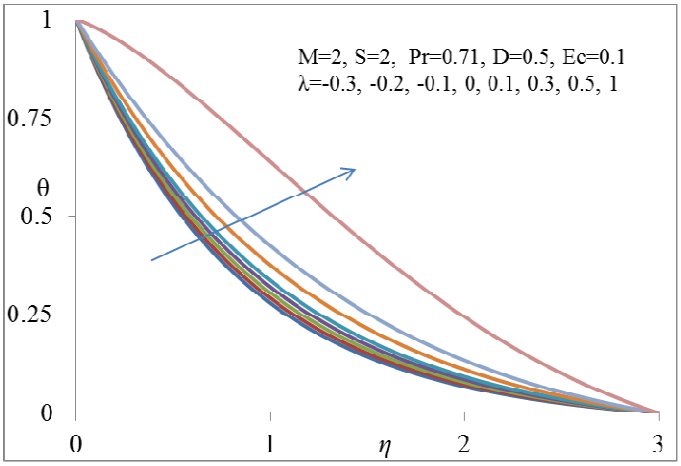

Fig.8: Graph of θ for different values of λ .

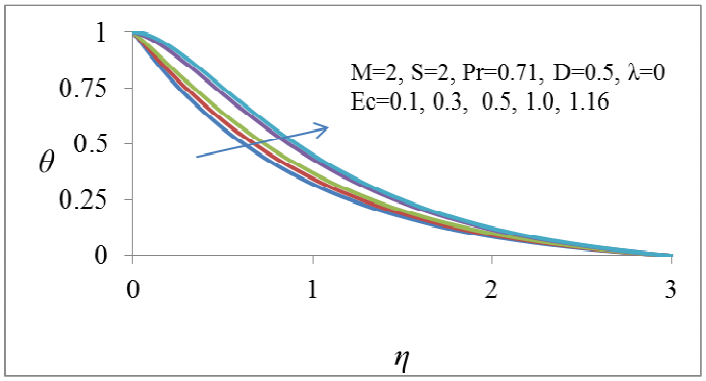

Fig.9: Graph of θ for different values of Ec .

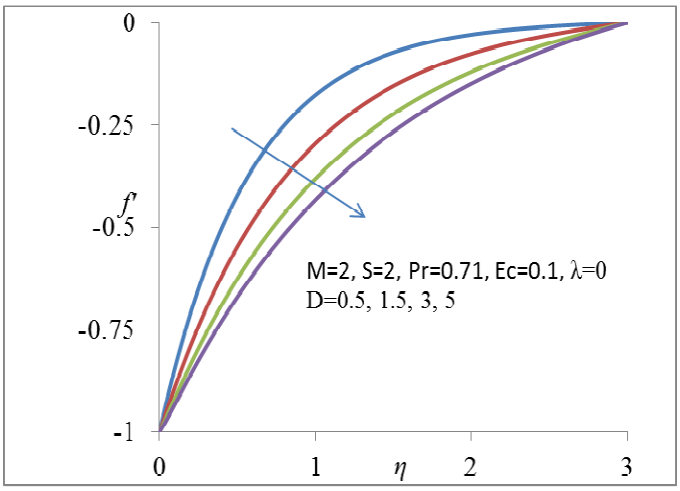

Fig.10: Graph of f' for different values of *D*.

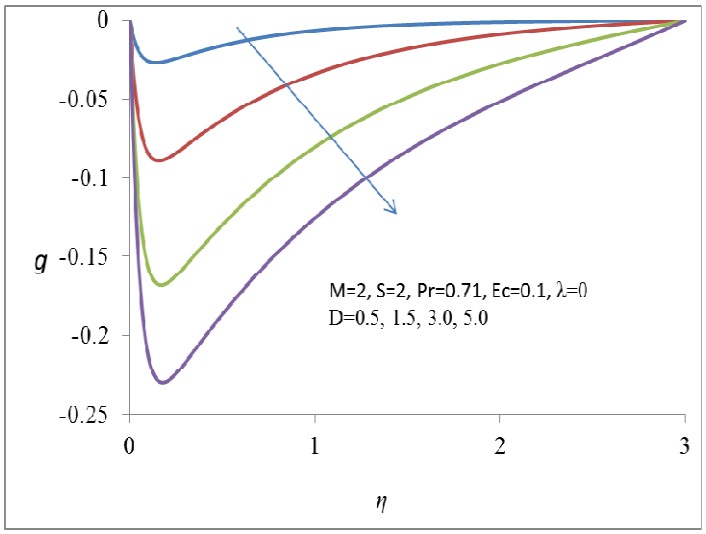

Fig.11: Graph of *g* for different values of *D*.

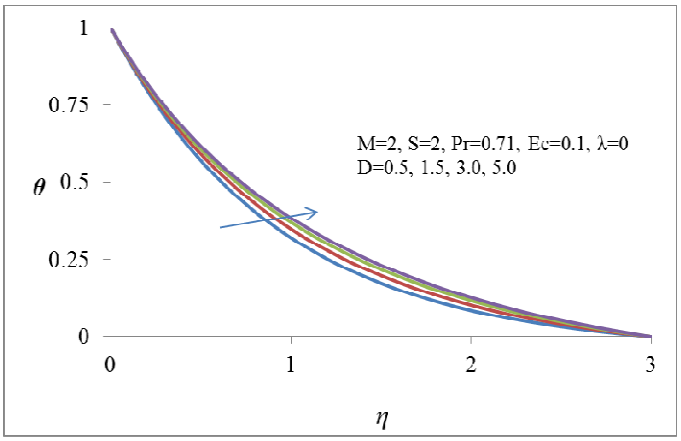

Fig.12: Graph of θ for different values of *D*.

4. CONCLUSION

Numerical solution of MHD boundary layer flow and heat transfer for micropolar fluids over a shrinking sheet has been obtained to examine the effects of physical parameters involved in this study. The main findings of this work are as follows:

- The magnitude of skin friction coefficient $-f''(0)$ is lesser for micropolar fluids than for Newtonian fluids.
- The micropolar parameter *D* decreases the magnitude of skin friction coefficient but it increases couple stress $-g'(0)$.
- Both the magnetic parameter M and suction parameter *S* increase f' and thus cause to decrease the boundary layer thickness.
- Both the magnetic field and suction at the surface decrease the non dimensional temperature function θ but the effect of *S* is more prominent
- The microrotation decreases near the boundary and then increases under the influence of *S*, as well as that of M.
- The temperature function increases with increase in the values of *Pr*, λ and *Ec*.
- The micropolar parameter *D* decreases f' but increases microrotation and temperature distribution.

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