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Analysis of Recirculation Flow Rate in Partially Plaque Deposited Capillaries by Power Law Model

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ABSTRACT

In this paper, response of non-Newtonian flow of blood due to plaque deposition in capillary segment has been investigated. The Power law model has been applied for describing shear thinning behaviour of blood. Numerical results, worked out using semi-implicit Taylor-Galerkin/pressure-corrector (TGPC) scheme and presented in streamline patterns to quantify vortex intensity during recirculation of blood at various inertial and deposition levels in capillary segment. The predicted results obtained are compared with analytical and experimental results, which show a good matching with each other. In future this research will be applicable towards medical devices design and treatment of vascular diseases.

KEYWORDS: Recirculation region, Atherosclerosis, Blood Flow, Capillaries, Non- Newtonian Fluid,

1. INTRODUCTION

The plaque deposition known as atherosclerosis in the capillary segment is treated as a main reason of cardiovascular diseases, which results huge figure of deaths all over the world. Most of the studies have been performed under Newtonian nature of blood numerically along axially symmetric level of deposition [1-2]. Whereas, some studies have been reported under irregular shape of deposition one of these are [3]. Whilst the impacts of atherosclerosis in the flow field of non-Newtonian nature of blood have been studied by many researchers, few of them are [4-5]. The distinction between Newtonian and non-Newtonian impacts of blood is due to the variation in viscosity, hence the effects of variations of viscosity are studied experimentally and numerically [6-8]. Some researchers have presented their work on the effects of atherosclerosis in the flow of blood as laminar [9], some as turbulent [10] or transitional [11] flows. Most of the studies are made toward the influences of wall shear stresses on wall parameters [12].

Towards flow structure of blood in terms of intensity in Power law model was employed for various deposition levels. The paper describes the study in 2-D axi-symmetric flow of blood in partially blocked capillary segment at various Reynolds numbers. To avoid the complexity of the constitutive equations only steady state case is considered. To simulate the non-Newtonian behaviour of blood in the capillary segment, Taylor-Galerkin/pressure-corrector scheme is incorporated with the application in the field of medical science related to the arterial diseases.

2. GOVERNING SYSTEM OF EQUATIONS

The laminar flow of blood in capillary segment described mathematically by employing continuity and generalized momentum equations in both radial and axial directions respectively. These equations are described as under:

Continuity Equation

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\,v_r\right)+\frac{\partial v_z}{\partial z}=0 \qquad (1)$$

Momentum Balance Equation

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r-component:

$$\rho\left(\frac{\partial v_r}{\partial t}+v_r\frac{\partial v_r}{\partial r}+v_z\frac{\partial v_r}{\partial z}\right)=\left(\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rr})-\frac{\tau_{\theta\theta}}{r}+\frac{\partial}{\partial z}\tau_{rz}\right)-\frac{\partial p}{\partial r} \quad (2)$$

z-component:

$$\rho\left(\frac{\partial v_z}{\partial t}+v_r\frac{\partial v_z}{\partial r}+v_z\frac{\partial v_z}{\partial z}\right)=\left(\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rz})+\frac{\partial}{\partial z}\tau_{zz}\right)-\frac{\partial p}{\partial z} \quad (3)$$

Where, $\tau_{rr}, \tau_{rz}, \tau_{zz}$ *and* $\tau_{\theta\theta}$ denotes the components of the extra stress tensor and defined as follows:

$$\tau_{rr}=2\mu(\dot{\gamma})\frac{\partial v_r}{\partial r}$$

$$\tau_{rz}=\mu(\dot{\gamma})\left\{\frac{\partial v_r}{\partial z}+\frac{\partial v_z}{\partial r}\right\}$$

$$\tau_{zz}=2\mu(\dot{\gamma})\frac{\partial v_z}{\partial z}$$

$$\tau_{\theta\theta}=2\mu(\dot{\gamma})\frac{v_r}{r}$$

3. PROBLEM SPECIFICATION

The deposition of plaque having parabolic form increasing in length as well as depth, in a capillary segment is mathematically modelled in terms of percentages by employing the following equation:

$$r_n=r_0\left[1+\frac{k}{100}\frac{(a-Z_c)(b-Z_c)}{h^2}\right], \quad (4)$$

Reader can refer [13] for more details.

The addressed problem can be specified by describing initial and boundary conditions. The velocity at the initial time is specified in terms of initial condition as; $v(z,0)=v_0(z)$ subject to $\nabla . v_0=0$. The boundary conditions at the inlet of the capillary are considered fully developed having maximum velocity profile given by $v_z=v_{\max}(1-r^2)$.Whilst, pressure is treated constant at upstream and zero at downstream at the exit.

4. NON-NEWTONIAN BEHAVIOUR OF BLOOD AND POWER LAW MODEL

To analysis the non-Newtonian behaviour of blood Power Law model has been tested to study the effect of shear-dependent functional viscosity of blood on partially blocked capillary, because the power law index *n* specifies extent of its behaviour.

4.1 Power- Law Model

The behaviour of blood along functional viscosity is given [14] as:

$$\mu(\dot{\gamma})=k(\dot{\gamma})^{n-1} \quad (5)$$

Where, *k, n;* $\dot{\gamma}$ treated as flow consistency index (k = 0.735), Power law index (n = 1); shear rate.
The analytical solution for Power law model [15] can be described as follows:

$$V_z = \frac{1}{4}\frac{6n+2}{(n+1)}\left(\frac{1}{R_0}\right)^2\left(1-\left(\frac{r}{R_0}\right)^{\frac{n+1}{n}}\right), \qquad 0 \le r \le R_0 \tag{6}$$

5. NUMERICAL SCHEME

For the numerical simulation, the choice of numerical algorithm is based on its accuracy, stability, efficiency and convergence rate. Literature shows that in the explicit schemes, the convergence rate is sluggish and requires small time-steps, which leads to apply the alternative approaches of semi-implicit techniques [16-18]. The semi-implicit technique is as under:

Stage-1(a):

$$\left[\frac{2}{\Delta t}M + \frac{1}{2\,\mathrm{Re}}S_{rr}^{j}\right](V_r^{j^{n+\frac{1}{2}}} - V_r^{j^n}) = \left[-\frac{1}{\mathrm{Re}}\left\{S_{rr}\,V_r^j + S_{rz}\,V_z^j\right\} - L_1^t P_k\right]^n - N(V)\,V_r^j, \tag{7}$$

$$\left[\frac{2}{\Delta t}M + \frac{1}{2\,\mathrm{Re}}S_{zz}^{j}\right](V_z^{j^{n+\frac{1}{2}}} - V_z^{j^n}) = \left[-\frac{1}{\mathrm{Re}}\left\{S_{rz}^{\dagger}\,V_r^j + S_{zz}\,V_z^j\right\} - L_2^t P_k\right]^n - N(V)\,V_z^j, \tag{8}$$

Stage-1(b):

$$\left[\frac{1}{\Delta t}M + \frac{1}{2\,\mathrm{Re}}S_{rr}\right](V_r^{j^*} - V_r^{j^n}) = \left[-\frac{1}{\mathrm{Re}}\left\{S_{rr}\,V_r^j + S_{rz}\,V_z^j\right\} - L_1^t P_k\right]^n - N(V)V_r^{j^{n+\frac{1}{2}}}, \tag{9}$$

$$\left[\frac{1}{\Delta t}M + \frac{1}{2\,\mathrm{Re}}S_{zz}\right](V_z^{j^*} - V_z^{j^n}) = \left[-\frac{1}{\mathrm{Re}}\left\{S_{rz}^{\dagger}\,V_r^j + S_{zz}\,V_z^j\right\} - L_2^t P_k\right]^n - N(V)\,V_z^{j^{n+\frac{1}{2}}}, \tag{10}$$

Stage- 2:

$$K\left(Q^{n+1}\right) = -\frac{2}{\Delta t}\left(L_1 V_r^{j^*} + L_2 V_z^{j^*}\right), \tag{11}$$

Stage 3:

$$M\left(V_r^{jn+1} - V_r^{j*}\right) = \frac{\Delta t}{2}L_1^{\,t}\left(p^{n+1} - p^n\right), \tag{12}$$

$$M\left(V_z^{jn+1} - V_z^{j*}\right) = \frac{\Delta t}{2}L_2^{\,t}\left(p^{n+1} - p^n\right), \tag{13}$$

The details of these matrices are described in [19-20].

6. RESULTS AND DISCUSSIONS

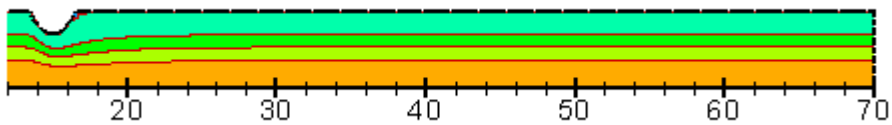
The predicted results are computed for different Reynolds numbers at various levels of deposition by employing Power law model for studying the impact of inertia on vortex intensity, displayed in Fig.1. It is observed that vortex intensity increases as level of deposition or Reynolds number increases. Further for comparing the intensity, computed from Power law model, one more well known, Carreau model is tested and the compared results are displayed again in Fig.1. It is observed that the computed results obtained from Carreau model are in good agreement with the Power law model, which confirms the validation of model. Whereas, Power law model occupies

slightly lesser vortex intensity than Carreau model but the trend is similar and is increasing in linear fashion with increasing Reynolds number as well as deposition level.

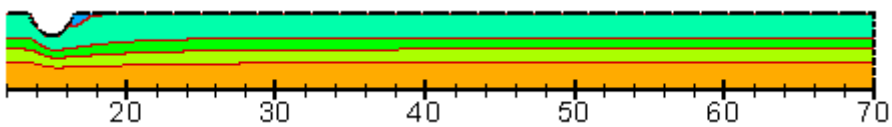
To study the further impacts of inertia on the flow field of blood, stream line projections presenting intensity of recirculation flow region at downstream in a capillary segment have been displayed in Figs. 1-3, by employing Power Law model by taking (Reynolds numbers = 100, 200 ; 300) and deposition levels (30%, 50% ; 70%) respectively. It is noted that the formation of vortex takes place at Reynolds number 100 having 30% level of deposition and its trend is increasing along increasing inertia and deposition levels. Further, it is observed that the intensity of recirculation flow region is significant at 50% level of deposition of Re = 200. Furthermore, it is observed that at 30% level of deposition the intensity is very small, moderate at 50% level and very high at 70 % level of deposition. Whereas, at 70% level of deposition of Re=300 is on over shooting side, therefore, stream line projections have not been displayed in Fig.3.

Table 1: Vortex intensity against Reynolds numbers for different percentages of deposition

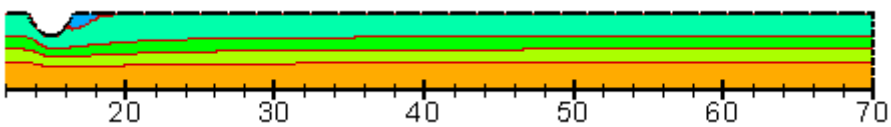
Re	Vortex Intensity					
	30% deposition		50% deposition		70% deposition	
	Power Law	Carreau model	Power Law	Carreau model	Power Law	Carreau model
100	0.0000	0.00015	0.0105	0.01109	0.0664	0.06746
200	0.0006	0.00099	0.0161	0.01718	0.0739	0.07500
300	0.0015	0.00201	0.0194	0.02019	--------	---------



(a) Re = 100

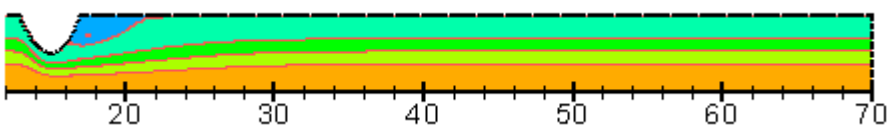


(b) Re = 200

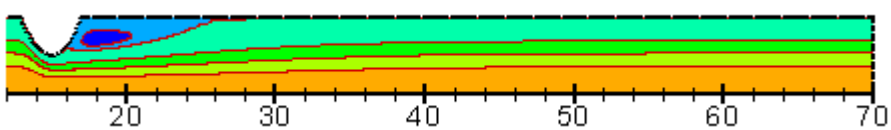


(c) Re = 300

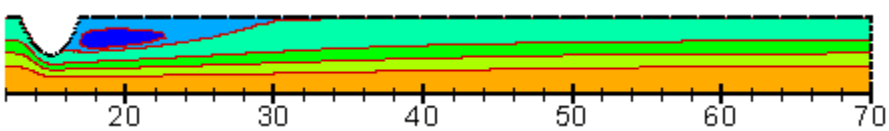
Figure-1: Vortex intensity during recirculation of blood at 30% level of deposition



(a) Re = 100

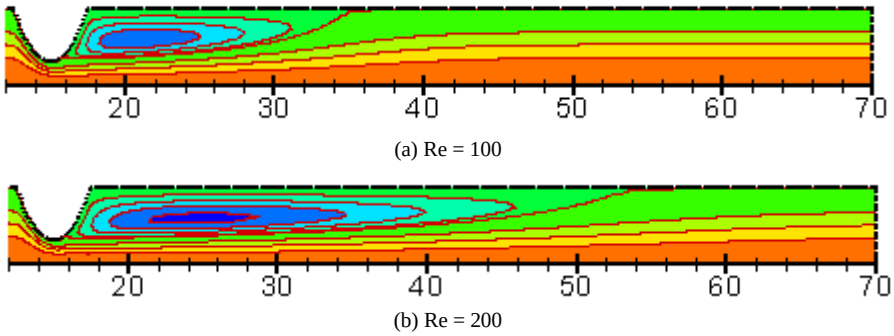


(b) Re = 200



(c) Re = 300

Figure-2: Vortex intensity during recirculation of blood at 50% level of deposition



(a) Re = 100

(b) Re = 200

Figure-3: Vortex intensity during recirculation of blood at 70% level of deposition

To validate the model further, computed numerical solutions of intensity are compared against analytical solution for Power law model [14], and experimental results [21] at various levels of inertia and deposition. It is interestingly observed that the addressed studies provide the identical trends and plots, which confirms the validation of the model [22].

7. CONCLUSIONS

It is concluded that the intensity of recirculation flow region at downstream in a capillary segment increases as level of deposition or Reynolds number increases. The intensity at 30% level of deposition from Power law model at Re =100, 200 ; 300) respectively is very small. Significant at 50% level of deposition of Reynolds number 200, whilst, very high at 70% deposition of 200 and 300 labelled Reynolds numbers. This also shows that the trend of recirculation flow rate is very high against increasing Reynolds number as well as level of deposition.

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