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Enhanced Newton-Raphson Method for Inverse Problems Solution of Electrical Impedance Tomography

Zahra Vahabi[1], Rassoul Amifattahi[1], Amir Salar Bidram[2], Mahdi Vahabi[3] and Ebrahim Ahmadifard[4]

[1]Digital Signal Processing Research Lab, Department of Electrical and Computer Engineering, Isfahan University of Technology, Isfahan, 84156-83111, Iran
[2]Technical and Vocational University, Mohajer Technical College
[3]Mechanical engineering department, AmirKabir University of Technology

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ABSTRACT

In this paper we study problems in reverse reconstruction of a wave equation with Gauss-Newton method and improve upon it. One of the most prominent engineering and computational science challenges is estimating of large scale, nonlinear parameters in the described systems with PDE (Partial Differential Equation).
These problems are called "Reverse Problems" as opposed to "Direct Problems" that describe large-scale simulations. Inverse problems are solved with more difficulty compared to direct problems because of many reason including ill-posedness, imperfect conditions of high density operators, coupled space-time and the need to solve direct problems for several times. We present a parallel algorithm for inverse problems described by temporal PDE and results of scalability of an inverse wave propagation problem in determining the material field of an acoustic media. We used total variation regularization, preconditioned matrix-free Gauss-Newton iteration, algorithmic check pointing and multiscale continuation to address the mentioned problems. Finally, we solved a synthetic inverse wave propagation problem.

KEYWORDS: Electrical Impedance Tomography, Inverse Problem, Partial Differential Equations, Newton-Raphson.

1. INTRODUCTION

one of the most famous computational science and engineering challenges is the estimating of large-scale nonlinear parameter in the described system with partial differential equations (PDE).These challenges are called inverse problems and they are different from direct problems which usually describe large-scale simulations[1-3].

In direct problems, the parameters of PDE (partial differential equations) including initial conditions, boundary and domain sources, coefficients of the material, the domain geometry are known; therefore the system state is determined by solving the PDE.

In inverse problems, the procedure is reversed. Some of the state parameters are observed, then a target function is formulated to minimize the difference between observed state and the predicted state by PDE, and finally

unknown parameters of the state are obtained by solving a non-linear optimization problem. An inverse problem is

harder to solve than a direct problem because:

1. Direct solutions are only a subroutine in inverse problems,
2. Inverse problems are often ill-posed whereas direct problems are always well-posed.
3. Unlike direct problems with direct operators, inverse operators couple the response time of the system.

An inverse problem can have infinite local answers. As mentioned above, solving an inverse problem is harder, that's why most large-scale simulation problems are inclined towards direct problems.

A very important challenge is faced when designing scalable parallel digital algorithms for solving inverse problems with non-linear PDEs. This is especially harder for those large-scale inverse problems that the number of inverse parameters is dependent on the mesh (for instance, when parameters provide unknown initial conditions, distributes resources, or heterogeneous material properties [1-4].

In this situation, scalability is crucial for both large-size problems and number of processors. Newton's Method:

Nonlinear scalable solver of initial pattern has the following problems:1. Newton's method needs Formally Dense type of Hessian matrix, which its grade is equal to inverse problem parameters, so it needs infinite

Corresponding author: Zahra Vahabi, Department of Electrical and Computer Engineering, Isfahan University of Technology, Isfahan, 84156-83111, Iran. z.vahabi@ec.iut.ac.ir

solutions of direct problems. Therefore, direct implementation of grid-based parameterization requires communication, work and inhibitor memory.

The second problem is related to objective function. Most of objective functions for inverse problems are divergent. Also, Newton's method, if not generalized properly, can become divergent too. In addition, for many of nonlinear inverse problems, the objective functions to be minimized have many local minima. General optimization methods cannot scale a large number of inverse parameters with grid-based optimizations. They also get stuck at local minima in Newton's method, unless they can converge the objective function which leads to definition of a new objective function.3. Density is the last problem that makes direct problems harder to solve. The struggle with direct solvers is caused by being dense, large size, and linear inverse operators. For the same reason, the matrix-based iterative solvers are not applicable either. Nevertheless, it is possible to perform implementation without matrix to extract the problem structure, so that each multiplication of the matrix vector requires solution of a few number of direct problems. Thus, this lack of matrix makes problems for preconditioned which is necessary to make a well-posed inverse problem in smarter regulators[2-7].

2. MATERIALS AND METHODS

Now we can say that such problems are not unseen in methods such as Newton's because many of large-scale inverse PDE methods are based only on gradient and avoid Hessian matrixes [1]. The most simple method is steepest descent in which current solution is optimized by moving along in the direction of negative gradient of the objective function. We can make better directions based on the gradient information by means of nonlinear conjugate gradient methods. Better than those are the pseudo-Newton methods with limited memory such as L-BFGS in which a limited amount of curvature information is produced based on changes in gradient and are reduced to nonlinear CGs with static boundary conditions. All these gradient-based methods avoid mentioned problems of Hessian of high density Newtonian. Furthermore, their parallel performance is very high because gradient calculations include PDE-like solutions which are accessible for many libraries and algorithms. However, these methods do not scale well with increasing size of the problem and are reduced from second grade to converging asymptote, thus performing poorly for ill-posed problems. Nevertheless, there is a high demand to solve large scale inverse problems in ocean modeling, transportation of air and water pollutions, earthquake simulation, attacks of dangerous materials, nondestructive tests, medical imaging, electromagnetic sensors, water reservoir simulations, etc. PDE simulations are the result of increasing trust in PDE models and powerful PDE solvers and computing hardware which is predicted to promote highly-resolved highly-parameterized models. For such large-scale problems, linear convergence methods such as L-BFGS are not consistent, but we need algorithms that are scalable with increasing problem size and number of processors. In this study, we present scalability of a parallel algorithm for inverse PDE problems in time. The goal is to determine coefficients of the parametric field of a heterogeneous media with observation of wave shape and source at the receivers at boundaries. Such inverse problems are posed in seismic explorations, earthquake simulations, ultrasound imaging, matched field processing [2], ocean acoustics, investigation of atomic treaties, obstacle illuminations, nondestructive evaluation, and structural health inspection. We present some results for inverse of the velocity field of a linear scalar wave equation as the representative of the class of inverse wave propagation with problems such as reduced grade, ill-posedness, large scale, time-space coupling, and local minima in an absorbent media that is reduced with increase in frequency[2-6].

Because there are about hundred problems of wavelength in inverse wave propagation (so that it is desired to create feature sizes to be hundreds times softer than characteristic dimension), it is very important that inverse algorithms to be able to scale the resolution of 1000 points per grid in each spatial dimension. Regarding local minima, we use multiscale continuation in which a series of estimation of the objective function (which are converging but oscillate accessional) are minimized by means of a chain of discrediting of state space and a parameter that softens ascension, which keep minimizing series within absorbent background of global minimum.

Total Variation (TV) regularization is used to avoid ill-posedness and steep variables in parametric inverse space. To solve undetermined Newtonian Hessian, dense and large, and expensive to build.

These methods require proper preconditions based on total variation regularization operator which causes lots of problems because of high heterogeneity and anisotropy. Our preconditioned is based on L-BFGS algorithm and in order to estimate curvature information by means of changes in directional differentiation. Newton's methods provide independence of mesh for nonlinear operators in softer grids (mesh grids). Inverse operators with proper preconditioning can produce spectra suitable for this method. However, conditions of total regularization operator become incomplete increasingly with decreasing mesh size, which often results in increase in Newtonian iteration with mesh size. In this paper, we study our algorithm performance in scalability problems in the field of inverse wave propagation.

The optimization approach to a general form of this inverse problem was elaborated in previous studies[1].

Consider an acoustic medium with predefined domain and boundary. The medium is excited with a known acoustic energy Source f(x; t) (for simplicity we assume a single source event), and the pressure u_(x; t) is observed at Nr receivers, corresponding to points xjin the domain. Our objective is to infer from these measurements the squared acoustic velocity distribution q(x), which is a property of the medium. From now on we refer to q as the material model and u(x; t) as the state variable. We can formulate the inverse problem as a nonlinear least squares parameter estimation problem with PDE constraints.

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We seek to minimize, over the period t = 0to T, an L2 norm difference between the observed state and that predicted by the PDE model of acoustic wave propagation, at the Nr receiver locations. The PDE-constrained optimization problem is:

$$\begin{aligned} &min(u,q)\sum_{j=1}^{Nr}\int_0^T[u^*-u]^2\,\delta(x-x_j)d\Omega dt+\beta F(q) \textit{Subject to } \ddot{u}-\nabla.q\nabla u=f \quad in\Omega * (0,T] \\ &q\nabla u.n=0 \qquad on \quad (0,T]u=\dot{u}=0 \qquad\qquad in\Omega*\{t=0\} \end{aligned} \tag{1}$$

The first term in the objective function is the misfit between observed and predicted states, the second term F (q) symbolizes the regularization functional with regularization parameter, and the constraints are the initial–boundary value problem for acoustic wave propagation. The form in which the acoustic equation is given assumes a constant bulk modulus and variable density; our method is equally applicable to a variable bulk modulus and constant density, or the case when both are variable. Here we have chosen zero initial conditions and Neumann boundary conditions, but the algorithms we discuss in this paper extend to more general conditions (as well as more general wave propagation PDEs). In the absence of regularization, the problem as formulated above is ill-posed: the solution may not be unique nor depend continuously on the given data [1,2]. Discretization of the inverse problem leads to inverse operators that are ill-conditioned and rank deficient. The null space of the inversion operator contains high frequency components of the material model q, which cannot be determined from the band-limited response u. Moreover, certain regions of the medium may be “hidden” as a result of source/receiver density and location, further enlarging the null space of the inverse operator. One remedy is to regularize with the $L2$ norm of the gradient of the material model:

$$F(q)=\frac{\beta}{2}\int_\Omega \nabla q.\nabla q\, d\Omega \tag{2}$$

3. **RESULTS**

Regularization does not cure all the problems. In addition to the null space problems described above, the objective function can be highly oscillatory in the space of material model q. Thus, there can be many local minima to contend with. Moreover, this oscillatory behavior becomes more pronounced with increasing frequency of propagating waves: the diameter of the basin of attraction of the global minimum varies as their wavelength, and local optimization methods become increasingly doomed to failure [3, 4]. To overcome this problem, we use multilevel grid and frequency continuation to generate a sequence of solutions that remain in the basin of attraction of the global minimum; that is, we solve for increasingly higher frequency components of the material model, on a sequence to optimize and repair the problems arising from the use multilevel grid frequency [5-9]. Figure 1 illustrates the effect of multiscale continuation and TV regularization. Without multiscale continuation, the optimization algorithm becomes trapped in a local minimum (Frame B) far from the global optimum. Frame C demonstrates the smoothing of interfaces due to the use of Tikhonov semi norm regularization. The equations that determine the solution to the optimization problem (1) can be derived by requiring stationary of a Lagrangian functional. These so-called optimality conditions take the form of a coupled nonlinear three-field system of Integra-partial-differential equations in the state variable u, the ad joint variable λ, and he material model q:

$$\ddot{\gamma}-\nabla.q\nabla\gamma-\sum_{\iota=1}^{Nr}[u^*-u]\delta\big(x-x_j\big)=0 \quad in \quad \Omega*(0,T]; \tag{3}$$

$$q\nabla\gamma.n = 0 on\ (0,t] \quad (4)$$

$$\gamma = \dot{\gamma} = 0 \qquad for\ \Omega * \{t = T\} \quad (5)$$

$$\int_0^T \nabla u.\nabla\gamma dt - \beta\nabla.(|\nabla q|_{\varepsilon}^{-1}\nabla q) = 0 \quad in \quad \Omega, \quad (6)$$

$$\nabla q.n = 0 on\ d\gamma \quad (7)$$

The ad joint equation (3) has the same form as the state equation, i.e. it is also an acoustic wave equation. However, there is a crucial difference: the ad joint wave equation is a final value problem, as seen in the final, as opposed to initial, conditions in (3), and it has a different source, which depends on the state variable u. Finally, the material equation (4) is integral-partial-differential and time-independent. This last equation is shown for the case of TV regularization [10-13] in figure 1:

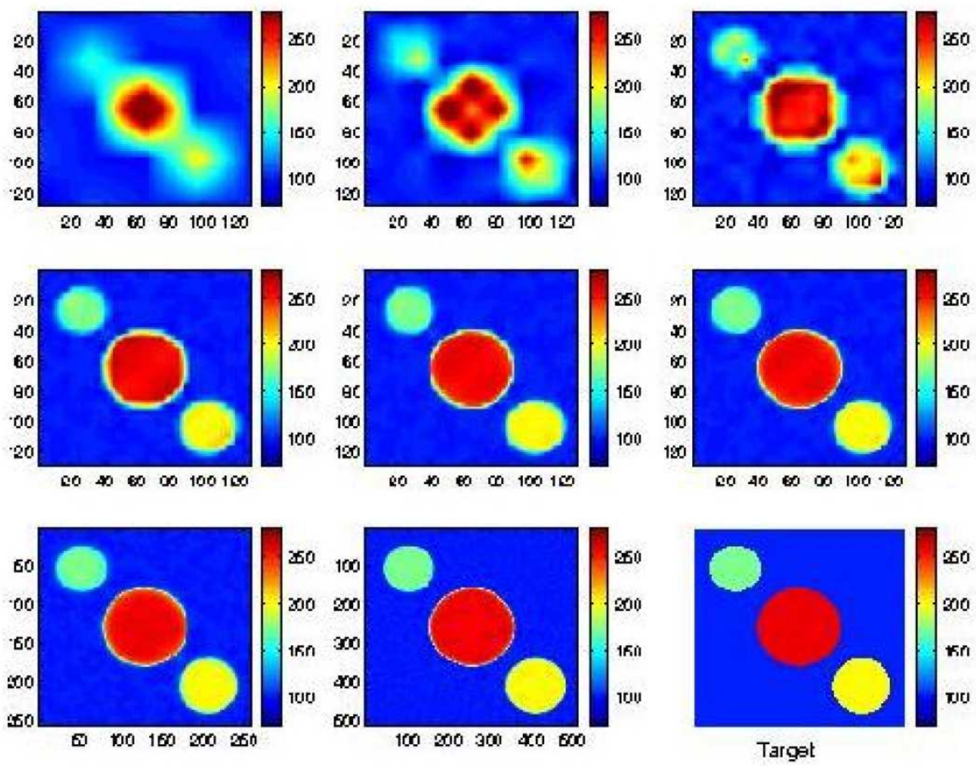

Figure 1.Inversion of a piecewise-homogeneous material model corresponding to three circular scatters using regularization. Receiver locations on top, sources on bottom, receiver data is synthesized based on forward solution of target model shown in last (naturally-ordered) frame.

This system is an extremely formidable system to solve. When appropriately discretized, the dimension of each of u and λ is equal to the number of grid points Ng multiplied by time stepsNt, q is of dimensionNg, and thus the total system is of dimension $2NgNt$ +Ng. This is can be very large for problems of interest. Time Next time stepping routine procedures are not hiding. The device comprises coupling the initial and final value problems are that is, both directions are evident in this machine. So in the next section we discuss the way in which the material model is reduced by eliminating the additional variables and equations of state of the machine, the problem solves.

figure. 1 .shows Effect of multiscale continuation and total variation regularization. Target field is in Frame A. Solution of inverse problem is on a single grid, without benefit of multiscale continuation (in grid size and source frequency), converges to a local minimum very far from the global, as shown in Frame B. Multiskilling, but with Tikhonov regularization, locates the global optimum, but smears the interfaces, as shown in Frame C.

Finally, Frame D depicts inversion with both multiskilling and TV regularization (further continuation produces the sharp reconstructed interfaces of Figure 1.

In summary, the method has the following nested components:

I. multiscale continuation on a sequence of finer grids with successively higher source frequencies, to capture increasingly finer components of the inversion model and keep the iterates in the basin of attraction of the global minimum;

ii. Gauss-Newton iteration for solving the nonlinear optimization problem at each grid and frequency level, with the option of maintaining a proper Newton reduced Hessian close to the solution for large misfit problems;

iii. Inexactly-terminated conjugate gradient solution of the Gauss-Newton linear system, the majority of which is dominated by a forward and an ad joint wave propagation solution, plus a sparse local rank three tensor product and time reduction involving gradients of the state and ad joint fields;

4. Conclusion

We have implemented the inversion method described in the previous section on top of the PETSclibrary . Spatial approximation is by Galerkin finite elements, in particular piecewise tri linear basis functions for the state u, ad joint λ, and material model q fields. So we took discretization through central differences which are the obvious choice. So the number of time steps is of the order of the cube root of the number of grid points. Since we require the precise time resolution of wave propagation phenomenon, the cost of repetition is higher than the overall job, and because the pre-conditioner is independent of time, the multiplication of hessian vector is dominant. With approximation of Gauss - Newton, CG matrices need to apply the same reduced gradient computation. These components are all "PDE-solver-like," and can be effectively parallelized in a fine grained domain-based way, using many of the building blocks of sparse PDE-based parallel computation: sparse grid-based matrix-vector products, vector sums, scaling, inner products, and etc. Indeed, we use routines from the PETSc parallel numerical library for these operations.

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