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Numerical Solution of Micropolar Fluid Flow over a Stretching Sheet in a Porous Medium

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ABSTRACT

Micropolar fluids flow over a permeable stretching wall with suction / injection is studied numerically. Suitable similarity functions have been employed to convert the highly nonlinear governing equations of motion into ordinary differential form. The effect of flow parameters namely Reynolds numbers $R,$ suction / injection parameter λ and the nondimensional material constants $C_{1,}$ C_2 and C_3 standing for micropolar behavior of the fluids are examined. The numerical results have been obtained by shooting method. The comparison between previous and present results found in good agreement and presented in tabular as well as in graphical form.

KEYWORDS: Micropolar fluids, Similarity Transformations, Hartmann number, Prandtl number, Decay parameter.

1. INTRODUCTION

Ever since, the introduction of micropolar fluid theory by Eringen [1], this research area has been expanding to investigate various important problems of the fluid dynamics. This class of fluids with microstructures has non-symmetric stress tensor and a couple stress. Micropolar fluids consist of rigid, randomly oriented or spherical particles with their own spins and microrotation, suspended in a viscous medium. Physically, micropolar fluids can be seen in ferrofluids, blood flows, bubbly liquids, liquid crystals, and so on, all of them containing intrinsic polarities. Excellent reviews about the applications of micropolar fluids have been written by Airman et al. [2, 3]. A comprehensive review of the subject and applications of micropolar fluid mechanics was given by Khonsari and Brewe [4], Chamkha et al. [5], Bachok et al. [6] and Kim and Lee [7].

The boundary layer problem due to a stretching sheet has relevance to extrusion problems. It has practical application to boundary layer control and thermal protection in high energy flow. Presently, it has received considerable interest. The surface stretching problem was first proposed and analyzed by Sakiadis [8] based on the boundary layer approximation. Crane [9] presented an exact solution of the two dimensional Navier-Stokes equations for a stretching sheet problem with the closed analytical form, where the surface stretching velocity was proportional to the distance from the slot. Magyari et al. [10] have studied Stokes' first problem for micropolar fluid and solved the problem analytically by Laplace transforms and numerically by Valk´o Abate procedure. Shafique and Rashid [11] obtained numerical solution of three dimensional micropolar fluid flows due to a stretching flat surface. Sastry and Rao [12] studied the effects of suction parameter on laminar micropolar fluid in a porous channel. Sajjad and Ahmad [13] considered MHD boundary layer flow of micropolar fluids over a stretching sheet. Takhar et al. [14] studied the mixed convective flow of a steady, incompressible micropolar fluid over a stretching sheet. Kamal and Sifat [15] studied 3-dimensional micropolar fluid motion caused by the stretching surface. Bhargava et al. [16] studied coupled fluid flow, heat and mass transfer phenomena of micropolar fluids over a stretching sheet with non-linear velocity.

Ojjela and Kumar [17] considered the unsteady two-dimensional incompressible MHD flow and heat transfer of a micropolar fluid in a porous medium between parallel plates with chemical reaction, Hall and ion slip effects. Al-Lawatia [18] examined micropolar fluid flow over a stretching sheet with variable concentration and solved numerically using the shooting method. Bhattacharyya et al. [19] studied the effects of chemical reaction on the boundary layer flow of viscous fluid and a numerical solution obtained by shooting method. Dayyan et al. [20] examined analytically the boundary layer flow through a porous medium over a stretching porous wall. El-Hakeem [21] studied heat transfer from moving surfaces in a micropolar fluid with internal heat generation. D. Pal [22] has also discussed hall current and MHD effects on heat transfer over an unsteady stretching permeable surface with thermal radiation.

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In present work micropolar fluid flow over a permeable stretching sheet is investigated. Numerical solution of the problem have been obtained for several values of flow parameters by using Shooting method. The effect of material constants is studied with special interest to explore the behavior of micropolar fluids.

2. MATHEMATICAL ANALYSIS

Considering micropolar fluid flow through a homogeneous porous medium of permeability K , over a porous stretching sheet. The linear velocity distribution of flow along the sheet is $U_w = U_0 x / L$. The fluid flow is steady, incompressible and two dimensional. The Cartesian coordinates are used. The sheet is stretching along $x - axis$ and $y - axis$ is perpendicular to the sheet. The origin is fixed. The u, v are velocity components along horizontal and vertical directions and w_3 is micro rotation vector component perpendicular to xy plane. The governing equations of the motion are:

$$\partial u / \partial x + \partial v / \partial y = 0 \tag{2.1}$$

$$(\mu + k)(\partial^2 u / \partial x^2 + \partial^2 u / \partial y^2) + k \partial w_3 / \partial y = \rho(u \partial u / \partial x + \mu \partial u / \partial y) \tag{2.2}$$

$$\gamma \partial^2 w_3 / \partial y^2 - 2k w_3 + k(\partial u / \partial x) = \rho j \upsilon \partial w_3 / \partial y \tag{2.3}$$

where μ is dynamic viscosity and ρ is fluid density.

The associated boundary conditions are:

$$U(x^*, 0) = u_0 x^*, \quad \upsilon(x^*, 0) = v_w, \; U(x^*, \infty) = 0 \tag{2.4}$$

where $x^* = x / L$ and L is length of the sheet.

Let us take the similarity transformations as:

$$\psi = u_0 x^* \sqrt{k} f(\eta) \tag{2.5}$$

$$w_3 = \frac{u_0 x^*}{\sqrt{k}} N(\eta) \tag{2.6}$$

where $\partial \psi / \partial y = u, \quad - \partial \psi / \partial y = \upsilon$ and $\eta = \frac{y}{\sqrt{k}}$ is dimensionless variable.

The equations (2.1) is readily satisfied and (2.2) and (2.3) are respectively transformed to ordinary differential equations,

$$(1 + C_1) f''' + R(ff'' - f'^2) - f' + C_1 N' = 0 \tag{2.7}$$

$$N'' - C_1 C_2 (f'' - 2N) = C_3 (f'N - fN') \tag{2.8}$$

where prime denotes the differentiation with respect to η. $R = \frac{\rho u_0 k}{L \mu}$ is the Reynolds number, $C_1 = \frac{k}{\mu}$, $C_2 = \frac{K \mu}{\gamma}$ and $C_3 = \frac{\rho j K \mu}{\gamma}$ are nondimensional material constants.

The boundary conditions (3.57) then become:

$$f(0) = f_w, f'(0) = 1, N(0) = 0, f'(\infty) = 0, \text{ and } N(\infty) = 0 \tag{2.9}$$

where $\lambda = -\frac{v_{wL}}{u_0 \sqrt{k}}$ is injection parameter. Also, $\lambda > 0$ shows suction and $\lambda < 0$ is for injection.

3. RESULTS AND DISCUSSIONS

The governing ordinary differential equations (2.7) and (2.8) are solved with the boundary conditions (2.9) using shooting technique in Fortran 90 coding. Numerical results for velocity and microrotation have been obtained for several values of parameters namely Reynold number R , suction/injection parameter λ and the nondimensional material constants namely vortex viscosity parameter C_1, spin gradient viscosity parameter C_2 and micro inertia density parameter C_3. In order to develop better understanding of micropolar fluid model, three different sets of these parameters have been chosen arbitrarily as given below:

Case	I	II	III
C_1	0.5	1.5	5.0
C_2	0.1	0.1	0.1
C_3	2.0	1.0	0.5

The results have been presented in tabular and graphical form for some representative values of the flow parameters. $Table-1$ and $Table-2$ respectively presents comparison of the present and previous results for velocity components f and f'. The value of both the velocity components becomes greater for micropolar fluids than for the Newtonion fluids. The results for skin friction coefficient $-f''(0)$ are given in Table-3. The magnitude of skin friction coefficient $-f''(0)$ becomes lesser for micropolar fluids than that of Newtonian fluids. The present results show a good comparison with previous results.

Fig-1 demonstrates the effects of R on velocity component f'. The increasing values of R has made the flow decreasing.

Fig-2 depicts that magnitude of velocity decreases for increase in suction and increases with the increase in injection.

The comparison of Newtonian fluids and micropolar fluids has been shown in Fig-3.

In Fig-4, both the velocity components f and f' are lesser in magnitude for micropolar fluids than that of Newtonion fluids. The magnitude of velocity decreases by increasing the values of material constants.

Fig-5 and Fig-6 respectively demonstrate the effect of C_1 and C_3 on f and f' where C_2 is kept fixed. These material constants have increasing effect on the microrotation of fluid particles.

Fig-7 shows that under the effect of R microrotation increases near the initial boundary and then decreases.

Fig-8 depicts that microrotation decreases in value a little away from the initial boundary with increasing suction. The value of microrotation is a bit larger near the initial boundary and then decreases with injection as shown in fig-9.

4. CONCLUSION

The main results of this work are summarised as follows: The magnetic field parameter M, has decreasing effect where as heat source parameter and Prandtl number have increasing effect on fluid flow velocity and angular velocity of microrotation of fluid particles. The material constants show decreasing effect on fluid flow.

$Table-1$

The results of f when $\lambda = 0$ and $R = 1$

	Results of f for Newtonian Fluid			Results of f for Micropolar Fluid		
Re	HAM	NS	Present Result	Case I	Case II	Case III
0	0	0	0	0	0	0
0.2	0.17042	0.17420	0.17420	0.17856	0.18302	0.18849
0.4	0.30564	0.30549	0.30549	0.32028	0.33596	0.35606
0.6	0.40571	0.40443	0.40443	0.43278	0.46389	0.50544
0.8	0.47505	0.47900	0.47900	0.52211	0.57101	0.63885
1.0	0.53802	0.53519	0.53520	0.59306	0.66076	0.75819
1.2	0.58010	0.57754	0.57755	0.64943	0.73602	0.86503
1.4	0.61161	0.60946	0.60946	0.69421	0.79916	0.96074

1.6	0.64096	0.63352	0.63352	0.72980	0.85216	1.04647
1.8	0.65374	0.65165	0.65164	0.75808	0.89665	1.12327
2.0	0.66952	0.66531	0.66530	0.78057	0.93400	1.19203
2.2	0.67812	0.67560	0.67560	0.79844	0.96537	1.25356
2.4	0.68567	0.68336	0.68336	0.81265	0.99170	1.30856
2.6	0.69054	0.68921	0.68920	0.82395	1.01379	1.35768
2.8	0.69675	0.69362	0.69360	0.83293	1.03232	1.40148
3.0	0.69842	0.69694	0.69692	0.84006	1.04785	1.44046

Table-2

The results of f' when $\lambda = 0$ and $R = 1$

	Results of f' for Newtonian Fluid			Results of f' for Micropolar Fluid		
Re	HAM	NS	Present Result	Case I	Case II	Case III
0	0.99999	1	1	1	1	1
0.2	0.75376	0.75363	0.75364	0.79359	0.83525	0.88767
0.4	0.56813	0.75363	0.56797	0.62992	0.69836	0.79031
0.6	0.42820	0.42804	0.42804	0.50012	0.58446	0.70527
0.8	0.32272	0.32259	0.32259	0.39716	0.48956	0.63045
1.0	0.24322	0.24311	0.24311	0.31547	0.41037	0.56423
1.2	0.18330	0.18322	0.18322	0.25064	0.34421	0.50532
1.4	0.13814	0.13808	0.13808	0.19917	0.28884	0.45269
1.6	0.1041	0.104060	0.10406	0.15829	0.24246	0.40554
1.8	0.07845	0.07842	0.07842	0.12582	0.20357	0.36319
2.0	0.05912	0.05910	0.05910	0.10002	0.17093	0.32509
2.2	0.04455	0.04454	0.04453	0.07951	0.14351	0.29077
2.4	0.03357	0.03357	0.03356	0.06321	0.12045	0.25981
2.6	0.02530	0.02529	0.02528	0.05024	0.10106	0.23184
2.8	0.01907	0.01906	0.01905	0.03993	0.08473	0.20656
3.0	0.01437	0.01436	0.01435	0.03172	0.07097	0.18367

Table-3

The results of $-f''(0)$ when $\lambda = 0$

	Results of $-f''(0)$ for Newtonian Fluid			Results of $-f''(0)$ for Micropolar Fluid		
Re	HAM	NS	Present Result	Case I	Case II	Case III
1.0	1.4198	1.4242	1.41422	1.15649	0.90268	0.60424
1.5	1.5799	1.5811	1.58114	1.29276	1.00827	0.67242
2.0	1.7234	1.7320	1.73205	1.41596	1.10371	0.73417
5.0	2.4394	2.4494	2.44949	2.00163	1.55727	1.02802

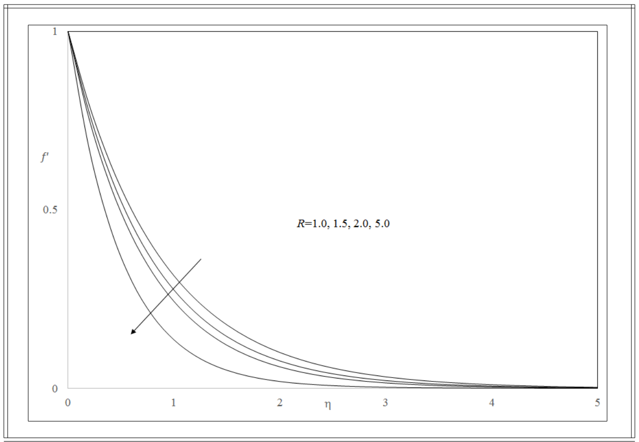

Fig-1. Graph of *f*❖for different values of *R*

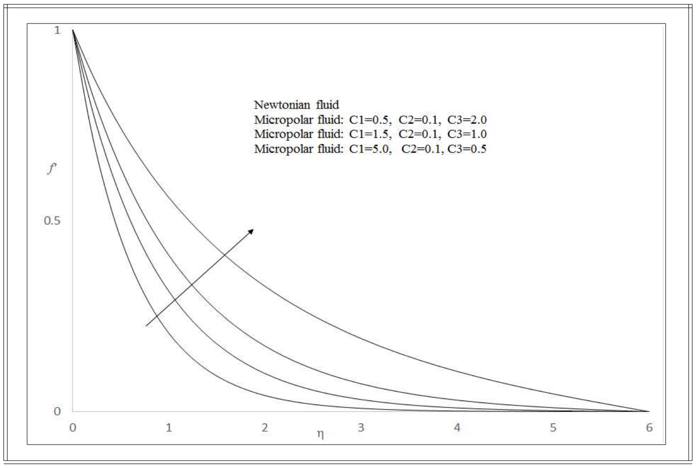

Fig-2. Graph of *f*❖for comparison of Newtonian Fluids and Micropolar fluids

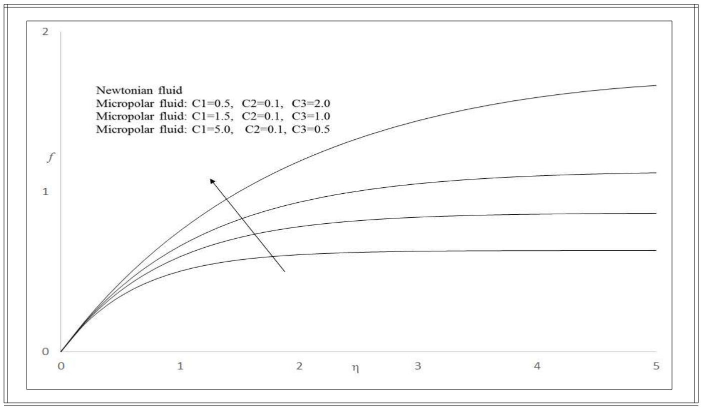

Fig-3. Graph of *f* for comparison of Newtonian Fluids and Micropolar fluids

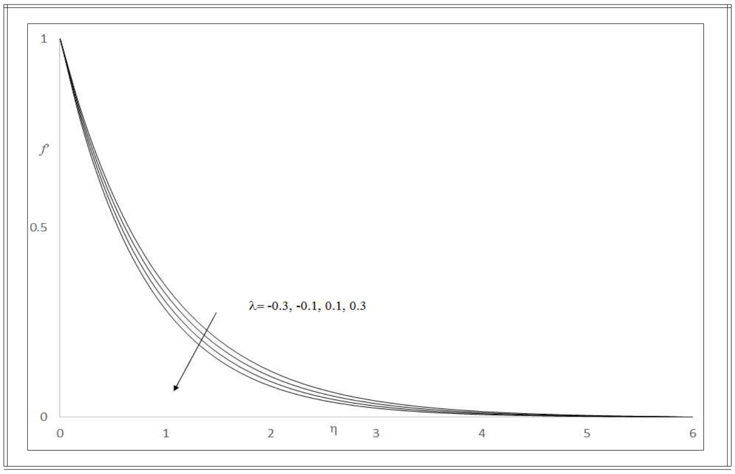

Fig-4. Graph of f❖for different values of ✞

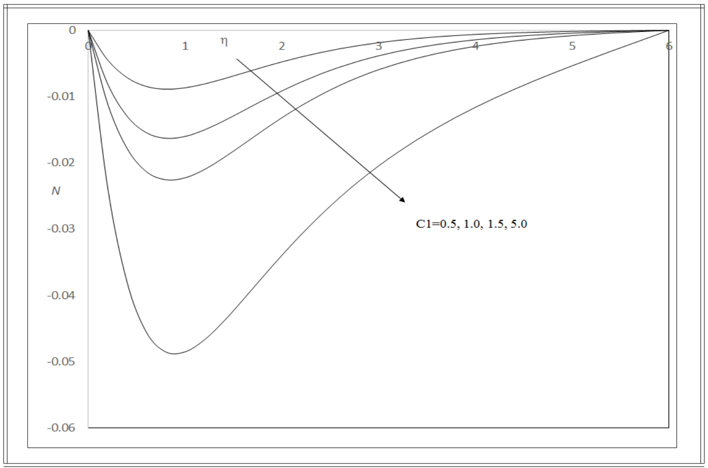

Fig-5. Graph of N for different values of c_1

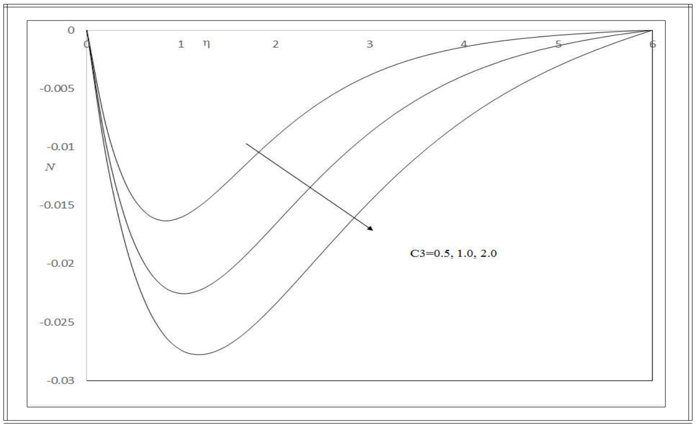

Fig-6. Graph of N for different values of c_3

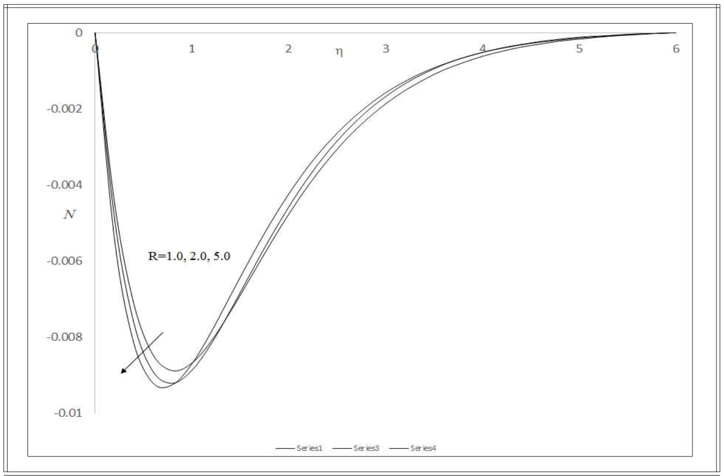

Fig-7. Graph of *N* for different values of *R*

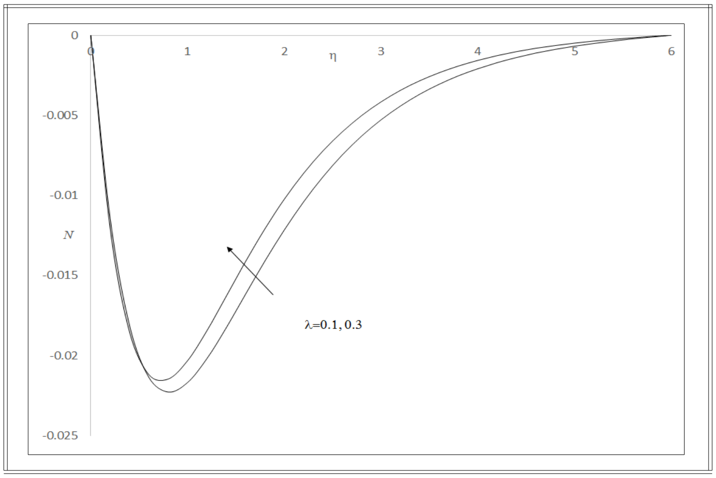

Fig-8. Graph of *N* for different values of ✞ ✇ 0

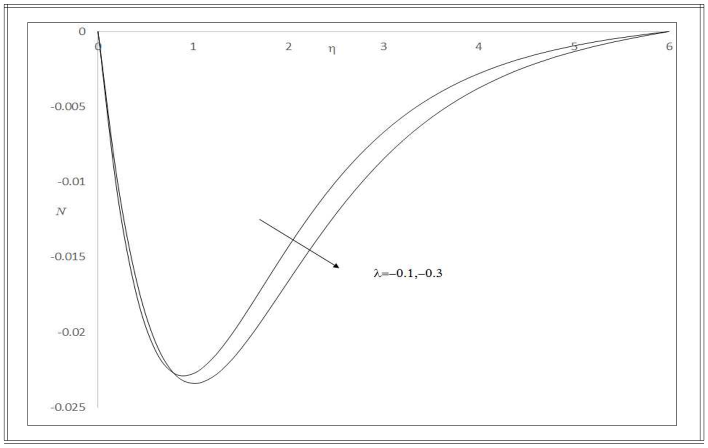

Fig-9. Graph of *N* for different values of ✞ 🖫 0

Nomenclature

[illegible] : Injection parameter,

K : Permeability of the porous medium,

R : Reynolds number,

u : Velocity in direction

u_0 : Wall velocity coefficient,

v : Velocity in direction

v_w : Injection velocity,

x : Coordinate system,

y : Coordinate system,

[illegible] : Density of the fluid

☟ : Stream function

[illegible] : Dynamic viscosity

[illegible] : Dimensionless similarity variable

[illegible] : Kinematic viscosity

C_1, C_2, C_3 : Dimensionless Material Constants

N : Dimensionless similarity function for microrotation

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