

Convective Heat Transfer for MHD Micropolar Fluids Flow through Porous Medium over a Stretching Surface

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ABSTRACT

This work examines heat transfer for MHD micropolar fluids flow through a porous medium over a stretching surface. The fluid flows due to a stretching sheet in presence of magnetic field and heat source. The parametric study of the problem provides results for velocity, microrotation and temperature distributions. The partial differential equations of fluid motion are transformed to ordinary differential form. The resulting equations have been then solved numerically for several values of the parameters involved in the study namely magnetic parameter, suction parameter, permeability parameter, heat source parameter, Prandtl number, Eckert number and material parameters C_1, C_2, C_3 . The comparison for Newtonian and micropolar fluids is presented. It is observed that fluid motion decelerates with increase in micropolar parameter C_1 .

KEYWORDS: Micropolar fluids, Similarity Transformations, Hartmann number, Prandtl number, Decay parameter,

1. INTRODUCTION

The concept of micropolar fluids, primarily introduced by Eringen [1] considers the fluids which exhibit certain microscopic effects arising from the local structure and micromotions of the fluid elements. Many classical flows are being re-examined to determine the effects of the fluid microstructure in the arena of the theory of micropolar fluids. Micropolar fluids consist of dilute suspension of rigid macromolecules with individual motions that support stress and body moments and are influenced by spin inertia. These fluids contain micro-constituents that can undergo rotation which affect the hydrodynamics of the flow. The dynamics of micropolar fluids provides some practical applications, for example turbulent shear flow, the flow of colloidal suspensions, polymeric fluids, liquid crystals, additive suspensions, human and animal blood, analyzing the behavior of exotic lubricants. Several researchers have considered models of polar and micropolar fluid flow through porous media (c.f.[2]-[7]). Peddission and McNitt [8] derived boundary layer theory for micropolar fluid which is important in a number of technical process and applied this equations to the problems of steady stagnation point flow, steady flow past a semi-infinite flat plate. Xinhui Si et al. [9] investigated the flow and heat transfer of an incompressible micropolar fluid in a channel with expanding or contracting walls and they employed Homotopy analysis method (HAM) to obtain the series solutions of the problem. Kamel et al. [10] derived field equations governing the steady flow of an incompressible micropolar fluid through isotropic porous sediments by using intrinsic volume averaging. Islam et al. [11] examined the MHD micro-polar fluid flow through a vertical porous plate. Alam et al. [12] investigated heat and mass transfer by mixed convection flow past a continuously moving infinite vertical porous plate under the action of strong magnetic field with constant suction velocity, constant heat and mass fluxes. Hamdan [13] considered the unsteady flow of a polar fluid through a porous sediment and developed governing equations that take in to account Forchheimer inertial effects in addition to the Darcy resistance.

The flow and heat transfer over a stretching surface bears important research interest due to its various applications in industries such as extrusion of a polymer in a melt spinning process, wire drawing, hot rolling, glass fiber production and manufacturing plastic films. Crane [14] investigated the flow caused by the stretching of a sheet. Many researchers such as Gupta and Gupta [15], Dutta et al. [16], Chen and Char [17] extended the work of Crane [14] by including the effect of heat and mass transfer analysis under different physical situations. Moreover, various aspects of such problem have been investigated by many authors such as Xu and Liao [18], Cortell [19, 20], Hayat et al. [21] and Hayat and Sajid [22]. Barik et al. [23] studied heat and mass transfer effect on the flow over a stretching sheet in the presence of a heat source.

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Hitesh Kumar [24] considered heat transfer over a stretching Porous sheet subjected to power law heat flux in presence of heat source. Sharma and Singh [25] studied effects of the Ohmic heating and viscous dissipation on steady MHD flow near a stagnation point on an isothermal stretching sheet. Anjali Devi and Ganga [26] studied effects of viscous and Joules dissipation on MHD flow, heat and mass transfer past a stretching porous surface embedded in a porous medium.

2. MATHEMATICAL ANALYSIS

Consider steady, two dimensional and incompressible laminar flow of viscous fluid through porous medium over a stretching/ shrinking surface. The fluid is electrically conducting. Magnetic field of strength B_0 is applied in normal direction to the sheet. A convective heat source with heat flux boundary conditions provides temperature T_w at the surface. The Cartesian coordinates are used. The x-axis is along the sheet and y-axis is perpendicular to it. The origin is fixed. Here u, v are velocity components along horizontal and vertical directions. The induced magnetic field is neglected. The permeability of medium is K_0 . The governing equations of the motion are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$(\mu + \kappa)\left(\frac{\partial^2 u}{\partial y^2}\right) + \kappa\left(\frac{\partial w_3}{\partial y}\right) - \rho\sigma B_0^2 u - \frac{\nu}{K_0} u = \rho(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}) \quad (2)$$

$$\gamma\left(\frac{\partial^2 w_3}{\partial x^2} + \frac{\partial^2 w_3}{\partial y^2}\right) + \kappa\left(-\frac{\partial u}{\partial y}\right) - 2\kappa w_3 = \rho j\left(u\frac{\partial w_3}{\partial x} + v\frac{\partial w_3}{\partial y}\right) \quad (3)$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{K'}{\rho C_p} \frac{\partial^2 T}{\partial y^2} \quad (4)$$

where w_3 is microrotation component. ρ is the density, μ is the coefficient of viscosity, κ is the vortex density, γ is the spin gradient viscosity coefficient, σ is the electrical conductivity of the fluid, T is the temperature, T_∞ is the free stream temperature, K' is the thermal conductivity of the fluid and C_p is the specific heat at constant pressure.

The associated boundary conditions are:

$$u = ax, \quad v = -v_0, \quad w_3 = 0 - K' \frac{\partial T}{\partial y} = q_w = E_0 x^2 \text{ at } y = 0$$

$$u \rightarrow 0, \quad w_3 \rightarrow 0, \quad T \rightarrow T_w \text{ as } y \rightarrow \infty \quad (5)$$

The similarity transformations are as follows:

$$u = axf'(\eta), \quad v = -\sqrt{\nu a} f(\eta), \quad w_3 = \sqrt{\frac{a}{\nu}} axg(\eta)$$

$$T - T_\infty = \frac{E_0 x^2}{K'} \sqrt{\frac{\nu}{a}} \theta(\eta) \quad (6)$$

where $\eta = y\sqrt{\frac{a}{\nu}}$ is dimensionless variable, E_0 is positive constant, T_∞ is temperature far away from the surface, q_w is ratio of heat transfer, ν is kinematic viscosity coefficient. The equation of continuity is identically satisfied and the equations (2) to (4) respectively yield:

$$(1 + C_1)f''' + ff'' - f'^2 - (M^2 + \frac{1}{K})f' + C_1g' = 0 \quad (7)$$

$$C_3g'' - C_1C_2f'' - 2C_1C_2g = (fg' - f'g) \quad (8)$$

$$\theta'' + \text{Pr} f\theta' - 2\text{Pr} f'\theta = 0, \quad (9)$$

where prime denotes the differentiation with respect to η , C_1, C_2 and C_3 are non dimensional material constants. $M^2 = \frac{\sigma B_0^2}{\rho c}$ is magnetic parameter, $K = \frac{K_0 c}{\nu}$ is permeability parameter and $Pr = \frac{\mu C_p}{K}$ is a Prandtl number.

The boundary conditions (4) and (5) then become:

$$\left. \begin{aligned} f(0) = \lambda, f'(0) = 1, g(0) = 0, \theta'(0) = -1, \\ f'(\infty) = 0, g(\infty) = 0, \theta(\infty) = 0. \end{aligned} \right\} \quad (10)$$

3. RESULTS AND DISCUSSIONS

The highly non-linear ordinary differential equations (7) to (9) together with boundary conditions (10) are solved numerically by using Mathematica software. The numerical results for velocity, microrotation and temperature functions have been obtained for several values of the physical parameters namely, magnetic parameter M^2 , Permeability parameter K , suction parameter λ and Prandtl number P_r . Curves have been drawn to elaborate the effects of these parameters on fluid flow, microrotation and heat transfer distributions. In order to elaborate the micropolar nature of the fluid motion, results have been obtained for different sets of the material constants C_1, C_2, C_3 chosen arbitrarily.

Figure-1 to Figure-3 respectively show that the horizontal velocity component f' decreases with increase in the values of magnetic, permeability and suction parameters. Fig.4 shows the comparison of fluid velocity component f' for Newtonian and micropolar fluids. It is to be mentioned that in absence of C_1 , the fluid motion relates to Newtonian fluids. Figure 5 demonstrates the microrotation that increases in the main flow stream with increase in the value of micropolar parameters.

The curves in figure 6 to figure 8 respectively, depict the pattern of temperature function θ under the effect of M^2, K and P_r . It is noticed that these parameters cause to increase the temperature distribution. Figure 9 demonstrates the effect of λ on θ . It is observed that the temperature decreases with increase in the values of λ .

4. CONCLUSIONS

Numerical solution for convective heat transfer in micropolar fluid flow through porous medium and over a stretching surface in the presence of magnetic field is considered. The main results of this work are as follows:

- The horizontal velocity component f' reduces in magnitude with increase in the values of M^2, K and λ .
- The velocity component f' is greater in magnitude for Newtonian fluids than for micropolar fluids
- The microrotation increases a little away from the boundary with increase in the values of micropolar parameters.
- The temperature increases with increase in the values of M^2, K and P_r but it decreases with increase in λ .

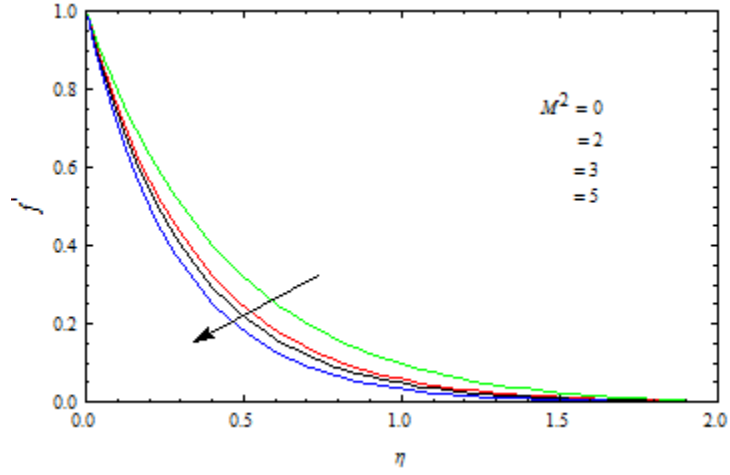


Fig-1. Graph of f' for different values of M^2

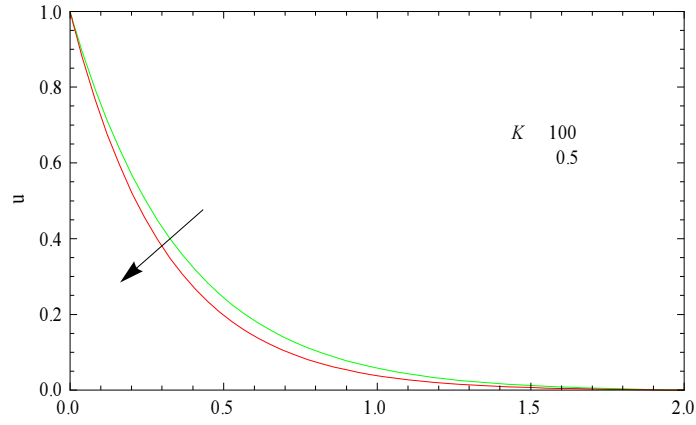


Fig-2. Graph of f' for different values of K

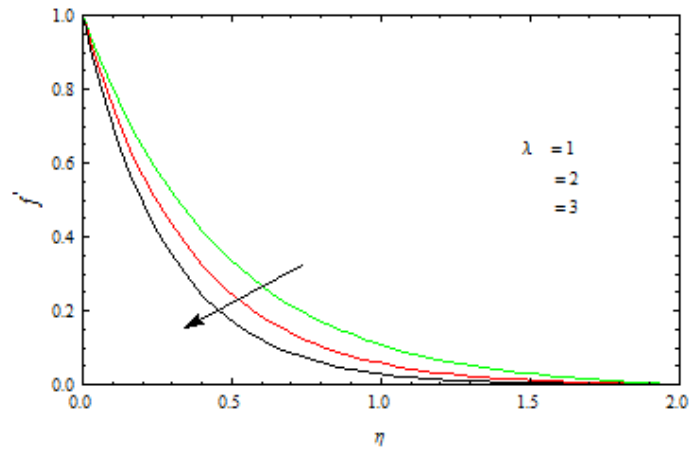


Fig-3. Graph of f' for different values of λ

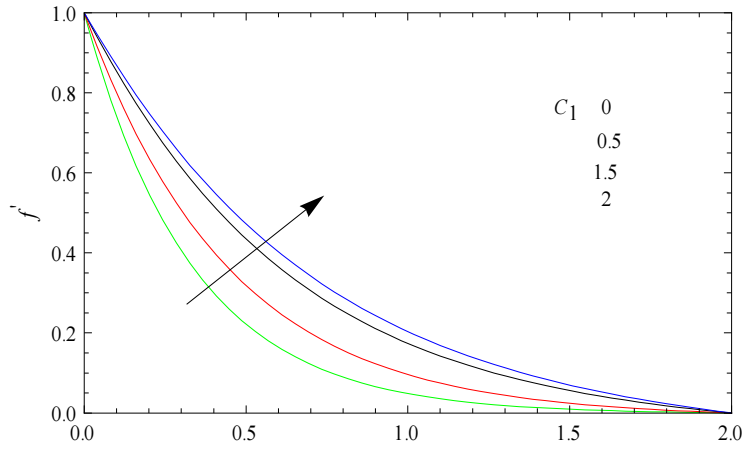


Fig-4. Graph of f' for different values of C_1

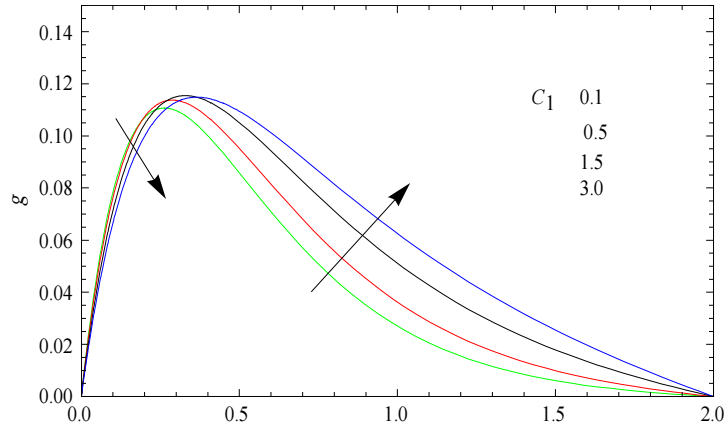


Fig-5. Graph of g for different values of C_1

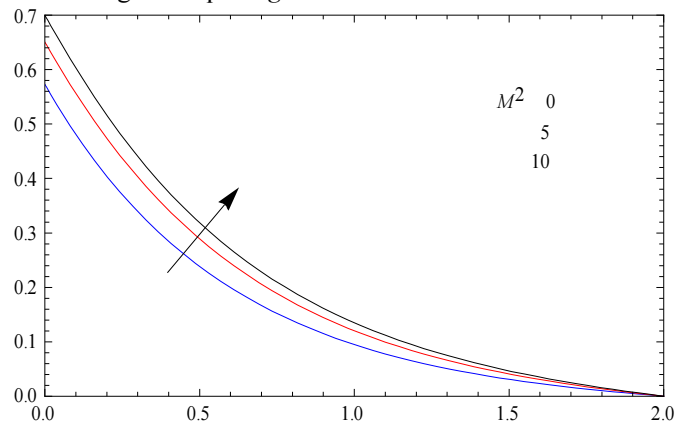


Fig-6. Graph of θ for different values of M^2

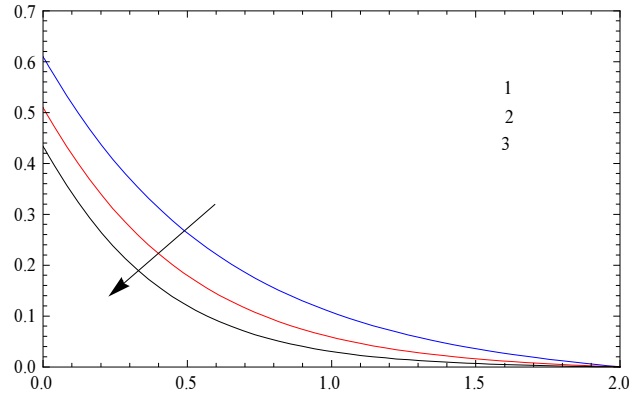


Fig-7. Graph of θ for different values of λ

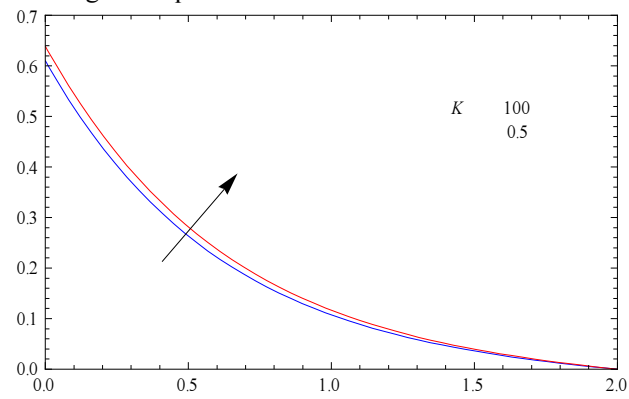


Fig-8. Graph of θ for different values of K

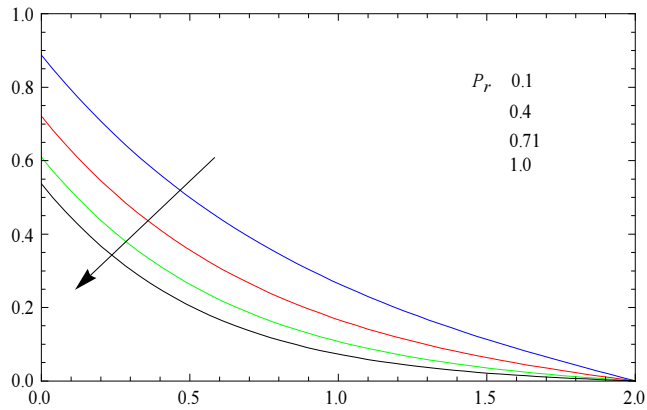


Fig-9. Graph of θ for different values of P_r

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