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Multi Dimensional Finite Element Method in Impedance Tomography Forward Problems

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ABSTRACT

This research is an attempt to describe the Electrical Impedance Tomography (EIT) in a different way. EIT is an undeveloped medical imaging technique which aims to clear the impedance structure of inside an object based on measuring its surface. Currently, using this method in business is not possible due to lack of access carefully enough. In this article, ignoring the classical viewpoints on this kind of tomography, the researcher tries to introduce new strategies to describe the issue using blocking out the object under study. Therefore, the object is considered as a set of similar blocks with a constant impedance that have electronic exchanges with each other and their surrounding environment. Then, calculating and simplifying the relations between each of these blocks, a series of equations are gained that can help to completely describe the object and tell how to directly solve the EIT issue.

KEYWORDS: blocking out the equations, Finite Element Method, Forward Problem, Impedance Tomography.

1. INTRODUCTION

In many medical issues, understanding the electrical features of the inside of human body and its changes is very important. By electrical features in the above sentence, we mean the electrical conductivity and permeability. Both of these two features are very significant in medical applications because on the one hand different tissues of the body have different modes of conductivity and permeability, but on the other, identifying this type of electrical features of the inside of body are very useful in medical issues like diagnosing Pulmonary embolism or blood coagulation in lung. The electrical map of human body can be useful in many other medical issues [1-5]. In practice, in order to register the data, some electrodes are attached to the skin around the body of patient and at their other ends, they are attached to a data collection unit whose output is plugged in a computer. Then, applying a weak current to the body, a series of potentials will be developed in other electrodes.

Since these applied currents select the circuits according to the impedance to flow through, the way this electrical circuit flows through the body (potential distribution among other electrodes) will be based on conductivity distributions. However, this changes reconstructing the image to a non-linear issue that can be tackled using several methods [1, 5, 7]. The result will be called Electrical Impedance Tomography (EIT) [2, 3, 5-9]. In the above sentence, the term Impedance is used with the same electrical concept that expresses the voltage ratio of both ends of an electrical element to the ampere [10-12]. Since the resistivity of different tissues ranges widely from 0.65 ohm meter in cerebrospinal fluid to 166 ohm meter in bone, it is possible to image every sections of the body and EIT, in fact, specifies the distribution map of electrical impedance of a specific section of the body. By now, different models such as continuous model have been suggested to describe and analyze the EIT issue each of which has unique features.

2. MATERIALS AND METHODS

In solving the two-dimensional EIT through blocking method, it is suggested that the object under study is a two-dimensional rectangular composed of m*n blocks of the same size (Figure. 1). In addition, it is suggested that in each of these blocks, all the parts have similar electrical impedance as in all the parts of a block, a linear electric field will be made due to applying the circuit to the object.

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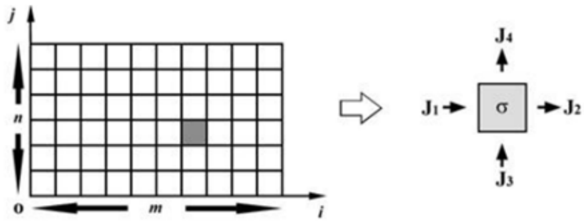

Figure 1- Ageneral design of a blocked-out object with a desired block of the object.

It is clear that as m, n→∞ increases; it is truer to suggest that the electrical impedance distribution inside the object is in a block form. As it is shown in figure 1, each block is described according to its position in relation to the point O, Cartesian coordinates with two horizontal and vertical components of *i* and *j* as *Bij* where *i* and *j* are changed from 1 to *m* and from 1 to *n* respectively. For each B_{ij} block, a special electrical conductivity is defined equal to σ*ij* that is similar for all parts of that block according to the suggested hypothesis. In this series, each block has electronic exchanges with adjacent blocks or injecting electrodes. Here, the hypothesis is that alternating current densities of $J_3[i, j]$ and $J_1[i, j]$ enter to the left and below dimensions of B_{ij} respectively and the alternating current densities of $J_4[i, j]$ and $J_2[i, j]$ exit from its right and above dimensions (Figure 2). Since the object is two-dimensional, the unit of measuring density is determined as ampere to the length (e.g. meter). Another important hypothesis, that will be considered in the block analysis of EIT, is the monotonicity of the current densities of each block and their linear changes in relation to the current flow direction. In other words, in each desired block of B_{ij}(Figure 2):

$$j_X[\mathrm{i,j}] = \mathrm{J}_2[\mathrm{i,j}] + \frac{\mathrm{x}-\Delta_\mathrm{x}}{\Delta_\mathrm{x}}(\mathrm{J}_2[\mathrm{i,j}] - \mathrm{J}_1[\mathrm{i,j}]) \quad (1)$$

$$j_y[\mathrm{i,j}] = \mathrm{J}_4[\mathrm{i,j}] + \frac{\mathrm{Y}-\mathrm{y}}{\Delta_\mathrm{y}}(\mathrm{J}_4[\mathrm{i,j}] - \mathrm{J}_3[\mathrm{i,j}]) (2)$$

Where $J_x[i, j]$ and $J_y[i,j]$ are respectively the horizontal and vertical components of current density of a part of block B_{ij}that are located at the distance x of the left dimension and y of the below dimension of the block (Δx and Δy are the sizes of horizontal and vertical dimensions of each block respectively). $J_1[i, j]$, $J_2[i, j]$, $J_3[i, j]$ and $J_4[i, j]$ are the corresponding current densities of the block B_{ij} identified before.

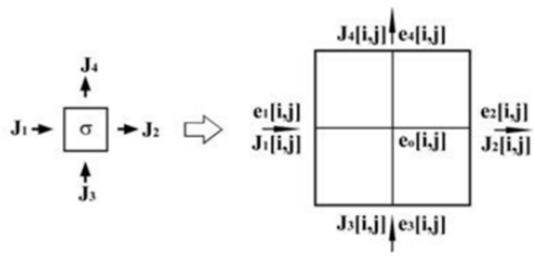

Figure 2- A desired block in the blocked-out object with the densities of input and output currents.

As suggested before, the special conductivity of each block is considered equal for all its parts; therefore, using ohm's law, *j*= σ*E*, the voltage of every desired part inside of the block can be obtained as the following:

$$e_x[i,j] - e_1[i,j] = -\int_0^x \frac{1}{\sigma_{ij}} J_x[i,j]dx = -\frac{1}{2}\left(\frac{J_2[i,j]-J_1[i,j]}{\sigma_{ij}\Delta_x}\right)x^2 - \frac{J_1[i,j]}{\sigma_{ij}}x (3)$$

$$e_y[i,j] - e_3[i,j] = -\int_0^y \frac{1}{\sigma_{ij}} J_y[i,j]dy = -\frac{1}{2}\left(\frac{J_4[i,j]-J_3[i,j]}{\sigma_{ij}\Delta_y}\right)y^2 - \frac{J_3[i,j]}{\sigma_{ij}}y (4)$$

In the above equations, the expression $e_x[i,j]$ - $e_1[i, j]$ shows the potential difference between the node $e_1[i, j]$ of the block B_{ij} and a node at the distance x located at the same height. Similarly, the expression $e_y[i, j]$ - $e_3[i, j]$ shows the potential difference between the node $e_3[i, j]$ of the block B_{ij} and a node at the distance y located on the same vertical line (Figure 2). Since the quantity of the potential is scalar, we can obtain the potential difference of both desired points inside the block using these equations. Other parameters have been identified before. Regarding the equations 3 and 4 and considering figure 2, the following results can be achieved:

$$e_1[\mathrm{i,j}] - e_\mathrm{o}[\mathrm{i,j}] = \frac{1}{8}\Delta_x\left(\frac{3J_1[\mathrm{i,j}]+\mathrm{J}_2[\mathrm{i,j}]}{\sigma_{ij}}\right) (5)$$

$$e_\mathrm{o}[\mathrm{i,j}] - e_2[\mathrm{i,j}] = \frac{1}{8}\Delta_x\left(\frac{J_1[\mathrm{i,j}]+3\mathrm{J}_2[\mathrm{i,j}]}{\sigma_{ij}}\right) (6)$$

$$e_3[\mathrm{i,j}] - e_\mathrm{o}[\mathrm{i,j}] = \frac{1}{8}\Delta_y\left(\frac{3J_3[\mathrm{i,j}]+\mathrm{J}_4[\mathrm{i,j}]}{\sigma_{ij}}\right) \quad (7)$$

$e_o[i,j] - e_4[i,j] = \frac{1}{8}\Delta_y\left(\frac{J_3[i,j]+3J_4[i,j]}{\sigma_{ij}}\right)$ (8)

In the above equations, the expressions $e1[i, j]$, $e2[i, j]$, $e3[i, j]$ and $e4[i, j]$ are respectively the potentials of the middle nodes of left, right, below and above dimensions of block B_{ij} and $e_O[i,j]$ is the potential of the central node of block B_{ij}. By solving the above coordinates according to the current densities, the following equations can be obtained:

$$J_1[i,j] = \sigma_{ij}\frac{3(e_1[i,j]-e_o[i,j])-(e_o[i,j]-e_2[i,j])}{\Delta_x} \quad (9)$$

$$J_2[i,j] = \sigma_{ij}\frac{3(e_o[i,j]-e_2[i,j])-(e_1[i,j]-e_o[i,j])}{\Delta_x} \quad (10)$$

$$J_4[i,j] = \sigma_{ij}\frac{3(e_o[i,j]-e_4[i,j])-(e_3[i,j]-e_o[i,j])}{\Delta_y} \quad (11)$$

$$J_4[i,j] = \sigma_{ij}\frac{3(e_o[i,j]-e_4[i,j])-(e_3[i,j]-e_o[i,j])}{\Delta_y} \quad (12)$$

Now that the KCL law is true, the sum of input currents in the common dimension of the block must be equal to 0; therefore, referring to the equations (5-12) and regarding the fact that in the common dimensions of the blocks, since the level of current flow is equal, the sum of currents is equal to sum of densities equal to 0, a set of equation is obtained as the following:

$\forall i \in [1, m-1], \forall j \in [1, n]: J_T[i,j] \triangleq J_2[i,j] = J_1[i+1,j]$(13)

$\forall i \in [1, m], \forall j \in [1, n-1]: J_N[i,j] \triangleq J_4[i,j] = J_3[i,j+1]$(14)

On the other hand, regarding the block model of the object, one can see that all the nodes in the adjacent dimensions are common with each other and thus, the following relations can be defined:

$\forall i \in [1, m-1], \forall j \in [1, n]: e_T[i,j] \triangleq e_2[i,j] = e_1[i+1,j]$(15)

$\forall i \in [1, m], \forall j \in [1, n-1]: e_N[i,j] \triangleq e_4[i,j] = e_3[i,j+1]$(16)

Simplifying the set of equations 11-14, n(m – 1) + m(n – 1) equations can be defined:

$$\forall i \in [1, m-1], \forall j \in [1, n]:$$
$$e_T[i,j] = \frac{4}{3}\frac{\sigma_{ij}e_o[i,j] + \sigma_{(i+1)j}e_o[i+1,j])}{\sigma_{ij} + \sigma_{(i+1)j}} - \frac{1}{3}\frac{\sigma_{ij}e_T[i-1,j] + \sigma_{(i+1)j}e_T[i+1,j]}{\sigma_{ij} + \sigma_{(i+1)j}} \quad (17)$$

$$\forall i \in [1, m], \forall j \in [1, n-1]:$$
$$e_N[i,j] = \frac{4}{3}\frac{\sigma_{ij}e_o[i,j]+\sigma_{i(j+1)}e_o[i+1,j])}{\sigma_{ij}+\sigma_{i(j+1)}} - \frac{1}{3}\frac{\sigma_{ij}e_N[i,j-1]+\sigma_{i(j+1)}e_N[i+1,j]}{\sigma_{ij}+\sigma_{i(j+1)}} \quad (18)$$

In addition, regarding the KCL law, the sum of input currents applied to each desired block should be equal to 0 and thus, another set of equations will be formed:

$\forall i \in [1, m], \forall j \in [1, n]: J_1[i,j].\Delta_x + J_3[i,j].\Delta_y = J_2[i,j].\Delta_x + J_4[i,j].\Delta_y$(19)

Simplifying this set of equations leads to n.m other equations as the following:

$\forall i \in [1, m], \forall j \in [1, n]: e_o[i,j] = \frac{e_T[i,j]+e_T[i-1,j]+e_N[i,j]+e_N[i,j-1]}{4}$(20)

In order to reduce the volume of calculations in the future, the central node variables will be eliminated by including the equation (20) in other equations and this leads to new equations as the following:

$$\forall i \in [1, m-1], \forall j \in [1, n]:$$
$$e_T[i,j] = \frac{1}{2}\frac{\sigma_{ij}(e_N[i,j] + e_N[i,j-1]) + \sigma_{(i+1)j}(e_N[i+1,j] + e_N[i+1,j-1])}{\sigma_{ij} + \sigma_{(i+1)j}} \quad (21)$$

$$\forall i \in [1, m], \forall j \in [1, n-1]:$$
$$e_N[i,j] = \frac{1}{2}\frac{\sigma_{ij}(e_T[i,j] + e_T[i-1,j]) + \sigma_{(i+1)j}(e_T[i,j+1] + e_N[i-1,j+1])}{\sigma_{ij} + \sigma_{i(j+1)}} \quad (22)$$

3. RESULTS

As it is described in the electrical impedance tomography theory, a current is applied to the object through electrode arrays; this means that the surrounding blocks of the object and thus the surrounding nodes of the object have their own KCL relations.

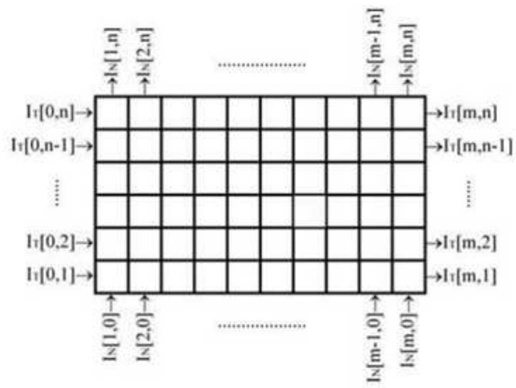

Figure 3- Calling the applied currents to surroundings of the object in analyzing the electrical impedance tomography via blocking method.

If we suppose that the currents $j\epsilon[1, n]$ and $I_T[0, j]$ are applied to the blocks of the left part of the object through electrodes and the currents $j\epsilon[1, n]$ and $I_T[m, j]$ exit from the electrodes of the blocks of the right part of the object (figure 3). Also, if we suppose that the currents $j\epsilon[1, n]$ and $I_N[i,0]$ are applied to the blocks of the below part of the object and the currents $i\epsilon[1, m]$ and $I_N[i, n]$ exit through the electrodes of the blocks of the above parts of the object (figure 3). Referring the equations (9-11), 2(n+m) relations will be formed as the following:

$\forall j \in [1,n]: I_T[0,j] = I_1[1,j].\Delta_x = \sigma_{ij}.(3(e_T[0,j] - e_o[1,j]) - e_o[1,j] - e_T[1,j]))$(23)

$\forall j \in [1,n]: I_T[m,j] = I_2[m,j].\Delta_x = \sigma_{mj}.(3(e_o[m,j] - e_T[m,j]) - e_T[m-1,j] - e_o[m,j]))$ (24)

$\forall i \in [1,m]: I_N[i,0] = I_3[i,1].\Delta_y = \sigma_{ij}.(3(e_N[i,0] - e_o[i,1]) - e_o[i,1] - e_T[i,1]))$(25)

$\forall i \in [1,m]: I_N[i,n] = I_4[i,n].\Delta_y = \sigma_{in}.(3(e_o[i,n] - e_n[i,n]) - e_N[i,n-1] - e_T[i,n]))$ (26)

In the above expression, $e_N[i,n]$, $e_T[m,j]$, $e_N[i, 0]$ and $e_T[0, j]$ are the meddle nodes of parietal dimensions of some blocks of the object located in its membranes. As the order shown in figure 3, the applied currents of $I_T[0, j]$, $I_N[m, j]$, $I_T[m, j]$ and $I_N[i, n]$ enter and exit from and to the object through these nodes. The above relations can be changed according to these nodes as the following:

$\forall j \in [1,n]: e_T[0,j] = \frac{1}{3}\frac{I_T[0,j]}{\sigma_{ij}} + \frac{4}{3}e_o[1,j] - \frac{1}{3}e_T[1,j]$(27)

$\forall j \in [1,n]: e_T[m,j] = -\frac{1}{3}\frac{I_T[m,j]}{\sigma_{mj}} + \frac{4}{3}e_o[m,j] - \frac{1}{3}e_T[m-1,j]$(28)

$\forall i \in [1,m]: e_N[i,0] = \frac{1}{3}\frac{I_N[i,0]}{\sigma_{ij}} + \frac{4}{3}e_o[i,1] - \frac{1}{3}e_N[i,1]$ (29)

$\forall i \in [1,m]: e_N[i,n] = -\frac{1}{3}\frac{I_N[i,n]}{\sigma_{in}} + \frac{4}{3}e_o[i,n] - \frac{1}{3}e_N[i,n-1]$ (30)

Central node variables of these equations will be eliminated by including the equation (20) and a new series of following equations will be formed:

$\forall j \in [1,n]: e_T[0,j] = \frac{1}{2}\frac{I_T[0,j]}{\sigma_{ij}} + \frac{1}{2}e_N[1,j] - \frac{1}{2}e_N[1,j-1]$(31)

$\forall j \in [1,n]: e_T[m,j] = -\frac{1}{2}\frac{I_T[m,j]}{\sigma_{mj}} + \frac{1}{2}e_N[m,j-1] - \frac{1}{2}e_N[m,j]$(32)

$\forall i \in [1,m]: e_N[i,0] = \frac{1}{2}\frac{I_N[i,0]}{\sigma_{ij}} + \frac{1}{2}e_T[i,1] - \frac{1}{2}e_T[i-1,1]$ (33)

$\forall i \in [1,m]: e_N[i,n] = -\frac{1}{2}\frac{I_N[i,n]}{\sigma_{in}} + \frac{1}{2}e_T[i,n] + \frac{1}{2}e_N[i-1,n]$ (34)

Now, considering the above formulae, when the EIT issue is studied from block point of view, there will be three kinds of potential nodes in the blocked-out object:

1. Horizontal nodes of $j\epsilon[1,n]$, $i\epsilon[0, m]$ and $e_T[i, j]$, N= n(m+1)
2. Vertical nodes of $j\epsilon[0,n]$, $i\epsilon[1, m]$ and $e_n[i, j]$, N= m(n+1)
3. Central nodes of $j\epsilon[1,n]$, $i\epsilon[1, m]$ and $e_o[i, j]$, N= m*n

Therefore, regarding eliminating the potentials of central nodes in the above equations, the general number of all our unknown nodes will be equal to m*(n+1) + n*(m+1), one of these nodes is the reference node whose potential is considered equal to 0; thus, the goal of solving the EIT issue is to obtain the m(n+1) + n(m+1) potentials of other nodes. On the other hand, referring to the equations (21-22) and (31-33), we will see that the number of these

equations will be equal to m*(n-1) + n*(m-1) + 2*(m+n) = m*(n+1) + n*(m+1). One of these equations is related to the reference node and according to the explanations offered in analyzing the nodes of electrical circuit, that equation will be ignored; thus, the number of other equations will be equal to the number of unknown quantities. All these equations are linear and independent. Hence, we can obtain all the unknown quantities of the problem including the potentials of horizontal and vertical nodes through the common methods to solve the linear equation system. Then, based on the potentials of these nodes and using the equations (9-12) and (20), all densities of the currents between the blocks- inside object- and the potentials of the blocks' central nodes will be calculated. In order to solve the above linear equation system, we can use different analytic methods such as the inverse matrix method or iterative methods like Gauss- Seidel Iteration Method. So, we can see how a current is distributed through a desired object by solving the equation system.
In this section, the process of direct solving the EIT issue was studied to see how the methods to solve it functions and the currents are distributed throughout an object. Therefore, a desired m*n matrix is considered as a special conductivity matrix of a given object. Here, the amount of each element of this matrix is considered as the special conductivity of a corresponding block of that element in the object. Thus, a pattern for impedance distribution was reconstructed by this matrix. Then, regarding the fact that some certain currents are applied to the object membrane through special points, all the currents and internal voltages of the object- the potentials of the given nodes- were calculated by solving the aforementioned problem through the inverse matrix method.

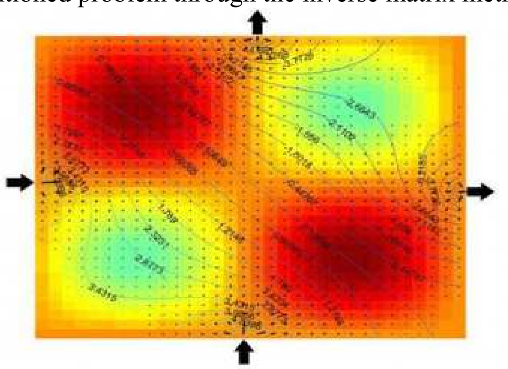

Figure 4- An image of how the current is distributed- the voltage inside the object in electrical impedance tomography.

The result has been shown in figure 4. This image, regarding applying 100-ampere currents, shows how the currents and voltages distribute throughout the object. The points of applying the current are marked by black arrows and the vectors in the picture indicate the distribution and the size of input currents. The continuous lines in the picture are the symbols of potential curves and their corresponding values are written on them. The background picture implies the impedance structure of the given object. The reference node is located on the top right corner of the object.

4. **Conclusion**

Electrical impedance tomography through the block method is a new approach to this technique that tries to change this problem to analysis of small elements rather than adopting the old methods that were mainly based on solving potential equations so that it would be possible to find the unknown quantities of the problem using only common simple precise solutions for linear equation system with *n* unknown quantities without any need to the old complex imprecise solutions, such as solving potential equations. In addition, we can increase the clarity by the optional increasing the number of given blocks of the object. As it was observed in the simulation process, this technique is a very efficient and quick method to directly solve the EIT issue, i.e. studying the current distribution in an object under electrical impedance that offer acceptable solutions. Therefore, the created equations system, and thus the block method, can describe the object under electrical tomography completely and precisely and calculate all its electrical features in each current application. The block method can also find the way to distribute the current-voltage in every environment. That is why this method can provide an inverse solution for the EIT issue too.

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