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Application of Finite Difference Method and Differential Quadrature Method in Burgers Equation

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ABSTRACT

In this paper, the Burgers equation will be solved by using the Finite Difference Method and Differential Quadrature Method. Some examples of Burgers equation were solved by these two methods for comparison with the numerical method to test its accuracy. In order to solve the Burgers equation, C- language program have been established based on the method of FDM, DQM and also numerical method. Generally, from the solutions of numerical method show that the DQM is better than the FDM in terms of accuracy.

KEYWORDS: Finite Difference Method (FDM), Ordinary Differential Equation (ODE), Differential Quadrature Method (DQM), Partial Differential Equation (PDE).

INTRODUCTION

We mostly notice the existence of a system or a distinct of Partial Differential Equations (PDEs) in various field of sciences and also engineering. PDEs correspondingly exemplify the phenomenal of fundamental law in mostly field of physical or chemical. This study employs the equation of Burgers-Huxley that can efficiently exemplary the association between convection effects, diffusion transports and reaction mechanisms[1]. Generally, there is a question in the inavailability or hard to get closed-form solution because most problems persist when the nonlinear partial differential equations was involved. With this, it has caused another alternative development to relatively proximate the elucidation of these PDEs.

The Burgers-Huxley is a nonlinear partial differential equation (PDE) in which it is significant to model diverse engineering field of mechanism such as the nonlinear wave process in economics, physics and ecology [2]. There is no general numerical approximation to obtain the solution for this non-linear PDE according to many researchers. They have adopted different numerical techniques in approximating the result of the Burgers-Huxley equation. However, in [3, 4] have been solved the exact solution for the Burgers-Huxley equation.

Finite Difference Method (FDM) is one of the practices used to determine the Burgers equation. FDM is generally known and acknowledged as the most simple method. The approximations of the derivatives by differences in these values were clarified by substituting the values at a particular grid points. From the neighborhood values, estimation was made for the partial derivatives in the PDE at every grid points.

Differential Quadrature Method (DQM) is the other method that we want to discuss here. DQM is a continuation of FDM which identified by [5] as the top rank of accurate difference scheme. The method epitomizes the way to summarize all the derivatives from the function at certain grid points. Later, the equation transfigures to a system of ordinary differential equations (ODEs) or a set of algebraic equations [6]. Finally the system of ODEs is solve using numerical method.

There are abundance of techniques to get the approximation solution for Burgers equation. The Burgers equation needs a set of preliminiary and boundary conditions to be solved. Hence, the use of FDM, eventhough it is quite simple to use, it lacks accuracy to solve the examples of Burgers equation. On the other hand, the usage of DQM to solve the problem acquires a little more calculation and a bit tediuous as it consumes a lot of times.

In the 1960s and 1970s the FDM have dominated in computational sciences since its inception and were the method of choice in that year. The characteristics of the finite difference method are this method utilizes uniformly spaced grids. Other characteristic is at each node, each derivative is approximated by an algebraic expression which references the adjacent nodes. Another of two characteristics are to get the system of algebraic equations by evaluating the previous step for each node and the system is solved for the dependent variable. FDM is used to approximate the derivative of a function values at one point by using the function value at discrete points. The solutions of the PDE or ODE can be approximated by substituting the derivatives expression with the FDM. This method is easy to be applied but it is less accurate compare to DQM.

In the year 1972, to overcome the problem from FDM, the DQM was first proposed by [10]. This method important in many fields such as biosciences, system identification, diffusion, transport process, fluid dynamics,

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chemical engineering, lubrication, acoustics and contact problem [7]. DQM was proven to be correspondent to the overall collocation method [8]. According to [9], it is also equivalent to the most high order finite difference scheme, which is in extension of the low order finite difference schemes.

The DQM is used to convert the partial differential equation to ordinary differential equation by replacing the derivative of a smooth function with a weighted linear combination value [10]. This method used the function values on all grid points, so that this method is more expensive and increase the time consumed to get the solution.

METHODOLOGY

Burgers-Huxley Equation

In [1] stated that the generalized Burgers-Huxley equation can be utilized to model the interaction between reaction mechanism, convection effects and diffusion transports as shown below

$$\frac{\partial u}{\partial t}-\varepsilon\frac{\partial^2 u}{\partial x^2}+\alpha u^{\delta}\frac{\partial u}{\partial x}=\beta\left(1-u^{\delta}\right)\left(u^{\delta}-\gamma\right)u$$

where α, β, γ and δ are parameters that β ≥ 0, δ > 0 with initial condition

$$u(x,0)=\left(\frac{\gamma}{2}+\frac{\gamma}{2}\tanh(A_1 x)\right)^{\frac{1}{\delta}}$$

and the boundary conditions

$$u(0,t)=\left(\frac{\gamma}{2}+\frac{\gamma}{2}\tanh\left(-A_1A_2x\right)\right)^{\frac{1}{\delta}}\qquad t\geq 0$$

$$u(1,t)=\left(\frac{\gamma}{2}+\frac{\gamma}{2}\tanh\left(A_1\left(1-A_2t\right)\right)\right)^{\frac{1}{\delta}}\qquad t\geq 0$$

where

$$A_1=\frac{-\alpha\delta+\delta\sqrt{\alpha^2+4\beta\left(1+\delta\right)}}{4\left(1+\delta\right)}\gamma,$$

$$A_2=\frac{\gamma\alpha}{\left(1+\delta\right)}+\frac{\left(1+\delta-\gamma\right)\left(-\alpha+\sqrt{\alpha^2+4\beta\left(1+\delta\right)}\right)}{2\left(1+\delta\right)}$$

with Dirichlet boundary conditions

$$u(a,t)=f(t)\ \text{ and }\ u(b,t)=g(t)$$

The example of the nonlinear Burgers equation is given by

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}-\frac{\partial^2 u}{\partial x^2}=e^{-t}(1+x^2)\qquad 0<x<1,\ t>0$$

with the initial condition:

$$u\left(x,0\right)=1+x^2\qquad 0<x<1$$

and the boundary equations:

$$u(0,t) = e^{-t}, \qquad u(1,t) = 2e^{-t} \qquad t > 0$$

RESULTS AND DISCUSSION

The example above has been solved by using C++ programming with different methods. Then, the results from FDM and DQM are compared in term of accuracy of the exact solution.

Table 1: The results for time, t = 0.001

u(x, t)\x		0	0.2	0.4	0.6	0.8	1
FDM		0.999	1.040272	1.159956	1.359504	1.638869	1.998001
DQM		0.999	1.038961	1.158841	1.358641	1.638361	1.998001
EXACT		0.999	1.038961	1.158841	1.358641	1.638361	1.998001
ERROR	FDM	0	0.001311	0.001115	0.000863	0.000508	0
	DQM	0	0	0	0	0	0

Table 2: The results for time, t = 0.002

u(x, t)\x		0	0.2	0.4	0.6	0.8	1
FDM		0.998002	1.040536	1.159911	1.359008	1.637739	1.996004
DQM		0.998002	1.037922	1.157682	1.357283	1.636723	1.996004
EXACT		0.998002	1.037922	1.157682	1.357283	1.636723	1.996004
ERROR	FDM	0	0.002614	0.002229	0.001725	0.001016	0
	DQM	0	0	0	0	0	0

From the table above, the errors between the result from FDM and DQM with the exact solution have been plotted in the graph.

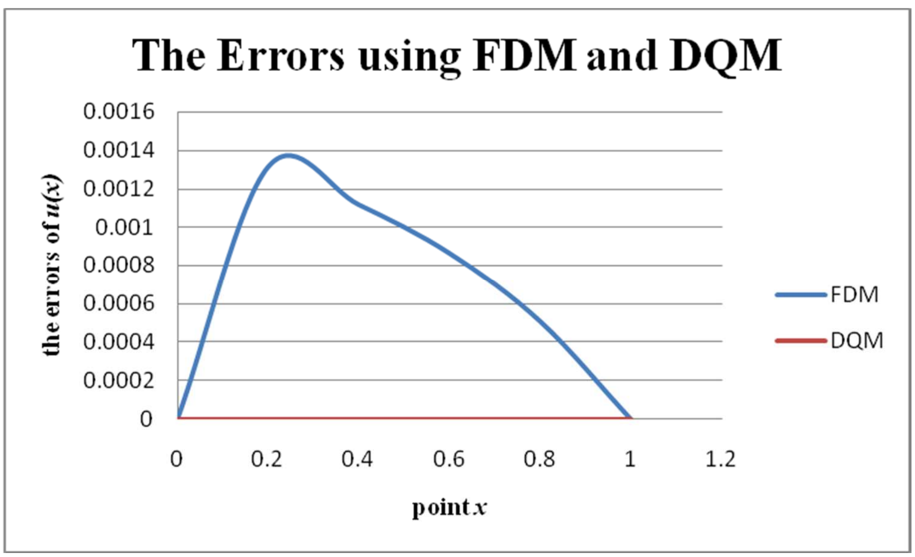

Figure 1 and 2 showed that the errors in graph form between FDM and DQM, u(x, t) against points x.

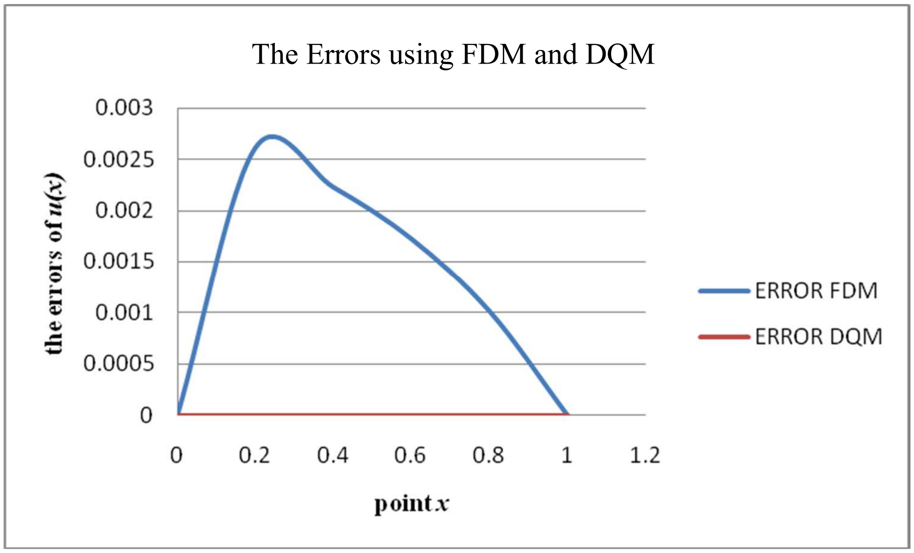

Figure 2: Result when time, t = 0.002

CONCLUSION

In conclusion, from the tables and figures above, the results from the DQM showed better than FDM at each of time tested. In term of errors, the errors from the DQM results are smaller than the results from the FDM. Therefore, the DQM is better than FDM in term of the accurate.

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