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The Application of Finite Element Method to Solve Heat Transfer Problem Involving 2D Irregular Geometry

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ABSTRACT

Finite Element Method (FEM) is one of a numerical technique applicable for solving boundary value problem and differential equations in engineering and mathematical physics by finding the approximate solutions. One of the most important partial differential equations is a heat transfer equation which explains energy in transition due to temperature differences, heat distribution and temperature variation over time in a given region. Heat transfer problems in industrial and engineering sector usually comprise of irregular and complex geometry domain. These require the use of FEM because even though FEM is a complex method thus it can solve complex problems. Since it is really difficult to get an analytical solution for most practical problems, the need for numerical methods increases. That is why FEM is essential for complicated geometries industrial problem when analytical solutions cannot be obtained. This research is aimed to solve heat transfer problem in simple 2D irregular geometry by applying FEM using the approach of Galerkin's method.
KEYWORDS: FiniteElement Method (FEM), 2D Irregular Geometry, Heat Transfer.

INTRODUCTION

Numerical method is a technique that able to find the approximation to a real solution for many sorts of engineering and science problems. Since the analytical solution for most practical problems is close to non-existence, the need for numerical methods increases. [1]. Complications caused by either irregular geometry or other discontinuities making it difficult to be solved analytically even when the customary equations and boundary conditions are resolved [2].

Heat transfer equation is one of the most important partial differential equations [3]. Heat transfer is the study of thermal energy transport within a medium of molecular interaction, fluid motion and electro-magnetic waves which resulting from a spatial variation in temperature [4]. The heat conduction problem was presented as

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+Q=0$$

where T is temperature and Q represents an equation for heat source [5]. A heat transfer analysis was made in earlier days by using analytical methods. However, this method is quite complicated. Later, it was improvise by using a numerical method which is very useful for analysis and result oriented with accuracy [6].

The approximation solutions to boundary value problems for differential equation can be answered using FEM [7]. Therefore, partial differential equation such as heat transfer can be seamlessly solved using FEM. FEM apply variation method to produce a stable solution and minimize an error function [8]. This resulting in its ability to deal with complex geometrical domains and existence of a huge set of approximation schemes adapted to various problems and local character of approximationsbut fixated in a unified formulation [9].

Fundamental idea of FEM is to discretize the domain into several subdomains or finite elements [10]. Finite element comes from the procedure in which a structure is divided into small, but finite size elements then the elements will be reconnecting at nodes [11]. Solution of FEM for numerical calculation is mesh generation which

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means a set of finite elements used to represent a geometric object for modeling or analysis [12]. FEM provides a formalism for generalizing discrete algorithms for approximating the solutions of differential equations. Such a task could be conceived automatically by a computer. However, it needs an amount of mathematical skill that requires human involvement [13]. The calculation of both methods is conducted numerically using MATLAB.

METHODOLOGY

2D Irregular Geometry Heat Transfer Problems

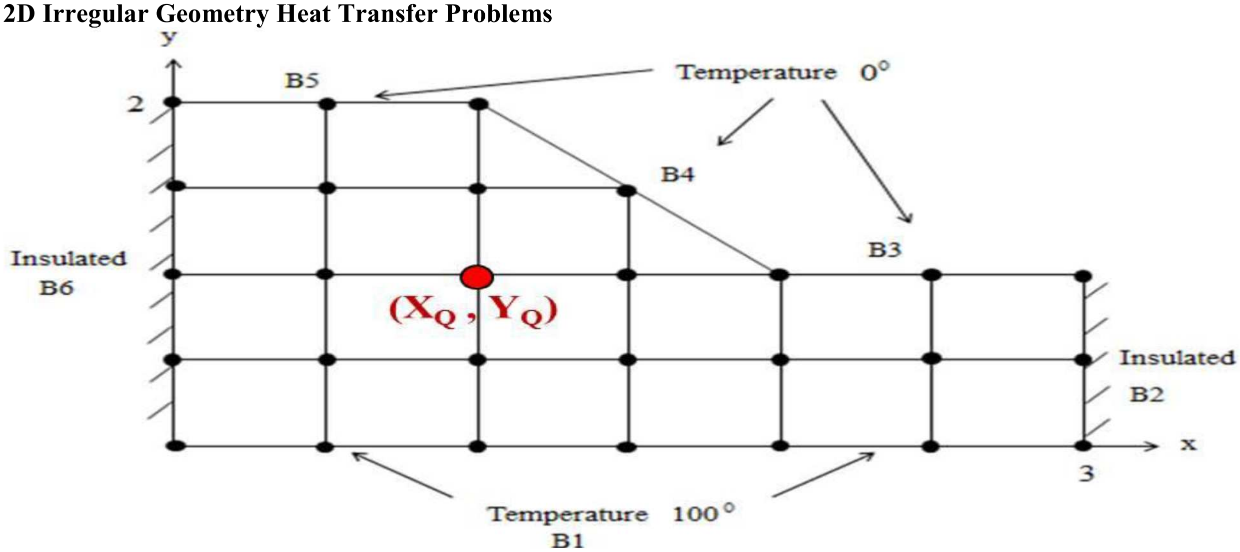

Figure 1: 2D simple irregular geometry heat transfer problem

The mathematical model of this problem is given as

$$\nabla^2 T + Q = 0$$

$$\nabla^2 T = -Q$$

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial x^2} = -Q$$

Range of x and y is given as $0 \le x \le 3,\ 0 \le y \le 2$. Equation Q and boundary conditions are given as

$$Q = f\frac{1}{\left(x - x_Q\right)^2 + \left(y - y_Q\right)^2 + 0.1}$$

$$\frac{\partial T}{\partial x} = 0 \qquad \text{for} \quad \text{B2 and B6}$$

$$T = 0 \qquad \text{for} \quad \text{B3, B4 and B5}$$

$$T = 100 \qquad \text{for} \quad \text{B1}$$

whereB represents boundary conditions and Q is an additional heat source that we set at the coordinate (X_Q, Y_Q). The boundary conditions are given and the temperature distribution or the unknown nodal values are needed to be obtained.

Finite Element Method: Heat Conduction using Galarkin's Approach

Application of FEM to solve 2D simple irregular geometry heat transfer problems can be simplified using the following flowchart.

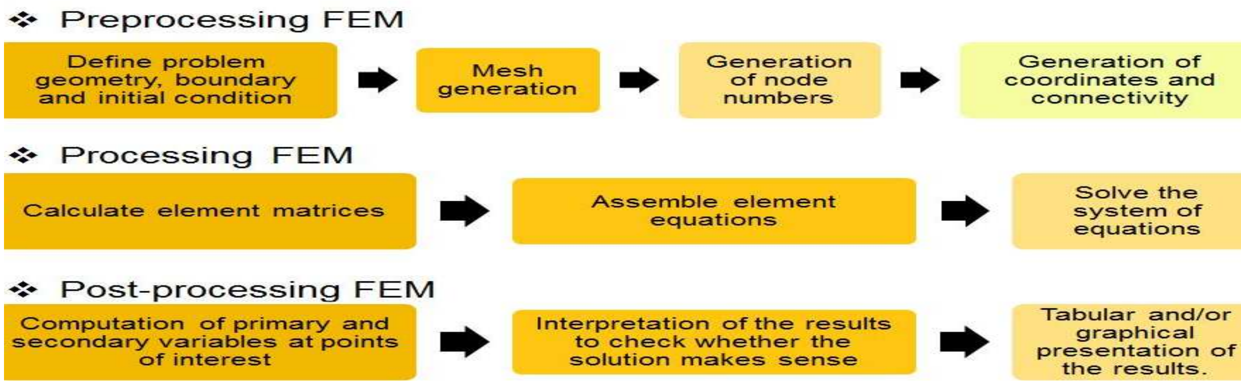

Figure 2:Flowchart of preprocessing, processing and post-processing of FEM

Step 1: Read geometry and boundary conditions.
Step 2: Generate mesh and node number (global node).

1. Note that mesh is represented by 3×nsize and 2×nsize where nsize is the mesh of segment per cell and we are using nsize = 2 for the solution of this problem.
2. Using triangular element concept on the mesh generation, all nodes and elements of the problem are generated.
3. The generation of local nodes with the global nodes is performed for all the calculation will be in global nodes.
4. The connectivity between nodes and element is shown in the table.

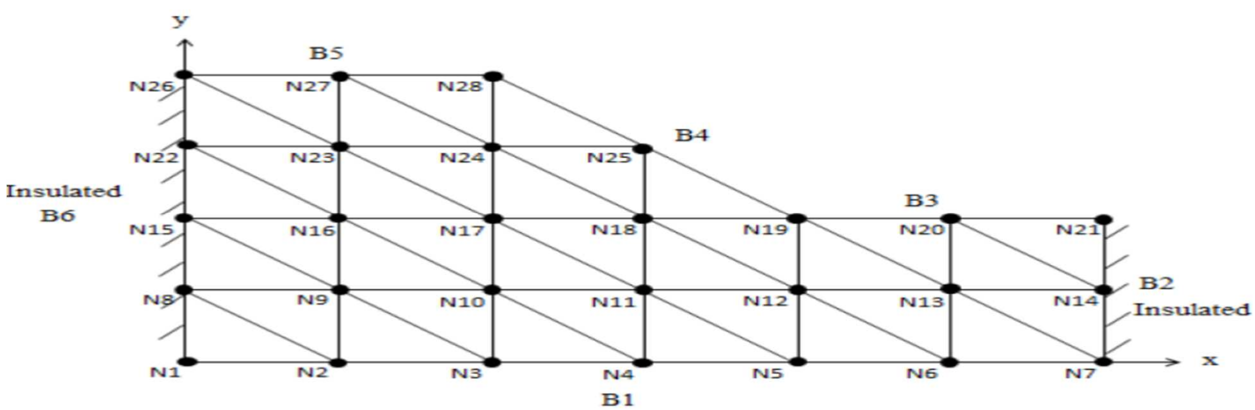

Figure 3: 28nodes generated for simple irregular problem using FEM

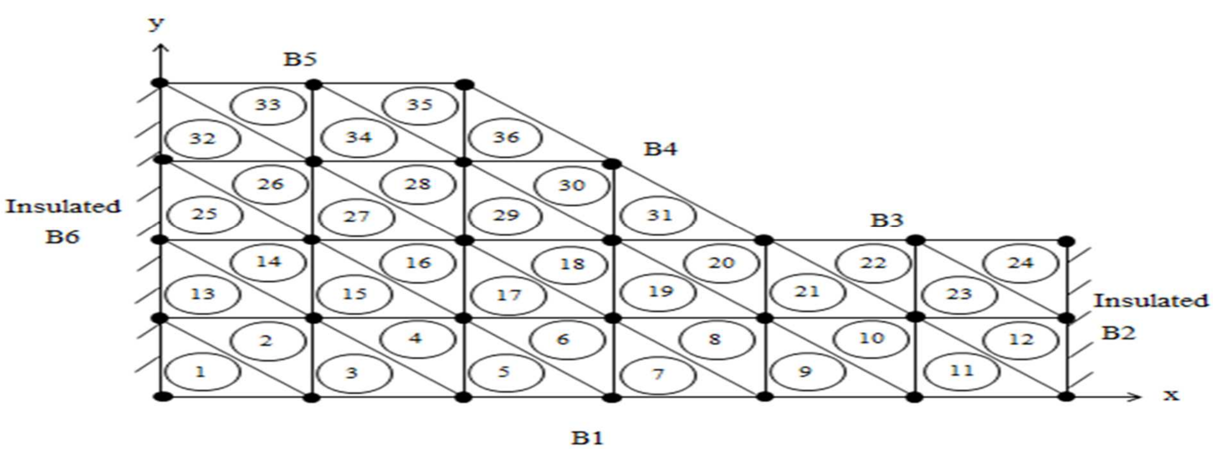

Figure 4: 36 elements generated for simple irregular problem using FEM

Table 1: Linear triangular element

Elements	1	2	3	Local nodes
1	1	2	8	
2	9	8	2	
3	2	3	9	
4	10	9	3	
5	3	4	10	
6	11	10	4	
7	4	5	11	
8	12	11	5	
9	5	6	12	
10	13	12	6	
11	6	7	13	
12	14	13	7	
13	8	9	15	
14	16	15	9	
15	9	10	16	
16	17	16	10	
17	10	11	17	
18	18	17	11	Global nodes
19	11	12	18	
20	19	18	12	
21	12	13	19	
22	20	19	13	
23	13	14	20	
24	21	20	14	
25	15	16	22	
26	23	22	16	
27	16	17	23	
28	24	23	17	
29	17	18	24	
30	25	24	18	
31	18	19	25	
32	22	23	26	
33	27	26	23	
34	23	24	27	
35	28	27	24	
36	24	25	28	

We can take one example for element 1, the nodes that connect the element are nodes 1, 2 and 8. While for element 2, the nodes that connect the element are nodes 9, 8 and 2. The nodes which acted like glue that connects the elements are in anti-clockwise notation.

Step 3: Calculate the element matrices.

1. Matrix B_T for all elements (e)

$$B_T = \nabla N = \nabla \left[N_1 \, N_2 \, N_3 \right] = \begin{bmatrix} \frac{\partial N_1}{\partial x} & \frac{\partial N_2}{\partial x} & \frac{\partial N_3}{\partial x} \\ \frac{\partial N_1}{\partial y} & \frac{\partial N_2}{\partial y} & \frac{\partial N_3}{\partial y} \end{bmatrix}$$

$$B_T = \frac{1}{\det J} \begin{bmatrix} \frac{\partial N_1}{\partial x} & \frac{\partial N_2}{\partial x} & \frac{\partial N_3}{\partial x} \\ \frac{\partial N_1}{\partial y} & \frac{\partial N_2}{\partial y} & \frac{\partial N_3}{\partial y} \end{bmatrix}$$

as detJ = $2A_e$ where orientation ±1 and A_e is the area of elementary

2. Matrix K_T for all elements (e)

$$\iint_A k\left(\frac{\partial\varphi}{\partial x}\frac{\partial T}{\partial x}+\frac{\partial\varphi}{\partial y}\frac{\partial T}{\partial y}\right)\partial A=\iint_A k\left(\frac{\partial\varphi}{\partial x}\ \frac{\partial\varphi}{\partial y}\right)\begin{bmatrix}\frac{\partial T}{\partial x}\\ \frac{\partial T}{\partial y}\end{bmatrix}\partial A$$

$$=\sum_e \phi^T\left[k_e\int_e B_T^T B_T dA\right]T^e=\sum_e \phi^T k_T T^e,$$

$$k_T=k_e\int_e B_T^T B_T dA=k_e A_e B_T^T B_T$$

Matrix h_T, r_q, r_∞, r_Q for element (e)

$$h_T=hl_{2-3}\int\begin{bmatrix}0\\ \eta\\ 1-\eta\end{bmatrix}\begin{bmatrix}0 & \eta & 1-\eta\end{bmatrix}d\eta=\frac{hl_{2-3}}{6}\begin{bmatrix}0&0&0\\0&2&1\\0&1&2\end{bmatrix}$$

$$r_\infty=hT_\infty l_{2-3}\int\begin{bmatrix}0\\ \eta\\ 1-\eta\end{bmatrix}d\eta=\frac{hT_\infty l_{2-3}}{2}\begin{bmatrix}0\\1\\1\end{bmatrix}\qquad r_Q=\frac{Q_e A_e}{3}\begin{bmatrix}1\\1\\1\end{bmatrix}$$

Step 4: Assemble element equation
1. The summation of the element mass matrices into the global mass matrix is called assembling.
2. The equation of global mass matrix given by

$$\psi^T\left(R_\infty-R_q+R_q\right)-\psi^T\left(H_T+K_T\right)T=0$$

Step 5: Solve the system of equation.
Step 6: Interpretation of result and validation: The result will be presented in graph and will be discussed.

RESULTS AND DISCUSSION

From $Q=f\dfrac{1}{\left(x-x_Q\right)^2+\left(y-y_Q\right)^2+0.1}$

First, we set the value of f to be equal to 10 and -10. We can see the heat distribution and heat flux is different when the value is positive and negative.

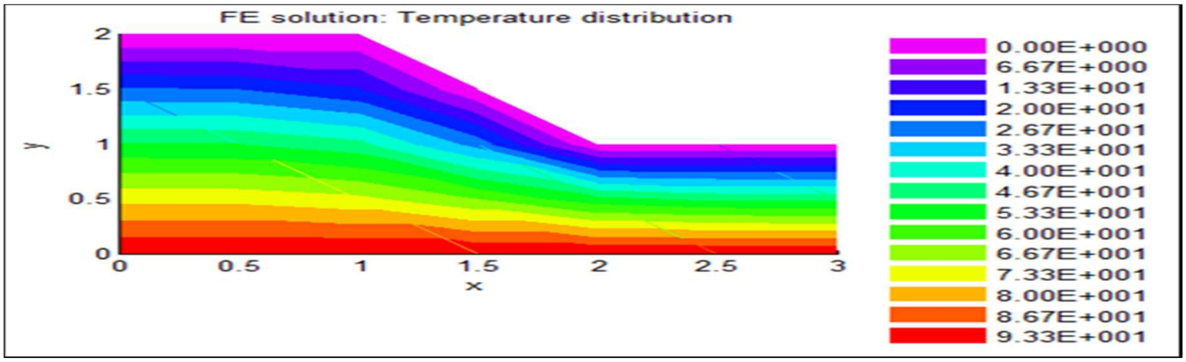

Figure 5:Heat distribution using FEM for simple irregular geometry problem when f = 10

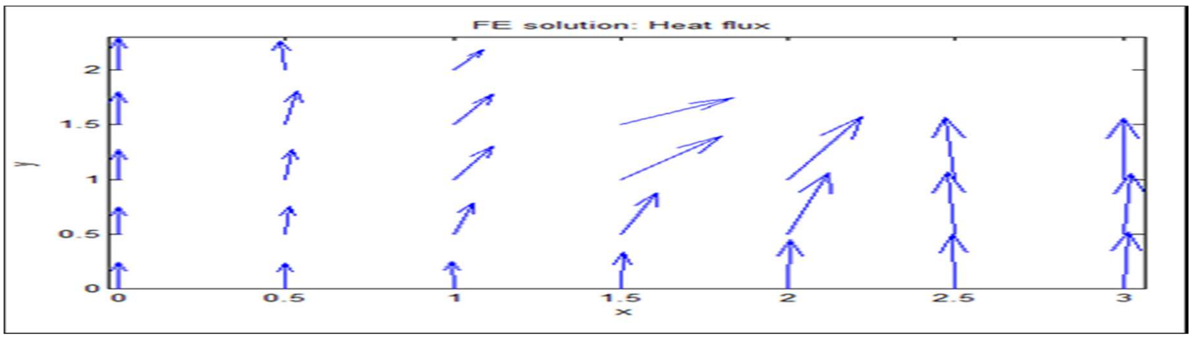

Figure 6:Heat flux using FEM for simple irregular geometry problem when f = 10

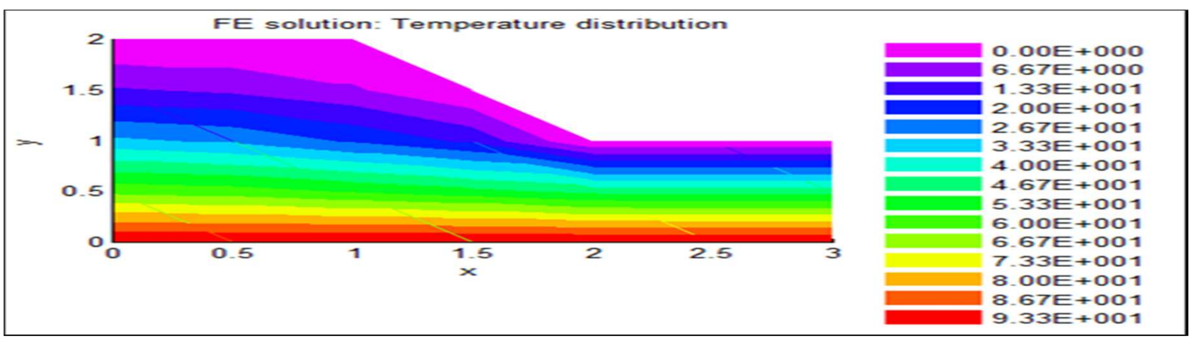

Figure 7:Heat distribution using FEM for simple irregular geometry problem when f = -10

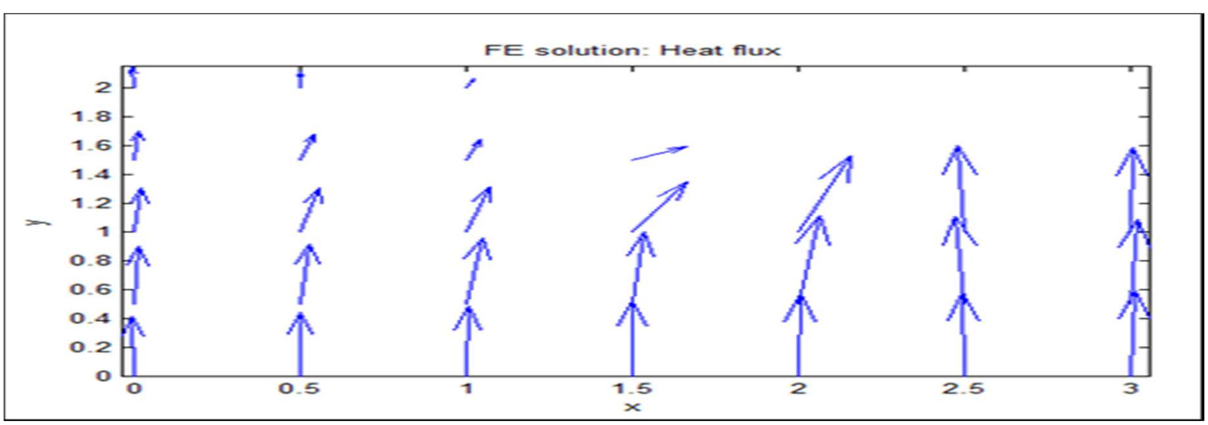

Figure 8:Heat flux using FEM for simple irregular geometry problem when f = -10

Referring to Figure 5and 7 aty = 1, the temperature is different because it is slightly hotter in Figure 5because we set a positive value of f. However, we cannot see much difference in the heat flux or the movement of the heat. All we can see is that the arrow is much longer in Figure 8 indicated that heat distributed much faster because f is negative. Therefore, we will increase the value of f so that we can see the changes clearer.

Then, we set the value of f to be equal to 100 and -100. We can see the heat distribution and heat flux is different when the value is positive and negative as positive value is the source for heat and negative value will absorb the heat from the bottom.

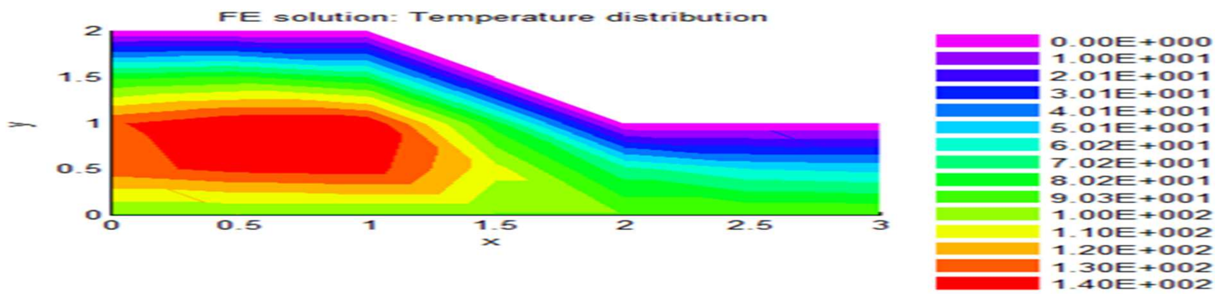

Figure 9:Heat distribution using FEM for simple irregular geometry problem when f = 100

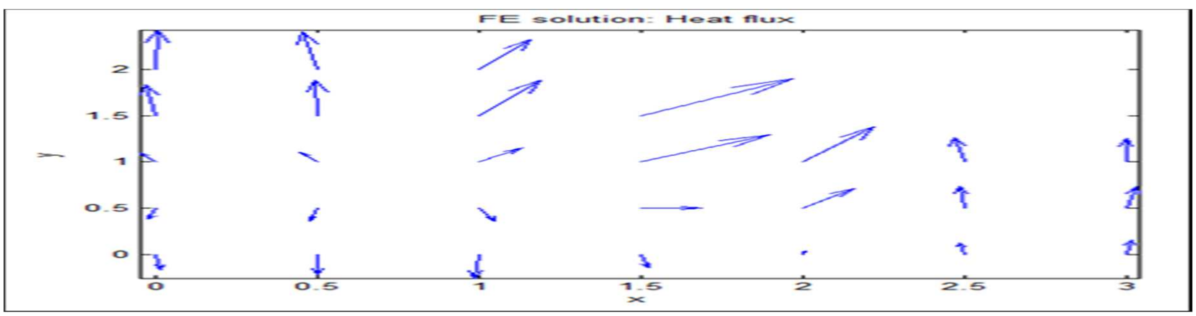

Figure 10:Heat flux using FEM for simple irregular geometry problem when f = 100

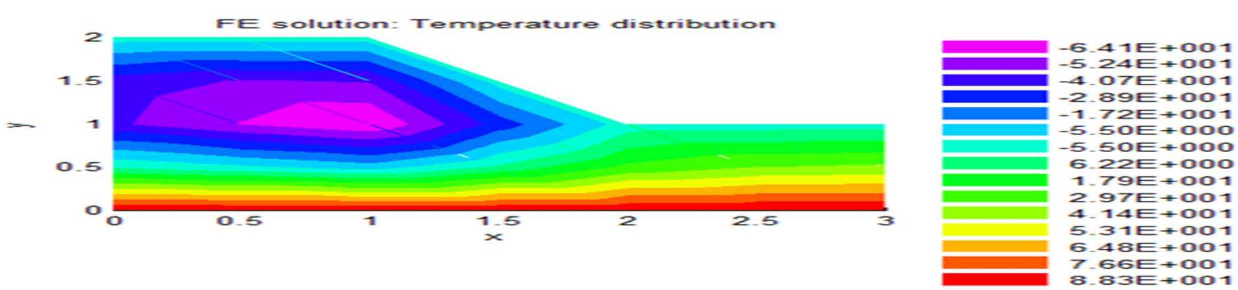

Figure 11:Heat distribution using FEM for simple irregular geometry problem when f = -100

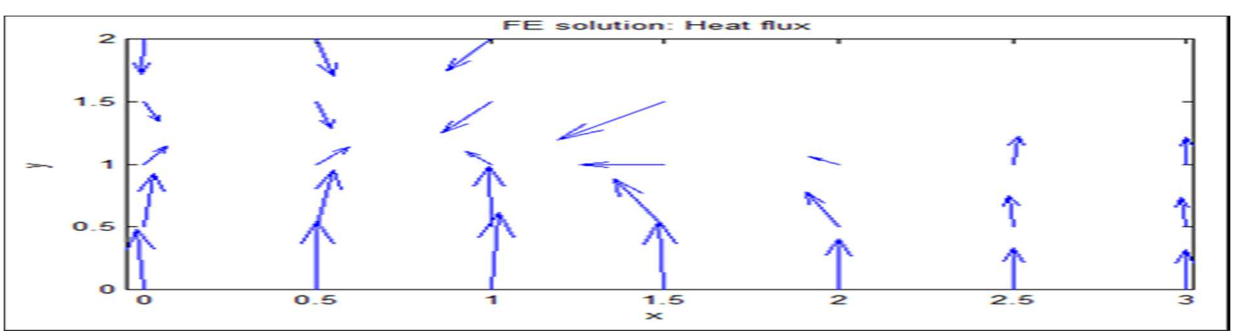

Figure 12:Heat flux using FEM for simple irregular geometry problem when f = -100

According to Figure 9, the highest temperature is 140^0C while the lowest is 0^0C. The temperature is very high in the area where we set the coordinate (X_Q, Y_Q). Figure 10 shows the movement of the heat which indicated by the arrow. The heat had been distributed not only from the bottom where we set temperature equal 100^0C but also from the middle because we have added another heat source in term of Q equation and alter the value of f to see the changes. The lowest temperature is only 0^0C because of the boundary condition.

Then let us see Figure 11 where the highest temperature is only 88.3^0c while the lowest is -64.1^0C. The temperature is very low in the area where we set the coordinate (X_Q, Y_Q). Even we set the boundary condition at the bottom x axis to be equal to 100^0C, but since the negative value of f had sink the heat making temperature drop greatly. Figure 12 shows the movement of the heat or heat flux which indicated by the arrow. We can see that the heat moved from the bottom to the opposite direction, but also directing to the (X_Q, Y_Q) area because of the low temperature.

CONCLUSION

The application of FEM can solve simple irregular geometry heat transfer problem. This method is numerically stable and the distribution of heat is more evenly. As the value of f changed the result is consistent thus we can conclude that the solution makes sense.

REFERENCES

1. P.N. Godbole, 2013. Introduction to finite element method. IK International Publishing House.
2. Kailani, N.H.A., 2014.The application of finite element method in 2D heat distribution problem for irregular geometry,Master thesis, Universiti Teknologi Malaysia, Johor.
3. Narasimhan, T.N., 1999. Fourier's Heat Conduction Equation: History, Influence, and Connections. Proceedings of the Indian Academy of Sciences-Earth and Planetary Sciences, 108(3): 117-148.
4. Roos, C.,2008. Principles of heat transfer: Energy efficiency fact sheet. Retrieved from http://www.energy.wsu.edu/documents/AHT_Principles%20of%20heat%20transfer.pdf.
5. Darrell W.Pepperand Juan C. Heinrich, 2005.The finite element method: basic concepts and applications. Taylor and Francis.
6. Mehta, N.C., V.B. Gondaliya andJ.V. Gundaniya, 2013. Applications of DifferentNumerical Methods in Heat Transfer-A Review. International Journal of Emerging Technology and Advanced Engineering,3(2): 363-368.
7. Yeak, S.H., 2012. Finite element method-Chapter 4: 2D heat equation. Universiti Teknologi Malaysia.
8. Daryl L. Logan, 2011. A first course in the finite element method. Cengage Learning.
9. Nayroles, B., G. Touzot and P. Villon, 1992. Generalizing the Finite Element Method: Diffuse Approximation and Diffuse Elements. Computational Mechanics, 10(5): 307-318.
10. Chao, T.Y. and W.K. Chow, 2002. A Review on the Application of Finite Element Method to Heat Transfer and Fluid Flow. International Journal on Architectural Science, 3 (1): 1-19.
11. Peir'o, J. and S. Sherwin, 2005. Finite difference, finite element and finite volume methods for partial difference equations. In: Handbook of Materials Modeling (ed S. Yip) pp. 2415-2446. Springer Netherlands
12. Mats G. Larson and F. Bengzon, 2013. The finite element method: Theory, implementation, and practice (Texts in computational science and engineering). Springer
13. S. Brenner and R. Scott, 2007. The mathematical theory of finite element methods. Springer Science and Business Media.