



## Influence of Local Inertia in the Forward Step Channel Filled with Porous Media

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### ABSTRACT

The semi-implicit pressure/correction finite element scheme is implemented for the steady-state solution of liquid laminar flow of constant viscosity Newtonian fluid through 4:1 contraction channel filled with and without porous media. The velocity streamline contours, pressure isobars, recirculation flow rate, excess pressure drop in the form of Couette-Correction through contraction domain of two-dimension are presented to examine the effect of Reynolds number varies from stoked flow to 50. The pressure drop is measured and was observed in growth. Furthermore, the vortex size and intensity is decreased in a salient corner due to increasing Reynolds number in the non-porous contraction channel and vortex intensity remains steady in the presence of porous material through 4:1 contraction channel.

**KEYWORDS:** Finite Element Method, Forward-Step Cannel Flow, Porous Media

### 1. INTRODUCTION

In computational fluid dynamics, Extensionally-dominated flows problem involving Newtonian and Non-Newtonian fluids including postponements of particles such as blood, ink material, colour paints, food materials, melts and solutions of polymers) related to applied sciences (i.e biosciences), engineering and industrial processes (injection moulding and film blowing) that can be modelled through mathematical models specify the non-linear behaviour of material functions through system of partial differential equation[xiii, xvii & xx]. To acquire the analytical solutions of complex problem are not straightforward, whilst, experimental approach is not cast effective. Therefore, best choice is to select numerical solution of complex problems of all areas. Now a day, there is an increasing trend in the field of computational fluid dynamics to validate the numerical results of fluid flow features with the advancement of computer technology. A series of papers on numerical results and experimental results have existed on that effect (Baloch, et al. [1994], Puangkird, et al. [2009], Mónica, et al.[2006], Lamidi [2012]).

Mitsoulis [2009] have presented the research work in the field of chemical technology and described as the Numerical simulation of contraction and expansion flows in the ratio 4:1 of Langmuir monolayers by adopted finite element scheme to obtain the steady-state and transient solution of Newtonian fluids.

The numerical results and experimental results both are examined and concluded that the small vortex activity appeared at low Reynolds number, with increasing Reynolds number the vortex activity reduced in the salient corner of the contraction channel. The recirculation flow rate is monotonically decreased and excess pressure loss due to increase a Reynolds number. Subsequently, Numerical simulations of complex isothermal and incompressible flows of Newtonian and viscoelastic fluids have described by Puangkird, et al. [2009]. The transient solution through the planar curved type corner contraction, lid-driven cavity and cross-slide devices geometries have obtained by using finite element Taylor-Galerkin scheme and hybrid schemes (finite element and finite volume). The various models such as Oldroyd-B, pom-pom models, Maxwell Model and Upper and Lower convicted Maxwell Models studied.

Many of the researchers had contributed the research work of fluid flow features through contraction geometries such as Aboubacar, M. and Webster, M. F. [2001] promoted the work on Newtonian fluid flow through Micro fabricated Hyperbolic Contractions through experimentally and numerically. The finite volume numerical scheme was employed for velocity fields by contraction and coupled pressure drops. Through experimentally, the

velocity contours was obtained in the vicinity of upstream of the contraction channel and good comparison of the experimental results were made by the exact solutions that suggested by White, F. M [1991].

The present research work is to examine the flow features of contraction channel and to investigate the effect of increasing Reynolds number, recirculation of flow rate, surplus of pressure drop and to indicate the elementary eddy structure through 4:1 contraction channel as viewed as [Baloch, et al. [1994], Zhang, et al.[2010] & Townsend, P. and Webster, M. F. [1987]).

## 2. GOVERNING SYSTEM OF EQUATIONS AND NUMERICAL SCHEME

For isothermal flow of liquid through forward-step channel, the governing equations for conservation of mass and momentum in dimensionless form can be written as follows:

$$\nabla \cdot \mathbf{u} = 0 \quad (1)$$

$$\frac{\partial \mathbf{u}}{\partial t} = \frac{1}{Re} \nabla^2 \mathbf{u} - \mathbf{u} \cdot \nabla \mathbf{u} - \nabla p \quad (2)$$

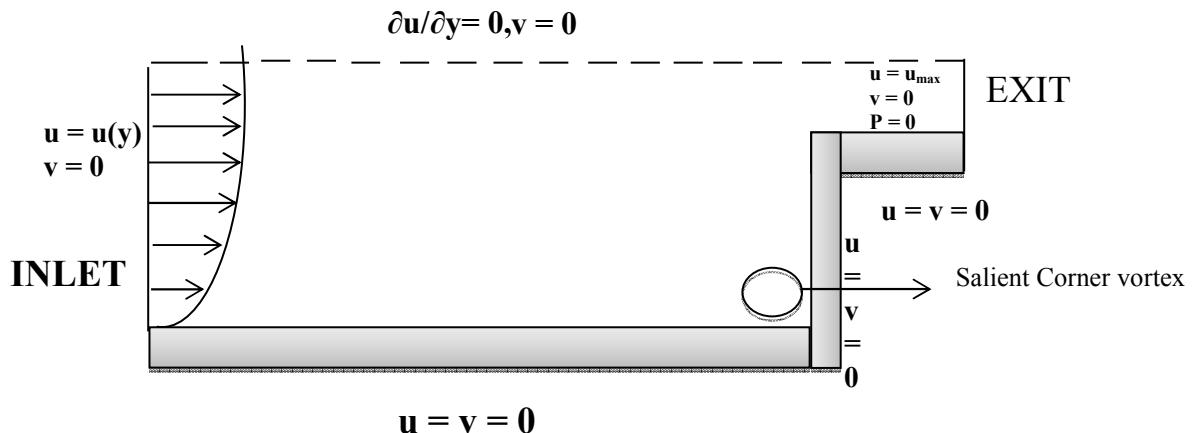
Where Re expressed as Reynolds number as  $Re = \frac{\rho UL}{\mu_0}$

In the present study, the numerical process deployed here and this process is extensively examined in our pioneer studies [Baloch, et al. [1994], Aboubacar, M. and Webster, M. F. [2001], P. Townsend and M. F. Webster. [1987] & Townsend, P and Walters, K. [1993]). For the sake of accuracy, correctness and completeness, review the key features of the numerical scheme to obtain the steady and transient solution. The present numerical scheme is called time stepping semi-implicit finite element pressure/correction scheme in second order. The scheme segregated into three stages (stage-1(a & b), stage-2 and stage-3). In stage-1, the momentum transport and continuity equation are solved to offer an auxiliary velocity. In stage-2, the pressure measurements develop against the continuity restriction on velocity and lastly, in stage-3, conservation of mass law is imposed on velocity through the envelopment from the pressure.

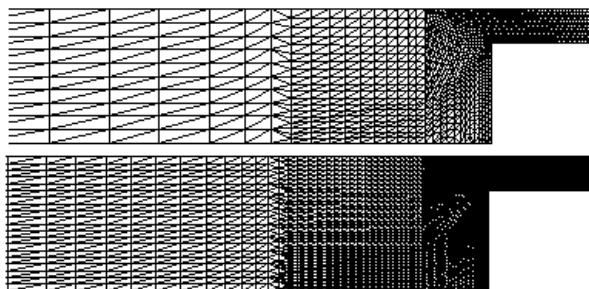
The non-linear terms are involved in governing equations and solved through stage-1, to adopt a predictor-corrector two-step Lax-Wendroff scheme. For the diffusion term, semi-implicit method is treated to increasing stability (Baloch, et al. [1994], Shaikh, et al. [2013] & Shah, et al. [2014]).

## 3. PROBLEM DESCRIPTION AND SOLUTION STRATEGY

Consider the liquid flow of constant viscosity fluid through 4:1 contraction channel by adopting a semi-implicit form of finite element scheme proposed by (Donea, j.[1984]). This benchmark problem is widely valuable and recognized to measure stability, accuracy and convergence properties, mainly at increasing levels of Reynolds number. Two channels small and large channels are presented and fluid passes through the long channel to cross-sectional small channel whose upstream length is 27.5L and downstream length is 49L, where L shows the length of the channel respectively. Complete well-posed Dirichlet conditions are imposed at inlet; outlet and no-slip boundary conditions on stationary walls see figure-1. The two coarse and refined meshes with triangular elements are displayed through figure-2. The interest has concentrated among the flow features such as the twelve stream lines contour figure-3 (a to f), vortex intensity, pressure isobar figure-5 (a to f) and loss of pressure drop across the contraction channel in terms of Reynolds numbers are described. Respectively the total number of elements, total number of nodes and degree-of-freedom are 2987, 6220 and 14057.



**Figure-1:** Schematic diagram 4:1 planer contraction channel with boundary conditions.



**Figure-2:** Finite Element Meshes

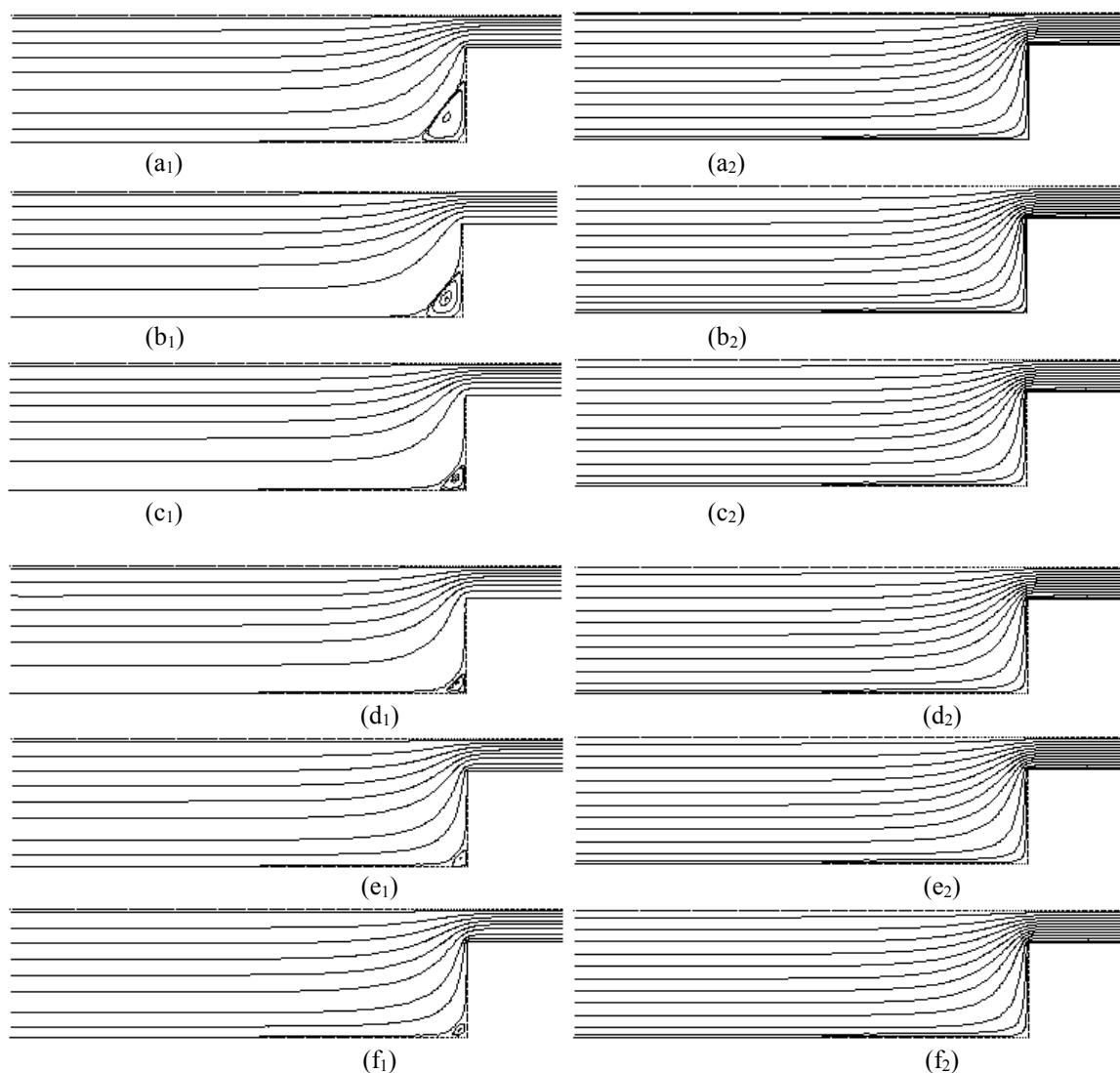
#### 4. NUMERICAL RESULTS AND DISCUSSIONS

##### 4.1 Influence of inertia

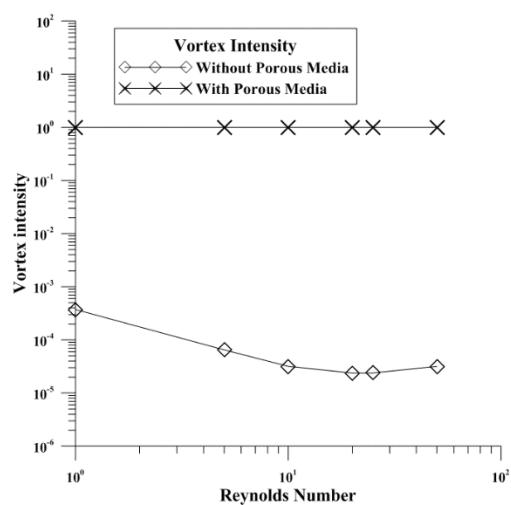
Here the numerical results for the constant viscosity fluid through 1:4 contraction channel filled with and without porous material are obtained by employing finite element code. The code was developed by (Donea, j.[1984]) and contributed by (Baloch, et al. [1994]) and then implemented in different fields by different researchers such as (Baloch, et al. [1994], Mitsoulis [2009], Aboubacar, M. and Webster, M. F. [2001] & Lamidi, O. T. and Ayeni, R. O. [2012]).

Consider the flow features by streamline contour figures and pressure contour figures of liquid flow through contraction channel filled with and without porous material. The figures (a<sub>1</sub>, b<sub>1</sub>, c<sub>1</sub>, d<sub>1</sub>, e<sub>1</sub>, f<sub>1</sub>) and (a<sub>2</sub>, b<sub>2</sub>, c<sub>2</sub>, d<sub>2</sub>, e<sub>2</sub>, f<sub>2</sub>) examined the streamline contours of contraction channel without porous material and with porous material. The various Reynolds number from stokes flow to Re = 50 are tested in both cases (without and with porous material) to investigate the salient corner vortex and lip vortex phenomena. Initially for stokes flow the salient corner vortex develop near the right wall of the upstream boundary (see figure-a<sub>1</sub>) and conversely when fill the porous material in the contraction channel the vortex on the right of the downstream boundary vanished completely(See figure-a<sub>2</sub>). With growth of Reynolds number up to Re = 50, the vortex size and intensity collapsed in the salient corner of the contraction channel without porous media (see a<sub>1</sub>, b<sub>1</sub>, c<sub>1</sub>, d<sub>1</sub>, e<sub>1</sub>, f<sub>1</sub>). Conversely, when filled the porous media the vortex vanished definitely up to Re = 50 in the salient corner of the contraction channel (see a<sub>2</sub>, b<sub>2</sub>, c<sub>2</sub>, d<sub>2</sub>, e<sub>2</sub>, f<sub>2</sub>).

The vortex intensity in terms of various Reynolds number (= 0 to 50) are illustrated of contraction channel filled with and without porous media (See figure-4) and clearly identified the vortex intensity is growing with increase Reynolds number for the flow through contraction channel without porous media and filled porous media, the vortex intensity with increase Reynolds number remain steady entirely. Present study is an extension of previous work on the flow through Expansion filled with and without porous media (Shaikh, et al.[2013] and Shah, et al.[2014]).



**Figure-3(a-f):** Stream lines contours for 4: 1 contraction flow.



**Figure-4:** Vortex Intensity is a function of Reynolds Number

#### 4.2 Excess Pressure drop

In standard contraction liquid flow are pushed through a contraction under a pressure differences. Pressure magnitudes are generated in the upstream and downstream of the contraction channel at precise locations on the walls. In the figure-5, the pressure contours of the liquid flow though contraction channel filled with and without porous material from stokes flow to  $Re = 50$  are presented. The different flow phenomena of pressure contours are examined.

The figures  $(a_1, b_1, c_1, d_1, e_1, f_1)$  and  $(a_2, b_2, c_2, d_2, e_2, f_2)$  shows a pressure contours of non-porous and porous channel in the ratio 4:1. The lip vortex in the right wall is developed initially and with increase Reynolds number the lip vortex is expanded in the non-porous contraction channel, but in the porous channel the lip vortex is totally vanished, with increasing Reynolds number the flow structure of pressure contours remain same.

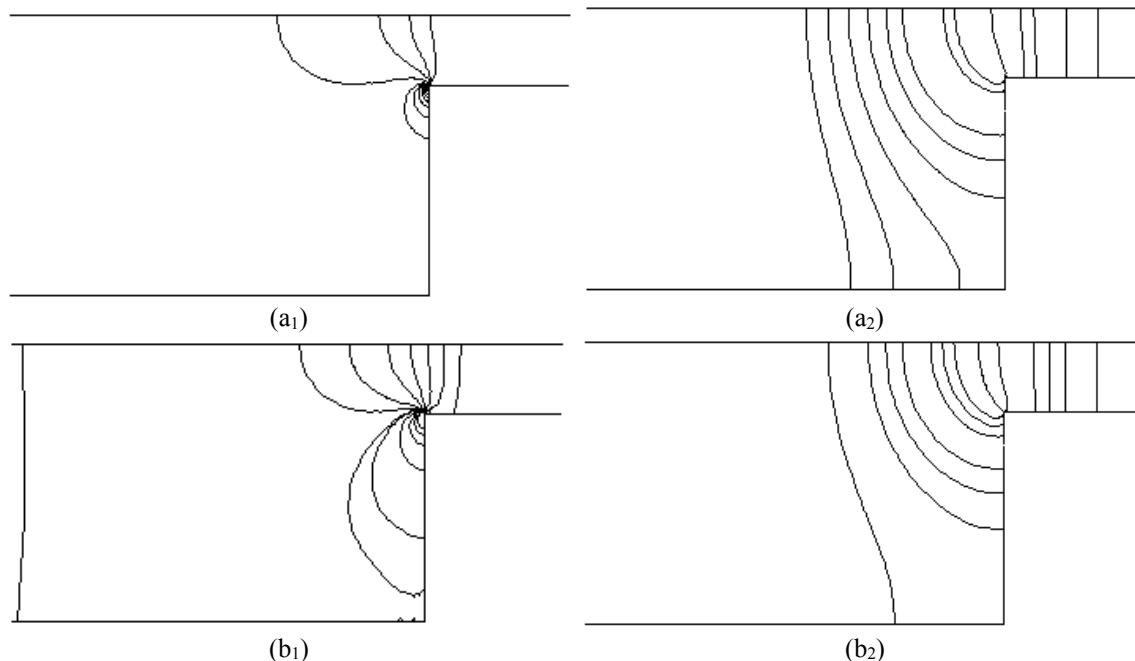
The flow through contraction channel with and without porous media plays a vital role in the field of computational fluid dynamics and various authors presented meaningful challenges, such as (Walter, K. and Webster, M.F. [2003]) especially presents the vortex developments, pressure gradients. Here considered specially the flow structure of pressure drop through so called Couette-Correction that is explained as

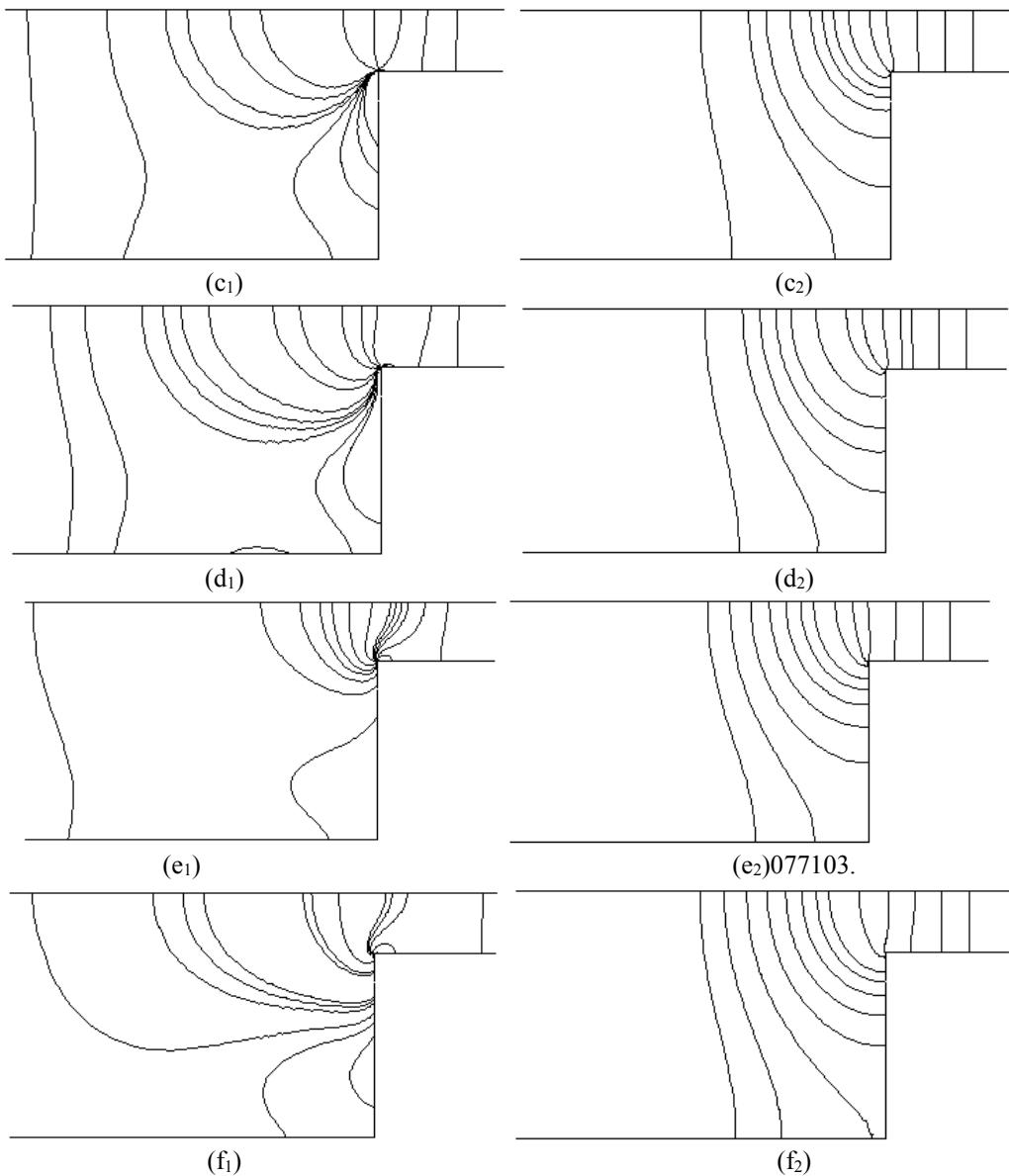
$$C = \frac{Re \delta p - (L_u \nabla P_u + L_d \nabla P_d)}{2\tau_w}$$

Where  $Re$  and  $\delta P$  is a fluid resistance and total pressure loss difference between inlet and exit of the domain,  $L_d$  and  $L_u$  are downstream and upstream lengths, whilst,  $\nabla P_d$  and  $\nabla P_u$  are downstream and upstream pressure differences

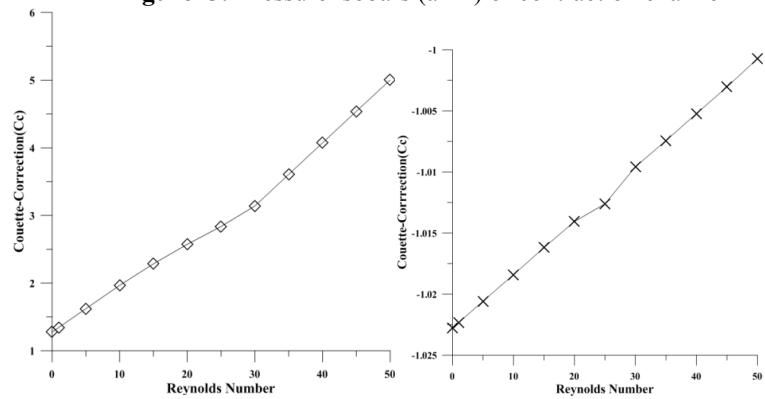
individually calculated analytically, and  $\tau_w$  is the wall shear stress  $\tau_w = \frac{\partial \mathbf{u}}{\partial y} \Big|_w$  in the fully advanced downstream flow.

In figure-6, the Couette-Correction in terms of various flow resistance are displayed and examined that with increase flow resistance the pressure gradient in enhanced for non-porous channel (see figure – 6(a<sub>1</sub>)) and for porous material the excess pressure drop in increased as well with increase flow resistance but the pressure magnitude occurred very high (see figure-6(a<sub>2</sub>)). The numerical results of excess pressure drop are compared with the (Tamaddon-Jahromi, et al.[2010]).



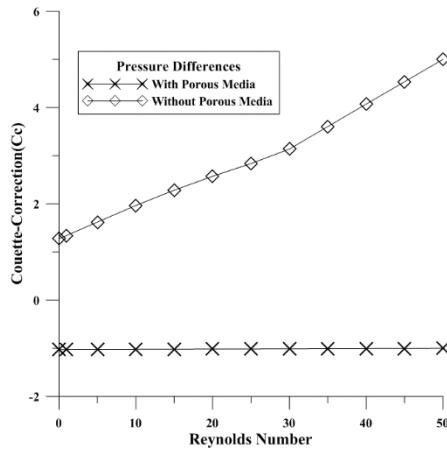


**Figure-5:** Pressure isobars (a – f) of contraction channel



(a<sub>1</sub>) Without Porous Media (a<sub>2</sub>) With Porous Media

**Figure-6:** Couette–Correction is a function of Reynolds Number (flow resistance)



**Figure–6:** Pressure drop is a function of Reynolds Number

## 5. Conclusion

The current research work has obscured the numerical approximations for streamline contours, pressure contours and excess pressure drop through Couette–Correction for the liquid flow through 4:1 contraction channel filled with and without porous media. The finite element Taylor–Galerkin pressure/correction with crank Nicolson process employed to obtain the steady solution. The vortex is developed at initial Reynolds number in the non–porous 4:1 contraction channel and with increasing flow resistance the vortex is declined. Figure–4 shows that the vortex size and intensity declining completely due to increase flow resistance for non–porous contraction channel and vortex intensity remains steady entirely in the presence of porous material through contraction channel.

In Figure–5, the pressure contours of the 4:1 contraction channel with and without porous material is described and small embryo lip vortex is initiated, with increasing flow resistance the lip vortex vanished entirely in the non–porous channel.

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