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Porous Effects on Second Grade Fluid in Oscillating Plate

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ABSTRACT

The main objective of this paper is to present exact solutions for incompressible second grade fluid under the effects of porosity due to sine and cosine oscillations of an infinite plate. The exact solutions are perused and obtained for velocity field and corresponding to shear stress by the techniques of integral transforms (Laplace and Fourier sine transforms). The general solutions are expressed as the convolution product, elementary functions and simple integrals. General solutions for the velocity field and corresponding shear stress satisfy the initial and boundary conditions $u(0,t) = U\,H(t)\,sin\omega t$or $u(0,t) = U\,H(t)\,cos\omega t$. Under the limiting cases, when $\phi \to 0$ the solutions are termed for second grade fluid under the absence of porosity and when$\lambda \to 0$ and $\phi \to 0$ the solutions are termed for Newtonian under the absence of porosity. Finally the effects of several rheological parameters and material limitations are interpreted for emerging interests on the motion of fluid. The comparison between four types of models is underlined as second grade fluid in presence of porous, second grade fluid in absence of porous, Newtonian fluid in presence of porous, and Newtonian fluid in absence of porous are analyzed by graphical illustrations with two distinct parameters for permeability.

KEY WORDS: Second grade fluid, Porous effects, Integral Transforms, Graphical illustrations.

INTRODUCTION

There is no denying fact that the modeling and understanding the flow of non-Newtonian fluids are of great interest because they are useful in several areas: particularly in chemical industries, processes of the petroleum, polymer composite factories, material processing, lubrication oils and guesses, paper coating, liquid metals, cosmetics, pharmaceutical, bio-engineering and in many scientific and technological advancements. Under the effects of absence and presence of porosity, the complex behavior of flowing fluids is involved in above mentioned application fields. Researchers, mathematicians, physicists and engineers have diverted their interest towards many porous problems of practical interest. In particular, the behavior of polymer solutions and melts, mercury amalgams, liquid crystals or biological fluids is challenging due to the involvement of non-linear terms among the governing equations, complex boundary conditions and field coupling. It is the known fact that non-linear behavior can not be simply described by the classical Navier-Stokes theory. Non-Newtonian fluids form a broad class of fluids linking the relationship to the shear rate and shear stress under the non-linearity that's why even no general governing model is present which describes all properties and characteristics of non-Newtonian fluids. Moreover, the governing models and equations come to be more tough and complex to handle the additional non-linear terms. We describe here the latest work regarding the non-Newtonian fluids [1-11] and the reference therein. Number of models has been purposed for non-Newtonian fluids to identify the rheological properties. They are mainly categorized as fluids of integral type, rate type and differential type. Viewing above categories of fluids, the rate type of fluids have memory effects (retardation phenomenon) and elastic effects (relaxation time) lying in the second grade fluid [12-13]. Seeing the literature, the behavior of most of flows for rate type fluids / models has gotten much attention by employing second grade fluid due to their infinite applications in science and technology. We include here few references [14-23] therein. The fundamental importance for study of flow through porous media lies in industries, biomechanics and geo-mechanics. Porosity plays significant role in geophysics, agriculture, astrophysics and in petroleum industries. It has vital importance, significance and applications in the field of thermal recovery of oil and exploration of oil, geological formations, geothermal reservoirs, the assessment of aquifers and underground nuclear waste storage sites. Water through rocks, filtration of fluids and regulation of skin reveals the interesting study of porous media. On the other hand, flows of non-Newtonian fluids always encounter porosity such as storage projects, rheology and petroleum engineering and under-ground natural resources etc. [24-35]. On contrast, in the literature, porous belt and flow past a vertical lubricating for the thin layer third order fluid has been studied by Aamer Khan et al [36]. Sanela Jamshad examined the fluid flow for second grade fluid for vertical stationary and oscillating plates [37].In the presence of heat transfer, porous and MHD second grade fluid has been discussed by

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Aneelashakir et al [38]. In brevity several study related to fluid problems for instance, mass and heat transfer for magneto hydrodynamics free convective, vertical cylinder for power law mode etc is analyzed in [39,40]. In this article, we are intended to explore porous flow of an upper-convicted second grade fluid under an infinite plate. Under the influence and usage of integral transforms (Fourier sine and Laplace transforms) the exact solutions are established. The velocity field and corresponding shear stress are presented in terms of multiple integrals, theorem of convolution product, simple integral and elementary functions which satisfy all imposed initial and boundary conditions. At the ending, effects and influences of pertinent parameters on the motion of fluid along with comparisons among above discussed different models, in the presence and absence of porous and permeability effects are discussed through graphical illustrations.

GOVERNING THE FLOW EQUATIONS

The Cauchy stress tensor $\mathbf{T}$ for an incompressible fluid of second grade or second-order fluid is related to the fluid motion by the constitutive equation,

$$\boldsymbol{T} = -\,\boldsymbol{p}\,\boldsymbol{I} + \boldsymbol{S}, \tag{1}$$

Under the component practice and form, we have

$$\boldsymbol{T} = \begin{pmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{yz} & \sigma_{zz} \end{pmatrix}, \tag{2}$$

and

$$\boldsymbol{S} = \mu \boldsymbol{A_1} + \alpha_1 \boldsymbol{A_2} + \alpha_2 \boldsymbol{A}_1^2, \tag{3}$$

where$-p\mathbf{I}$ denotes the indeterminate spherical stress for the constraint of incompressibility, μ is the dynamic viscosity, $\boldsymbol{S}$is extra-stress tensor, α_1 and $\alpha_2$denote the material moduli or normal stress moduli, $\boldsymbol{A_1}$ and $\boldsymbol{A_2}$show the kinematic tensors or first two Rivlin-Ericksen tensors. Since the fluid is incompressible, it can undergo only isochoric motions and hence

$$\frac{du}{dx} + \frac{dv}{dy} + \frac{dw}{dz} = 0 \tag{4}$$

For the problem under consideration we shall assume a velocity field of the form

$$\boldsymbol{S} = \boldsymbol{S}(y,t), \qquad \boldsymbol{V} = \boldsymbol{V}(y,t) = u(y,t)\boldsymbol{i}\,, \tag{5}$$

Where $\boldsymbol{I}$ is the unit vector along the x-coordinate direction. For these flows the constraint of incompressibility is automatically satisfied. If the fluid is at rest up to the moment $t = 0$, then

$$\boldsymbol{S} = (y,0) = 0, \qquad \boldsymbol{V} = (y,0) = 0\,, \tag{6}$$

employing equation (5) and (6) imply

$$\tau_{xz} = \tau_{yz} = \tau_{zx} = \tau_{zy} = 0, \quad \sigma_{zz} = -p, \tag{7}$$

under the absence of body force, the balance of linear momentum reduce to

$$\frac{\partial p}{\partial y} = \frac{\partial p}{\partial z} = 0, \qquad \frac{\partial \tau(y,t)}{\partial x} - \frac{\partial p}{\partial x} = p\left(\frac{\partial u}{\partial t}\right) \tag{8}$$

Meanwhile the flow is to be considered as a unsteady, the relations depend upon the virtual mass and drag effect. The expression for the velocity and pressure drop for porous media in a second grade fluid is

$$\boldsymbol{\nabla} p = -\frac{\emptyset}{k}\left(\mu + \alpha_1 \frac{\partial}{\partial t}\right) u \tag{9}$$

Eliminatingτ equations (7) and (8), we get the following system of partial differential equations governing the second grade flow in porous media

$$\frac{\partial u}{\partial t} = \left(\nu + \alpha \frac{\partial}{\partial t}\right)\frac{\partial^2 u}{\partial^2 t} - \frac{\emptyset}{k}\left(\mu + \alpha_1 \frac{\partial}{\partial t}\right) u, \tag{10}$$

$$\tau = \left(\mu + \alpha_1 \frac{\partial}{\partial t}\right)\frac{\partial u}{\partial y}\,. \tag{11}$$

Under our consideration the motion of second grade fluid is at rest over an infinitely extended plate. The plane is moving with constant velocity $u(0,t) = UH(t)\ cos\omega t\ \ or\ \ UH(t)\ sin\omega t$, where $H(t)$ is Heaviside function, (Heaviside function is used due to the fact that at the time $t = 0$ the plane is at rest). The fluid above the plane is slowly and gradually moved under the effects of shear stress gained by the fluid due to the motion of the plane.

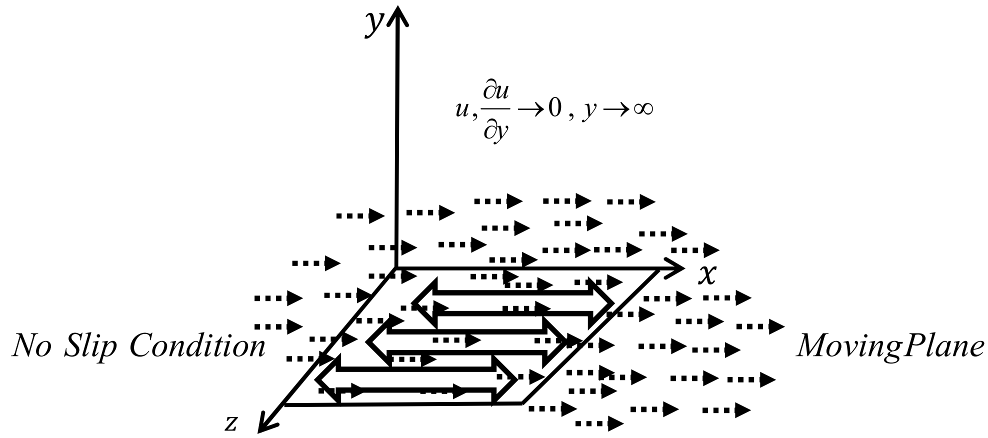

Figure 1: Geometry of the Problem.

The initial and boundary conditions with respect to relevant problem are described as under

$$\frac{\partial u(y,0)}{\partial t} = 0, \quad u(y,0) = 0, \qquad \tau(y,0) = 0, \qquad y > 0, \tag{12}$$

$$u(0,t) = UH(t)cos\omega t \quad or \quad UH(t)sin\omega t \qquad t \geq 0. \tag{13}$$

Naturally and obviously the vivid conditions described below are satisfied,

$$u(y,t), \qquad \frac{\partial u(y,t)}{\partial t} \to 0 \qquad as \qquad y \to \infty \quad and \quad t > 0, \tag{14}$$

can be also fulfilled.

COMPUTATION OF VELOCITY FIELD

The case $u(0,t) = UH(t)sin\omega t$

For finding the solution of velocity field, partial differential equation governs the fluid (10), keeping in concentration for the boundary and initial conditions (12) and (13), and utilizing the Fourier sine transforms with respect to spatial variable [34], we have

$$\frac{\partial u_s(\xi,t)}{\partial t} = -\xi^2\left(\nu + \alpha\frac{\partial}{\partial t}\right)u_s(\xi,t) + U\xi\sqrt{\frac{2}{\pi}}\left(\nu + \alpha\frac{\partial}{\partial t}\right)H(t)sin\omega t - \frac{\mu\emptyset}{k}\left(1 + \frac{\alpha_1}{\mu}\frac{\partial}{\partial t}\right)u_s, \tag{15}$$

solving equation (15) and $H(t)$ is the Heaviside function,

$$\frac{\partial u_s(\xi,t)}{\partial t} + \xi^2\left(\nu + \alpha\frac{\partial}{\partial t}\right)u_s(\xi,t) = U\xi\sqrt{\frac{2}{\pi}}\{\nu H(t)sin\omega t + \alpha\delta(t)sin\omega t + \alpha\omega H(t)cos\omega t\} - \frac{\mu\emptyset}{k}\left(1 + \frac{\alpha_1}{\mu}\frac{\partial}{\partial t}\right)u_s, \tag{16}$$

Where $\delta(t)$ and $\delta'(t)$ is delta function and derivative of delta function respectively. $u_s(\xi,t)$is the Fourier sine transform of $u(y,t)$ which can be shown as

$$u_s(\xi,t) = \sqrt{2/\pi}\int_0^\infty \sin(y\xi)\, u(y,t)\, d\xi, \tag{17}$$

and the initial conditions have to full fill and satisfy,

$$u_s(\xi,0) = 0, \qquad \frac{\partial u_s(\xi,0)}{\partial t} = 0, \qquad \xi > 0\,, \tag{18}$$

By utilizing the Laplace transform to (12) and keeping in concentration for the initial conditions (13), we have

$$\overline{u_s}(\xi,q) = U\xi\sqrt{2/\pi}\left[\frac{\omega(\nu+\alpha q)}{(q^2+\omega^2)\left\{\left(1+\alpha\xi^2+\frac{\alpha_1\emptyset}{k}\right)q+\nu\xi^2+\frac{\mu\emptyset}{k}\right\}}\right], \tag{19}$$

Now, for further proper representation of the (19), we rewrite (19) in the blow comparable form

$$\overline{u_s}(\xi,q) = \frac{U\,\omega\,\nu\,\xi}{\left(\nu\xi^2+\frac{\mu\emptyset}{k}\right)}\sqrt{\frac{2}{\pi}}\left[\frac{1}{(q^2+\omega^2)} - \frac{q\left(1+\alpha\xi^2+\frac{\alpha_1\emptyset}{k}\right)}{(q^2+\omega^2)\left\{\left(1+\alpha\xi^2+\frac{\alpha_1\emptyset}{k}\right)q+\nu\xi^2+\frac{\mu\emptyset}{k}\right\}}\right] + U\,\omega\,\xi\,\alpha\sqrt{\frac{2}{\pi}}$$

$$\times\left\{\frac{q}{(q^2+\omega^2)\left\{\left(1+\alpha\xi^2+\frac{\alpha_1\emptyset}{k}\right)q+\nu\xi^2+\frac{\mu\emptyset}{k}\right\}}\right\} \tag{20}$$

Inverting (20) by employing of the Fourier sine transform, we express as

$$\bar{u}(\xi,q) = \frac{2\,U\,\omega\,\nu}{\pi}\int_0^\infty \frac{\xi sin(y\xi)}{\left(\nu\xi^2+\frac{\mu\emptyset}{k}\right)}\left[\frac{1}{(q^2+\omega^2)} - \frac{q\left(1+\alpha\xi^2+\frac{\alpha_1\emptyset}{k}\right)}{(q^2+\omega^2)\left\{\left(1+\alpha\xi^2+\frac{\alpha_1\emptyset}{k}\right)q+\nu\xi^2+\frac{\mu\emptyset}{k}\right\}}\right]d\xi + U\,\omega\,\alpha\sqrt{\frac{2}{\pi}}$$

$$\times\int_0^\infty \xi sin(y\xi)\left\{\frac{q}{(q^2+\omega^2)\left\{\left(1++\alpha\xi^2+\frac{\alpha_1\emptyset}{k}\right)q+\nu\xi^2+\frac{\mu\emptyset}{k}\right\}}\right\}d\xi, \tag{21}$$

On simplification equation (21) and using the fact of integral,

$$\int_0^\infty \frac{\xi\ sin(y\xi)}{a^2+b^2}d\xi = \frac{\pi}{2}e^{-ay} \tag{22}$$

$$\bar{u}(\xi,q) = \frac{U\omega}{(q^2+\omega^2)}e^{-y\sqrt{\frac{\mu\emptyset}{\nu k}}} - \frac{2\,U\,\omega\,\nu}{\pi}\int_0^\infty \frac{\xi\ sin(y\xi)}{\left(\nu\xi^2+\frac{\mu\emptyset}{k}\right)}\left\{\frac{q}{(q^2+\omega^2)\left(q+\frac{\nu\xi^2+\frac{\mu\emptyset}{k}}{1+\alpha\xi^2+\frac{\alpha_1\emptyset}{k}}\right)}\right\}d\xi + \frac{2\,U\,\omega\,\alpha}{\pi}$$

$$\times\int_0^\infty \frac{\xi\ sin(y\xi)}{\left(1++\alpha\xi^2+\frac{\alpha_1\emptyset}{k}\right)}\left\{\frac{q}{(q^2+\omega^2)\left(q+\frac{\nu\xi^2+\frac{\mu\emptyset}{k}}{1+\alpha\xi^2+\frac{\alpha_1\emptyset}{k}}\right)}\right\}d\xi, \tag{23}$$

To reveal velocity field, we use the inverse Laplace transform to (23), we get

$$u_S(y,t) = U\,H(t)\ sin\omega t\ e^{-y\sqrt{\frac{\mu\emptyset}{\nu k}}} - \frac{2\,U\,\omega\,\nu}{\pi}\int_0^\infty \frac{\xi\ sin(y\xi)}{\left(\nu\xi^2+\frac{\mu\emptyset}{k}\right)}cos\omega t * e^{-\left(\frac{k\nu\xi^2+\mu\emptyset}{k+k\alpha\xi^2+\alpha_1\emptyset}\right)t}d\xi + \frac{2\,U\,\omega\,\alpha}{\pi}$$

$$\times\int_0^\infty \frac{\xi\ sin(y\xi)}{\left(1++\alpha\xi^2+\frac{\alpha_1\emptyset}{k}\right)}\ cos\omega t * e^{-\left(\frac{k\nu\xi^2+\mu\emptyset}{k+k\alpha\xi^2+\alpha_1\emptyset}\right)t}d\xi, \tag{24}$$

We find the velocity field, the following simple expression in terms of integral form is

$$u_S(y,t) = UH(t)sin\omega t\ e^{-y\sqrt{\frac{\mu\emptyset}{\nu k}}} - \frac{2U\nu\omega}{\pi}\int_0^\infty\int_0^t \frac{\xi\ sin(y\xi)}{\left(\nu\xi^2+\frac{\mu\emptyset}{k}\right)}cos\omega(t-z)e^{-\left(\frac{k\nu\xi^2+\mu\emptyset}{k+k\alpha\xi^2+\alpha_1\emptyset}\right)z}d\xi dz$$

$$+\frac{2\,U\,\omega\,\alpha}{\pi}\int_0^\infty\int_0^t \frac{\xi\,sin(y\xi)}{\left(1++\alpha\xi^2+\frac{\alpha_1\emptyset}{k}\right)} cos\omega(t-z)e^{-\left(\frac{k\nu\xi^2+\mu\emptyset}{k+k\alpha\xi^2+\alpha_1\emptyset}\right)z}d\xi dz, \qquad (25)$$

COMPUTATION OF SHEAR STRESS

On differentiating equation (24) with respect to "y" partially, we have

$$\frac{\partial u_s}{\partial y} = -\sqrt{\frac{\mu\emptyset}{\nu k}}\,UH(t)sin\omega t\,e^{-y\sqrt{\frac{\mu\emptyset}{\nu k}}} - \frac{2U\nu\omega}{\pi}\int_0^\infty \frac{\xi^2\,cos(y\xi)}{\left(\nu\xi^2+\frac{\mu\emptyset}{k}\right)} cos\omega t * e^{-\left(\frac{k\nu\xi^2+\mu\emptyset}{k+k\alpha\xi^2+\alpha_1\emptyset}\right)t}d\xi + \frac{2\,U\,\omega\,\alpha}{\pi}$$
$$\times\int_0^\infty \frac{\xi^2\,cos(y\xi)}{\left(1++\alpha\xi^2+\frac{\alpha_1\emptyset}{k}\right)}\,cos\omega t * e^{-\left(\frac{k\nu\xi^2+\mu\emptyset}{k+k\alpha\xi^2+\alpha_1\emptyset}\right)t}d\xi, \qquad (26)$$

Again differentiating equation (26) with respect to "t" partially, we get,

$$\frac{\partial}{\partial t}\left(\frac{\partial u_s}{\partial y}\right) = -\sqrt{\frac{\mu\emptyset}{\nu k}}\,U(\delta(t)sin\omega t+\omega H(t)cos\omega t)\,e^{-y\sqrt{\frac{\mu\emptyset}{\nu k}}} - \frac{2U\nu\omega^2}{\pi}\int_0^\infty \frac{\xi^2\,cos(y\xi)}{\left(\nu\xi^2+\frac{\mu\emptyset}{k}\right)} sin\omega t\, * e^{-\left(\frac{k\nu\xi^2+\mu\emptyset}{k+k\alpha\xi^2+\alpha_1\emptyset}\right)t}d\xi$$
$$+\frac{2U\omega\alpha}{\pi}\int_0^\infty \frac{\xi^2\,cos(y\xi)}{\left(1++\alpha\xi^2+\frac{\alpha_1\emptyset}{k}\right)} cos\omega t * e^{-\left(\frac{k\nu\xi^2+\mu\emptyset}{k+k\alpha\xi^2+\alpha_1\emptyset}\right)t}d\xi, \qquad (27)$$

using equations (26) and (27) in governing equation (11), we get

$$\tau = -U\mu\sqrt{\frac{\mu\emptyset}{\nu k}}\,H(t)sin\omega t\,e^{-y\sqrt{\frac{\mu\emptyset}{\nu k}}} - \frac{2U\nu\omega\mu}{\pi}\int_0^\infty \frac{\xi^2\,cos(y\xi)}{\left(\nu\xi^2+\frac{\mu\emptyset}{k}\right)} cos\omega t * e^{-\left(\frac{k\nu\xi^2+\mu\emptyset}{k+k\alpha\xi^2+\alpha_1\emptyset}\right)t}d\xi + \frac{2U\omega\alpha\mu}{\pi}$$
$$\times\int_0^\infty \frac{\xi^2\,cos(y\xi)}{\left(1++\alpha\xi^2+\frac{\alpha_1\emptyset}{k}\right)} cos\omega t * e^{-\left(\frac{k\nu\xi^2+\mu\emptyset}{k+k\alpha\xi^2+\alpha_1\emptyset}\right)t}d\xi - U\alpha_1\sqrt{\frac{\mu\emptyset}{\nu k}}\,e^{-y\sqrt{\frac{\mu\emptyset}{\nu k}}}\{\delta(t)sin\omega t+\omega H(t)cos\omega t\} - \frac{2U\nu\omega^2\alpha_1}{\pi}$$
$$\times\int_0^\infty \frac{\xi^2\,cos(y\xi)}{\left(\nu\xi^2+\frac{\mu\emptyset}{k}\right)} sin\omega t * e^{-\left(\frac{k\nu\xi^2+\mu\emptyset}{k+k\alpha\xi^2+\alpha_1\emptyset}\right)t}d\xi + \frac{2U\omega\alpha\alpha_1}{\pi}\int_0^\infty \frac{\xi^2\,cos(y\xi)}{\left(1++\alpha\xi^2+\frac{\alpha_1\emptyset}{k}\right)} cos\omega t * e^{-\left(\frac{k\nu\xi^2+\mu\emptyset}{k+k\alpha\xi^2+\alpha_1\emptyset}\right)t}d\xi, \qquad (28)$$

Under simplification, the expression for corresponding shear stress in the integral form

$$\tau_S(y,t) = -U\,\mu\sqrt{\frac{\mu\emptyset}{\nu k}}\,H(t)\,sin\omega t\,e^{-y\sqrt{\frac{\mu\emptyset}{\nu k}}} - \frac{2\,U\,H(t)\nu\,\omega\,\mu}{\pi}\int_0^\infty\int_0^t \frac{\xi^2\;cos(y\xi)}{\left(\nu\xi^2+\frac{\mu\emptyset}{k}\right)} cos\omega(t-z)e^{-\left(\frac{k\nu\xi^2+\mu\emptyset}{k+k\alpha\xi^2+\alpha_1\emptyset}\right)z}d\xi\,dz$$
$$+\frac{2\,U\,H(t)\,\omega\,\alpha\,\mu}{\pi}\int_0^\infty\int_0^t \frac{\xi^2\,cos(y\xi)}{\left(1++\alpha\xi^2+\frac{\alpha_1\emptyset}{k}\right)} cos\omega(t-z)e^{-\left(\frac{k\nu\xi^2+\mu\emptyset}{k+k\alpha\xi^2+\alpha_1\emptyset}\right)z}d\xi\,dz - U\alpha_1\sqrt{\frac{\mu\emptyset}{\nu k}}\;e^{-y\sqrt{\frac{\mu\emptyset}{\nu k}}}$$
$$\times\{\delta(t)\,sin\omega t+\omega\,H(t)\,cos\omega t\} - \frac{2\,U\,H(t)\nu\,\omega^2\alpha_1}{\pi}\int_0^\infty\int_0^t \frac{\xi^2\,cos(y\xi)}{\left(\nu\xi^2+\frac{\mu\emptyset}{k}\right)} sin\omega(t-z)e^{-\left(\frac{k\nu\xi^2+\mu\emptyset}{k+k\alpha\xi^2+\alpha_1\emptyset}\right)z}d\xi\,dz$$
$$+\frac{2\,U\,H(t)\,\omega\,\alpha\alpha_1}{\pi}\int_0^\infty\int_0^t \frac{\xi^2\,cos(y\xi)}{\left(1++\alpha\xi^2+\frac{\alpha_1\emptyset}{k}\right)} cos\omega(t-z)e^{-\left(\frac{k\nu\xi^2+\mu\emptyset}{k+k\alpha\xi^2+\alpha_1\emptyset}\right)z}d\xi\,dz \qquad (29)$$

is obtained.

The case $u(0,t) = UH(t)cos\omega t$

By applying similar process, we have investigated velocity field and corresponding shear stress for the case of cosine oscillation under effects of porosity.

$$u_C(y,t) = UH(t)cos\omega t\,e^{-y\sqrt{\frac{\mu\emptyset}{\nu k}}} - \frac{2\,U\,H(t)\,\nu\,\omega}{\pi}\int_0^\infty\int_0^t \frac{\xi\,sin(y\xi)}{\left(\nu\xi^2+\frac{\mu\emptyset}{k}\right)} sin\omega(t-z)e^{-\left(\frac{k\nu\xi^2+\mu\emptyset}{k+k\alpha\xi^2+\alpha_1\emptyset}\right)z}d\xi dz\; + \frac{2\,U\,\omega\,\alpha}{\pi}$$

$$\times \int_0^\infty \int_0^t \frac{\xi \, sin(y\xi)}{\left(1 + +\alpha\xi^2 + \frac{\alpha_1 \emptyset}{k}\right)} sin\omega(t-z) e^{-\left(\frac{k\nu\xi^2+\mu\emptyset}{k+k\alpha\xi^2+\alpha_1\emptyset}\right)z} \, d\xi dz, \tag{30}$$

$$\tau_C(y,t) = -\mu \, U \sqrt{\frac{\mu\emptyset}{\nu k}} \, H(t) \, cos\omega t \;\; e^{-y\sqrt{\frac{\mu\emptyset}{\nu k}}} - \frac{2 \, U \, H(t) \nu \, \omega \, \mu}{\pi} \int_0^\infty \int_0^t \frac{\xi^2 \;\, cos(y\xi)}{\left(\nu\xi^2 + \frac{\mu\emptyset}{k}\right)} sin\omega(t-z) e^{-\left(\frac{k\nu\xi^2+\mu\emptyset}{k+k\alpha\xi^2+\alpha_1\emptyset}\right)z} d\xi \; dz$$
$$+ \frac{2 \, U \, H(t) \, \omega \, \alpha \, \mu}{\pi} \int_0^\infty \int_0^t \frac{\xi^2 \, cos(y\xi)}{\left(1 + +\alpha\xi^2 + \frac{\alpha_1\emptyset}{k}\right)} sin\omega(t-z) e^{-\left(\frac{k\nu\xi^2+\mu\emptyset}{k+k\alpha\xi^2+\alpha_1\emptyset}\right)z} \, d\xi \; dz - U\alpha_1 \sqrt{\frac{\mu\emptyset}{\nu k}} \; e^{-y\sqrt{\frac{\mu\emptyset}{\nu k}}}$$
$$\times \; \{\delta(t) \; cos\omega t - \omega \; H(t) \; sin\omega t\} \; - \frac{2UH(t)\nu\omega^2\alpha_1}{\pi} \int_0^\infty \int_0^t \frac{\xi^2 \, cos(y\xi)}{\left(\nu\xi^2 + \frac{\mu\emptyset}{k}\right)} cos\omega(t-z) e^{-\left(\frac{k\nu\xi^2+\mu\emptyset}{k+k\alpha\xi^2+\alpha_1\emptyset}\right)z} d\xi \; dz$$

$$+ \frac{2 \, U \, H(t) \, \omega \, \alpha \, \alpha_1}{\pi} \int_0^\infty \int_0^t \frac{\xi^2 \, cos(y\xi)}{\left(1 + +\alpha\xi^2 + \frac{\alpha_1\emptyset}{k}\right)} sin\omega(t-z) e^{-\left(\frac{k\nu\xi^2+\mu\emptyset}{k+k\alpha\xi^2+\alpha_1\emptyset}\right)z} d\xi \; dz, \tag{31}$$

LIMITING CASE

5.1. SECOND GRADE FLUID IN THE ABSENCE OF POROUS EFFECTS$\emptyset \rightarrow 0$

Letting $\emptyset \rightarrow 0$ in equations (25), (29), (30) and (31), we get

$$u_S(y,t) = UH(t) sin\omega t - \frac{2 \; U \; H(t) \; \omega}{\pi} \int_0^\infty \int_0^t \frac{sin(y\xi)}{\xi} e^{-\left(\frac{\nu\xi^2}{1+\alpha\xi^2}\right)z} cos\omega(t-z) \; dz \; d\xi + \frac{2 \; U \; \omega \; \alpha}{\pi} \int_0^\infty \int_0^t \frac{\xi \; sin(y\xi)}{(1+\alpha\xi^2)}$$
$$\times \, e^{-\left(\frac{\nu\xi^2}{1+\alpha\xi^2}\right)z} cos\omega(t-z) \; dz \; d\xi, \tag{32}$$

$$u_C(y,t) = UH(t) cos\omega t - \frac{2 \; U \; H(t) \omega}{\pi} \int_0^\infty \int_0^t \frac{sin(y\xi)}{\xi} e^{-\left(\frac{\nu\xi^2}{1+\alpha\xi^2}\right)z} sin\omega(t-z) \; dz \; d\xi + \frac{2 \; U \; \omega \; \alpha}{\pi} \int_0^\infty \int_0^t \frac{\xi \; sin(y\xi)}{(1+\alpha\xi^2)}$$
$$\times \, e^{-\left(\frac{\nu\xi^2}{1+\alpha\xi^2}\right)z} sin\omega(t-z) dz \; d\xi, \tag{33}$$

$$\tau_S(y,t) = -\frac{2 \; U \; H(t) \omega \; \mu}{\pi} \int_0^\infty \int_0^t cos(y\xi) \, e^{-\left(\frac{\nu\xi^2}{1+\alpha\xi^2}\right)z} cos\omega(t-z) \; dz \; d\xi + \frac{2 \; U \; H(t) \omega \; \alpha \; \mu}{\pi} \times \int_0^\infty \int_0^t \frac{\xi^2 \; cos(y\xi)}{(1+\alpha\xi^2)}$$
$$\times \, e^{-\left(\frac{\nu\xi^2}{1+\alpha\xi^2}\right)z} cos\omega(t-z) \; dz \; d\xi - \frac{2UH(t)\omega^2\alpha_1}{\pi} \int_0^\infty \int_0^t cos(y\xi) \, e^{-\left(\frac{\nu\xi^2}{1+\alpha\xi^2}\right)z} sin\omega(t-z) \; dz \; d\xi + \frac{2UH(t)\omega\alpha\alpha_1}{\pi}$$
$$\times \int_0^\infty \int_0^t \frac{\xi^2 \; cos(y\xi)}{(1+\alpha\xi^2)} e^{-\left(\frac{\nu\xi^2}{1+\alpha\xi^2}\right)z} cos\omega(t-z) \; dz \; d\xi \tag{34}$$

$$\tau_C(y,t) = -\frac{2UH(t)\omega \; \mu}{\pi} \int_0^\infty \int_0^t cos(y\xi) \, e^{-\left(\frac{\nu\xi^2}{1+\alpha\xi^2}\right)z} sin\omega(t-z) \; dz \; d\xi + \frac{2UH(t)\omega\alpha\mu}{\pi} \int_0^\infty \int_0^t \frac{\xi^2 \; cos(y\xi)}{(1+\alpha\xi^2)} e^{-\left(\frac{\nu\xi^2}{1+\alpha\xi^2}\right)z}$$
$$\times \, sin\omega(t-z) \; dz \; d\xi - \frac{2 \; U \; H(t) \omega^2 \alpha_1}{\pi} \int_0^\infty \int_0^t cos(y\xi) \, e^{-\left(\frac{\nu\xi^2}{1+\alpha\xi^2}\right)z} cos\omega(t-z) \; dz \; d\xi + \frac{2 \; U \; H(t) \; \omega \; \alpha \; \alpha_1}{\pi}$$
$$\times \int_0^\infty \int_0^t \frac{\xi^2 \; cos(y\xi)}{(1+\alpha\xi^2)} e^{-\left(\frac{\nu\xi^2}{1+\alpha\xi^2}\right)z} \; sin\omega(t-z) \; dz \; d\xi \tag{35}$$

5.2. NEWTONIAN FLUID IN THE PRESENCE OF POROUS EFFECTS $\boldsymbol{\alpha_1} \rightarrow \mathbf{0}$

Substituting $\alpha_1 \rightarrow 0$ in equations (25), (29), (30) and (31), we get

$$u_S(y,t) = U \; H(t) \; sin\omega t \; e^{-y\sqrt{\frac{\mu\emptyset}{\nu k}}} - \frac{2 \; U \; H(t) \; \nu \; \omega}{\pi} \int_0^\infty \int_0^t \frac{\xi \; sin(y\xi)}{\left(\nu\xi^2 + \frac{\mu\emptyset}{k}\right)} e^{-\left(\frac{k\nu\xi^2+\mu\emptyset}{k+k\alpha\xi^2}\right)z} cos\omega(t-z) \; dz \; d\xi + \frac{2 \; U \; H(t) \; \omega \; \alpha}{\pi}$$

$$\times \int_0^\infty \int_0^t \frac{\xi \, sin(y\xi)}{(1++\alpha\xi^2)} cos\omega(t-z) e^{-\left(\frac{k\nu\xi^2+\mu\emptyset}{k+k\alpha\xi^2}\right)z} dz \, d\xi, \tag{36}$$

$$u_C(y,t) = UH(t)cos\omega t \, e^{-y\sqrt{\frac{\mu\emptyset}{\nu k}}} - \frac{2\,U\,H(t)\,\nu\,\omega}{\pi} \int_0^\infty \int_0^t \frac{\xi \, sin(y\xi)}{\left(\nu\xi^2+\frac{\mu\emptyset}{k}\right)} sin\omega(t-z) e^{-\left(\frac{k\nu\xi^2+\mu\emptyset}{k+k\alpha\xi^2}\right)z} d\xi dz + \frac{2\,U\,H(t)\,\omega\,\alpha}{\pi}$$
$$\times \int_0^\infty \int_0^t \frac{\xi \, sin(y\xi)}{(1++\alpha\xi^2)} sin\omega(t-z) e^{-\left(\frac{k\nu\xi^2+\mu\emptyset}{k+k\alpha\xi^2}\right)z} d\xi dz, \tag{37}$$

$$\tau_S(y,t) = -\mu U \sqrt{\frac{\mu\emptyset}{\nu k}}\, H(t)\, sin\omega t \, e^{-y\sqrt{\frac{\mu\emptyset}{\nu k}}} - \frac{2UH(t)\nu\omega\mu}{\pi} \int_0^\infty \int_0^t \frac{\xi^2 \; cos(y\xi)}{\left(\nu\xi^2+\frac{\mu\emptyset}{k}\right)} cos\omega(t-z) e^{-\left(\frac{k\nu\xi^2+\mu\emptyset}{k+k\alpha\xi^2}\right)z} d\xi dz$$
$$+ \frac{2UH(t)\omega\alpha\mu}{\pi} \int_0^\infty \int_0^t \frac{\xi^2 \, cos(y\xi)}{(1++\alpha\xi^2)} cos\omega(t-z) e^{-\left(\frac{k\nu\xi^2+\mu\emptyset}{k+k\alpha\xi^2}\right)z} d\xi \, dz, \tag{38}$$

$$\tau_C(y,t) = -\mu\, U \sqrt{\frac{\mu\emptyset}{\nu k}}\, H(t)\, cos\omega t \, e^{-y\sqrt{\frac{\mu\emptyset}{\nu k}}} - \frac{2\,U\,H(t)\nu\,\omega\,\mu}{\pi} \int_0^\infty \int_0^t \frac{\xi^2 \; cos(y\xi)}{\left(\nu\xi^2+\frac{\mu\emptyset}{k}\right)} sin\omega(t-z) e^{-\left(\frac{k\nu\xi^2+\mu\emptyset}{k+k\alpha\xi^2}\right)z} d\xi \, dz$$
$$+ \frac{2\,U\,H(t)\,\omega\,\alpha\,\mu}{\pi} \int_0^\infty \int_0^t \frac{\xi^2 \, cos(y\xi)}{(1++\alpha\xi^2)} sin\omega(t-z) e^{-\left(\frac{k\nu\xi^2+\mu\emptyset}{k+k\alpha\xi^2+\alpha_1\emptyset}\right)z} d\xi \, dz \tag{39}$$

5.3. NEWTONIAN FLUID IN THE ABSENCE OF POROUS EFFECTS$\emptyset \to 0$, $\alpha \to 0$ and $\alpha_1 \to 0$

Taking the $\emptyset \to 0, \alpha \to 0$ and $\alpha_1 \to 0$into equations (25), (29), (30) and (31) the solutions for velocity field and the shear stress under the form of Newtonian fluid are

$$u_S(y,t) = U\,H(t)\, sin\omega t - \frac{2\,H(t)\,U\,\omega}{\pi} \int_0^\infty \int_0^t \frac{sin(y\xi)}{\xi} e^{-\nu\xi^2 z} cos\omega(t-z)\, dz\, d\xi \tag{40}$$

$$u_C(y,t) = UH(t)cos\omega t - \frac{2\,U\,H(t)\,\omega}{\pi} \int_0^\infty \int_0^t \frac{sin(y\xi)}{\xi} e^{-\nu\xi^2 z}\, sin\omega(t-z)\, dz\, d\xi \tag{41}$$

$$\tau_S(y,t) = -\frac{2\,U\,H(t)\omega\,\mu}{\pi} \int_0^\infty \int_0^t cos(y\xi)\, e^{-\nu\xi^2 z}\, cos\omega(t-z)\, dz\, d\xi \tag{42}$$

$$\tau_C(y,t) = -\frac{2\,U\,H(t)\omega\,\mu}{\pi} \int_0^\infty \int_0^t cos(y\xi)\, e^{-\nu\xi^2 z} sin\omega(t-z)\, dz\, d\xi \tag{43}$$

are found out.

NUMERICAL RESULTS AND CONCLUSION

In this research article, we have perused the exact solutions under the effects of porosity for flow problem of second grade fluid. In order to have few related physical features and aspects for found results; many graphical illustrations are discussed and depicted in this regard. Profiles of velocity field and corresponding shear stress for second grade fluid with porous effects are illustrated numerically with different results, in which the interpretation is made with respect to the discrepancies of emerging parameters ($U,\ \mu,\ \alpha,\ \omega,\ k,\ t$and ν) of interest. Different diagrams are drawn against different values of distinct pertinent parameters. The contrast and comparisons is shown for four types of models such as second grade, second grade with porous effects, Newtonian with porous effects and Newtonian fluids. Under effects of porosity between two models second grade and Newtonian fluids have great interesting situations relevant to the increasing and decreasing behavior of fluid on motion. Due to the effects of porosity motion of fluid has become sometimes slowest and fastest depending upon rheological parameters. Here the study is made for finding the new exact solutions under the effects of porosity for second grade fluid. The exact solutions are obtained by utilizing integral transforms (Laplace and Fourier sine transforms) which are written as multiple integrals, the product of convolution, elementary functions and simple integral. The general solutions are verified for satisfaction of the initial and boundary conditions. Under the limiting cases, when $\phi \to 0$ the solutions are termed as for second grade fluid in the absence of porosity, when $\alpha \to 0$ the solutions are termed as for Newtonian fluid in the presence of porosity and when$\alpha \to 0$ and $\phi \to 0$ the solutions are termed as for Newtonian in the absence of porosity. At the ending, to bring the light on main consequences and outcomes for over all this study is followed below:

(i) Equations (25) and (29) are the general solutions of second grade fluid with porosity which are expressed in terms of multiple integrals form and elementary functions, both solutions can be particularized to produce similar solutions for the effects under absence and presence of porosity.

(ii) For the check of correctness, different values of time$t$ and relaxation time α have produced the motion of fluid for the velocity field$u_s(y,t)$ and corresponding shear stress as a decreasing function.

(iii) It can be pointed out, the motion of fluid is oscillating with respect to distinct value of kinematics viscosity ν.

(iv) It is seen that velocity field and corresponding shear stress are decreasing function of fluid with regard to different values of amplitude ω.

(v) Increasing the range of permeability $K$is fixed for the interval $0.4 \leq K \leq 1.0$, for which figure 6 shows the motion of fluid is some sort scattered as compared to other figures among the graphs.

(vi) The comparisons among the four type of models leads as the second grade fluid, second grade fluid with porous, Newtonian fluid with porous and Newtonian fluid, the motion of fluid has reciprocal behavior for velocity field and shear stress; i-e velocity field is decreasing and corresponding shear stress is increasing with respect to distinct increasing effects of permeability parameter K.

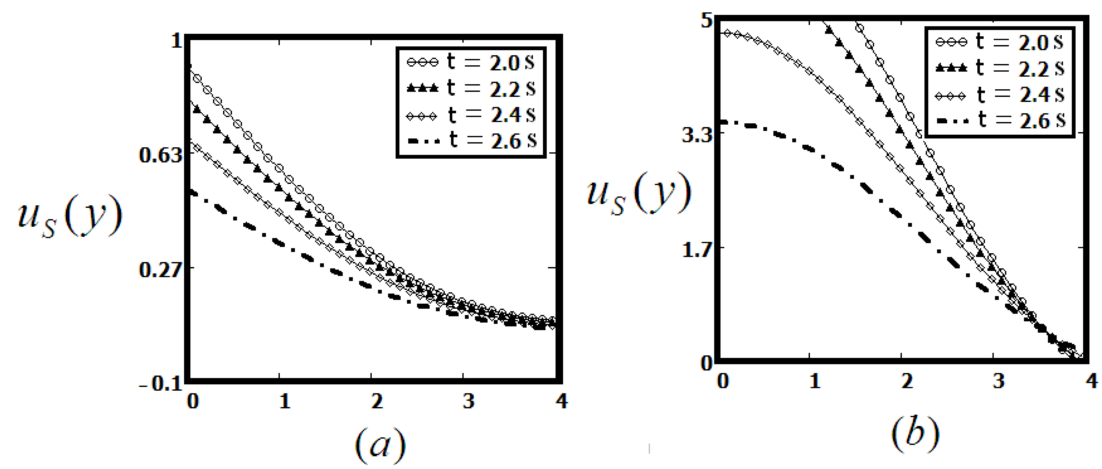

Figure 2: Fordistinct values of *t* and $\nu = 0.63, U = 1, \alpha = 1.5, \omega = 2, \mu = 32.52$**,** Profiles for Second grade fluid over velocity field and shear stress are given by equations (25) and (29).

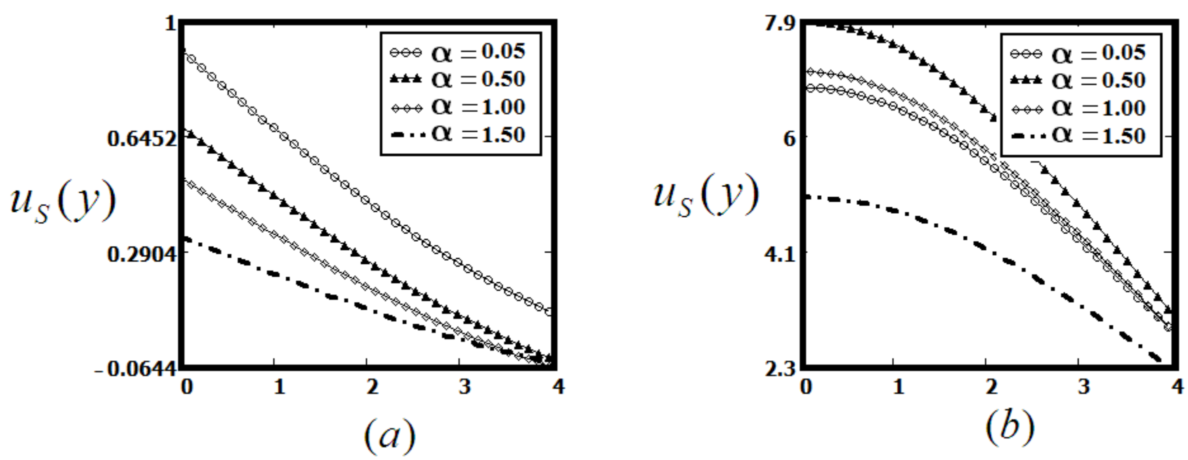

Figure 3: For distinct values of $\alpha$and $\nu = 0.63, U = 1, t = 2\ s, \omega = 2, \mu = 32.52$, Profiles for Second grade fluid over velocity field and shear stress are given by equations (25) and (29).

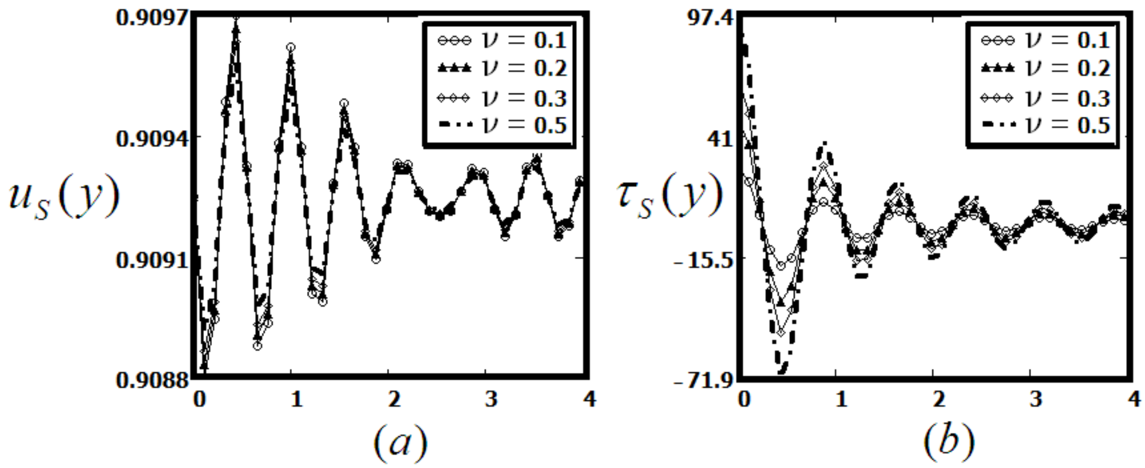

Figure 4: For distinct values of $\nu$and$U = 1,\ \rho = 2.413,\ \mu = 32.52,\ t = 2\ s,\ \alpha = 1.5$, Profiles for Second grade fluid over velocity field and shear stress are given by equations (25) and (29).

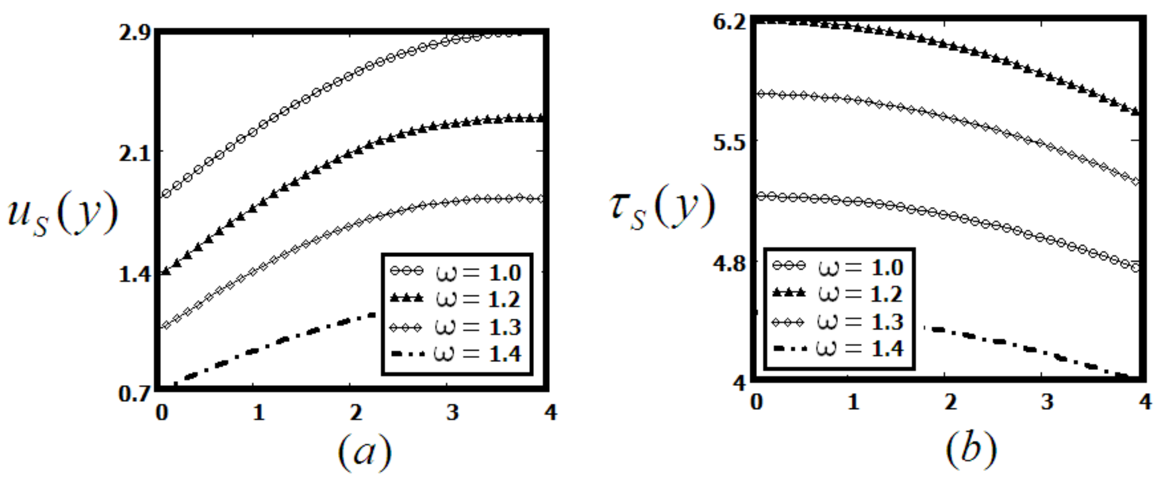

Figure 5: For distinct values of $\omega$and $U = 1,\ \nu = 0.63,\ \mu = 32.52,\ t = 2\ s,\ \alpha = 1.5$, Profiles for Second grade fluid over velocity field and shear stress are given by equations (25) and (29).

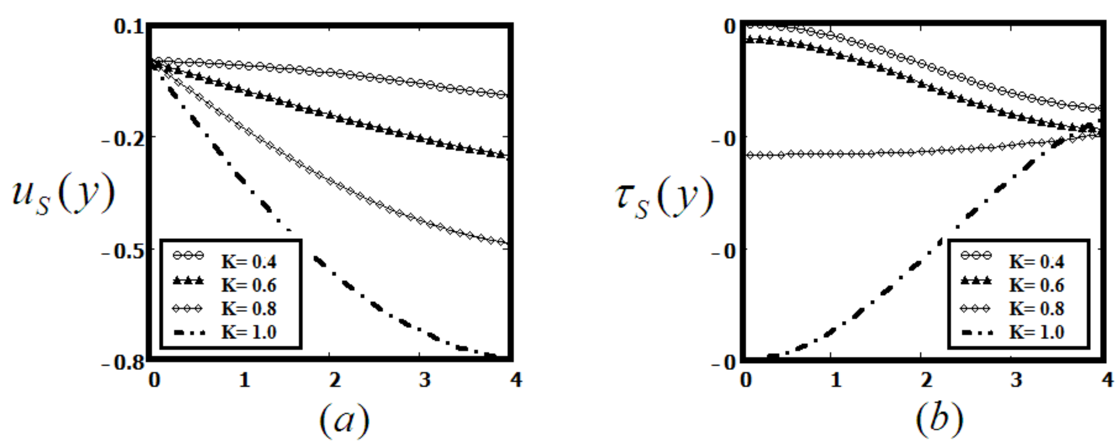

Figure 6: For distinct values of $K$and $U = 1,\ \nu = 0.63,\ \mu = 32.52,\ t = 2\ s,\ \omega = 2$, Profiles for second grade fluid with porous effects for Velocity field and shear stress given by equations (25) and (29).

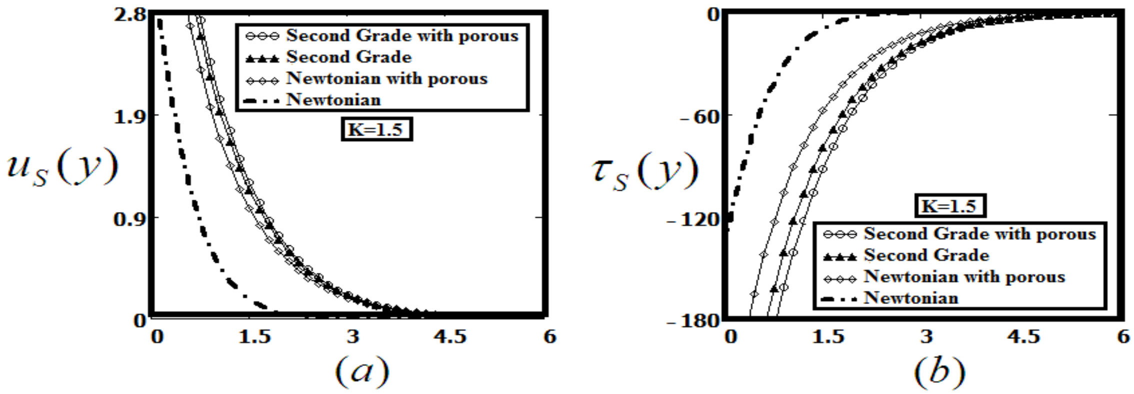

Figure 7: For distinct values of $K$and $U = 1,\ \nu = 0.63,\ \mu = 32.52,\ t = 5\ s,\ \omega = 2, K = 1.5$, Profiles for second grade fluid with porous effects for Velocity field and shear stress given by equations (25) and (29).

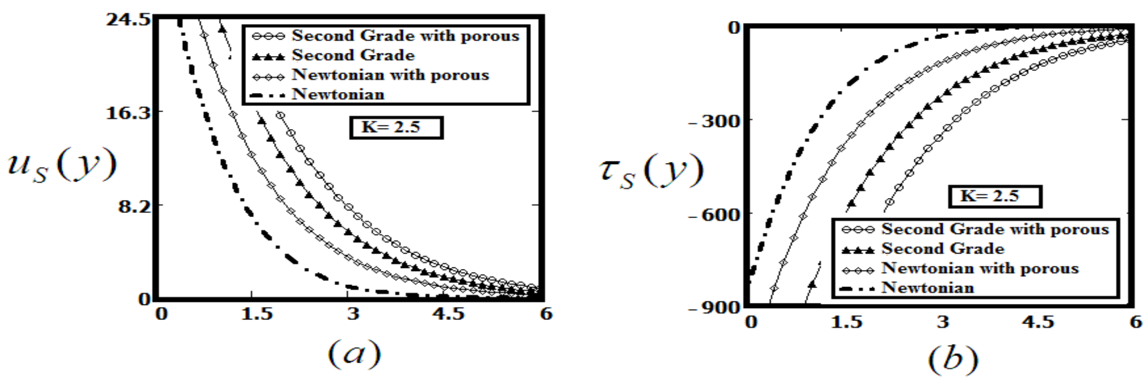

Figure 8: For distinct values of $K$and $U = 1,\ \nu = 0.63,\ \mu = 32.52,\ t = 5\ s,\ \omega = 2, K = 2.5$, Profiles of second grade fluid with porous effects for velocity field and shear stress are given by equations (25) and (29).

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