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Analytical Solution for Metallic Wire Coating Using Sisko Fluid Flow

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ABSTRACT

The present study explores the analytical analysis of MHD flow of a visco-elastic Sisko fluid arising in the wire coating analysis. A pressure type coating die is used for this purpose. The objective is to examine the effect of emerging parameters such as the power index (n), the radii ratio δ, the material parameter(λ) and the speed of the wire Von the flow characteristics through graphs. The nonlinear equations are solved analytically by utilizing the Adomian Decomposition Method (ADM). Additionally, the Optimal Homotopy Asymptotic Method (OHAM) has been used to verify and strengthen the results obtained by ADM. The convergence of the series solution is established. For some special cases of the present work, a comparison with the previously published results has been presented.
KEYWORDS: OHAM and ADM solutions, Sisko fluid, Wirecoating, Analytical solution.

1. INTRODUCTION

Non-Newtonian fluids [1-4] have gained a deep interestby researchers because of its applications in industries like oil, polymer, plastic, etc. Various models, both analytical and numerical, have been discussed in the study of non-Newtonian fluids. Fluids models are characterized by the underlying fluid grades such as second grade, third grade, etc., generalizing to n-grade fluids [4]. It includes shear-thinning, shear-thickening, yields-stress, viscoelasticityetc. An individual case of indiscriminate Newtonian liquids known as Sisko model is considered [5]. Themodelof the sisko fluid is used to envisage the Pseudoplastic and Dilatant performance. Although, prevailing use in industry and engineering little research work has been reportedin this area. Cobble et al. [6] investigated incompressible non-Newtonian fluid in orthogonal coordinates.Akyildiz at el. [7] studied the sisko fluid flow and gave an implicit differential equation. Siddiqui at el. [8] studied sisko fluid for Taylor's scraping problem. Thin film of non-Newtonian fluid flow was studied by Siddiqui at al. [9].Wan et al. [10] investigated MHD sisko fluid.Khan et al. [11] studied the Sisko fluid in porous media.The Sisko fluid investigated by Abelman et al. [12]in a rotating frame for Rayleigh problem. The MHD (magneto-hydrodynamic)flow of as sisko fluid is investigated numerically by Khan et al. [13] in an annular pipe.

Different types of fluids are used for wire and fiber optics coating. The wire coating depends upon the temperature, geometry, fluid viscosity and polymer. It depends on the coating die, fluid viscosity, temperature of the wire and the molten polymer. Most relevant work on wire coatings are thus summarized in the following.

Shah et al. [14] investigated wire coating analysis with linearly varying temperature. Han and Rao [15] carried out an analysis on wire coating extrusion. Non-Newtonian fluid model was used byAkter and Hashmi [16, 17] for wire coating. Siddiqui et al. [18] investigatedthe extrusion in wire coating in a pressurized type die. Fenner and Williams [19] investigated the coating flow in a pressurized die. Mitsoulis [20] studied the wire coating flow with heat transfer. Unsteady second grade fluid with oscillating boundary condition inside the wire coating die was investigated by Shah et al. [21]. Exact solution was obtained for unsteady second grad fluid for wirecoating by Shah et al. [22]. Oldroyd 8-constant fluid was used for wire coating analysis by Shah et al.[23]. Shah et al. [24] studied wire coating using third grade fluid flow along with heat transfer analysis. Recently Sajid et al. [25] used Sisko fluid for wire coating analysis by applying HAM. Recently, Zeeshan et al. [26] used PhanThien Tanner fluid in double-layer optical fiber coating. The same author [27] investigated optical fiber coating using wet-on-wet coating process. In the process the authors have used PTT fluids of different viscosities for the constant pressure gradient. Zeeshan et al. [28] investigated an approximate solution for optical fiber coating in a pressure type die using two immiscible Oldroyd 8-constant fluids using OHAM. Flow and heat transfer of two immiscible fluids in double-layer optical fiber coating is investigated by Zeeshan et al. [29].

In scrutiny of the above incentive, in the present study, weanalyze the wire coating analysisusingSisko fluid flow in an annular die.Well known mathematical techniques, namelyADM and OHAM are used for a series solution. The

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ADM [30-33]is broadly used by the researchers to solvenonlinear problems. Additionally the results are also verified by using OHAM [21, 23, 24, 28, 34-36]. Further, the comparison of the present work and published work [25] is also made for clarity. The paper is organized as in the following.Section 2 presents formulation of the problem. Analysis of ADM is given in section 3. Section 4 is reserved for analysis of OHAM. ADM solution of the problem is given in Section 5.Section 6 and 7 are given for results and discussion and concluding remarks respectively.

2. Modeling of the Problem

Manufacturing process of wire coating is depicted in figure 1.A metallic wire is dragged with velocity V insidethe die of length L. The direction of the flow is represented along the z-axis and r are taken perpendicularto z, where R_w and R_d are radius of the wire and the die respectively.

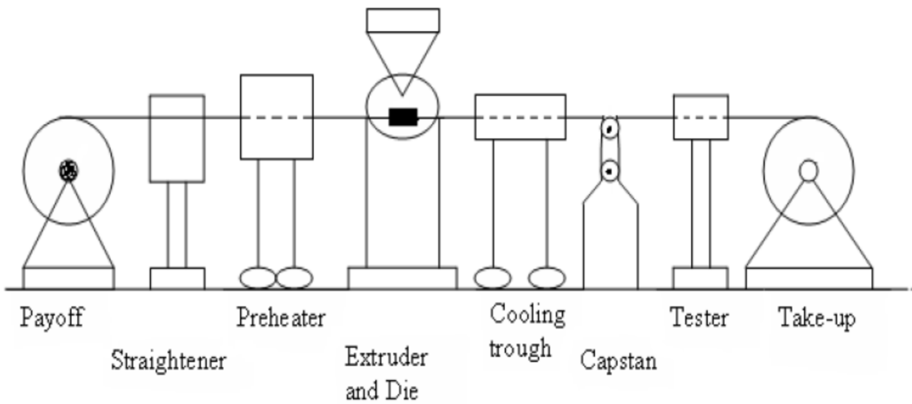


Figure1. Typical manufacturing process of wire.

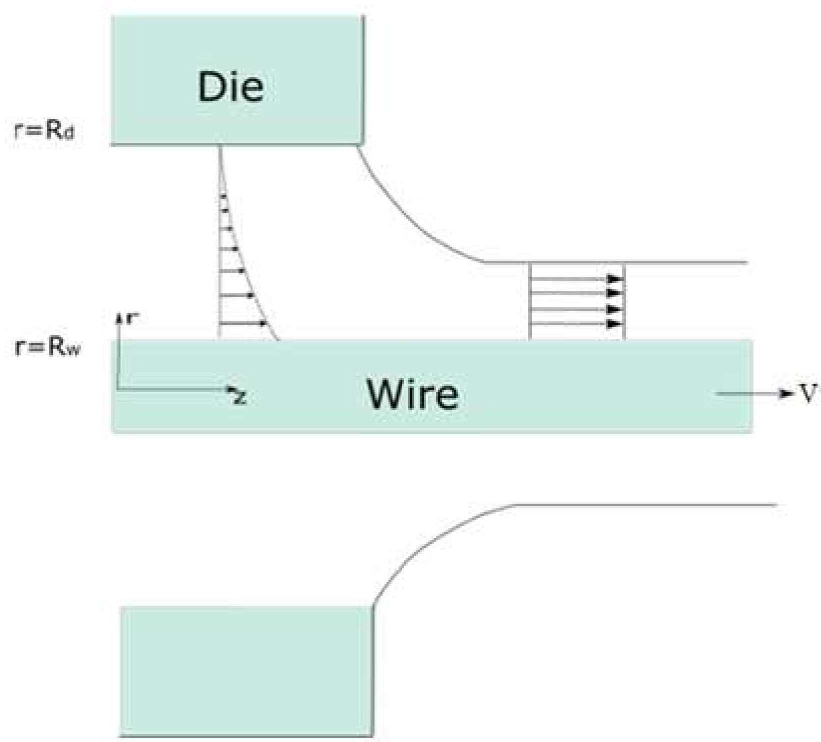


Figure2. Flow model in coating die.

The continuity and momentum equations for incompressible flow are [8-10]

$$\nabla.\widehat{w}=0, \tag{1}$$

$$\rho\frac{d\widehat{w}}{dt}=\nabla.\widehat{\mathbf{T}}. \tag{2}$$

With

$$\widehat{\mathbf{T}}=-p\mathbf{I}+\widehat{\mathbf{S}}, \quad \widehat{\mathbf{S}}=\left[\widehat{a}+\widehat{b}\left|\sqrt{\frac{1}{2}tr\left(\widehat{\mathbf{A}}^{2}\right)}^{\,n-1}\right|\right]\widehat{\mathbf{A}}, \tag{3}$$

$$\widehat{\mathbf{A}}=gradu+\left(gradu\right)^{T}. \tag{4}$$

here, $\widehat{w}$, $\widehat{\mathbf{T}}$, d/dt, p, $\mathbf{I}$, $\widehat{\mathbf{S}}$, $\widehat{\mathbf{A}}$, are the velocity, Stress tensor, the material time derivative, the pressure, the identity tensor, the extra stress tensor and First Revilin Erickson tensor, and *T,* $\widehat{a},\widehat{b}$ are matrix transpose and the material parameters respectively.

Velocity and Stress fields are:

$$\widehat{w}=\left[0,0,\widehat{w}\left(r\right)\right],\widehat{\mathbf{S}}=\widehat{\mathbf{S}}\left(r\right). \tag{5}$$

In view of Eqs. (3)-(5), the governing equation for velocity field is as follows:

$$\widehat{a}\left(r\frac{d^{2}\widehat{w}}{dr^{2}}+\frac{d\widehat{w}}{dr}\right)+\widehat{b}\left[\left(\frac{d\widehat{w}}{dr}\right)^{n}+nr\left(\frac{d\widehat{w}}{dr}\right)^{n-1}\frac{d^{2}\widehat{w}}{dr^{2}}\right]=0. \tag{6}$$

Boundary conditions on the velocity are

$$\widehat{w}=V \text{ for } r=R_{w}, \text{ And } \widehat{w}=0, \text{ for } r=R_{d} \tag{7}$$

The average velocity as in [21-24] is

$$\widehat{w}_{ave}=\frac{2}{R_{d}^{2}-R_{w}^{2}}\int_{R_{w}}^{R_{d}}r\widehat{w}\left(r\right)dr. \tag{8}$$

The volume flow rateat any control surface of the coating is [21-24]

$$Q=\pi V\left(R_{c}^{2}-R_{w}^{2}\right). \tag{9}$$

Where R_c is thickness ofthe coated wire.

The volume flow rate (flux) is[21-24]

$$\widehat{\mathrm{Q}}=\int_{R_{w}}^{R_{d}}r\widehat{w}\left(r\right)dr. \tag{10}$$

From Eqs. (9) and (10), thickness of the coated wire is [21-24]

$$\widehat{R}_{c}=[R_{w}^{\;2}+\frac{2}{V}\int_{R_{w}}^{R_{d}}r\widehat{w}(r)dr]^{\frac{1}{2}} \tag{11}$$

The shear stress is

$$\widehat{s}_{rz}\Big|_{r=R_{w}}=\left(\widehat{a}+\widehat{b}\left|\frac{d\widehat{w}}{dr}\right|^{n-1}\right)\frac{d\widehat{w}}{dr}. \tag{12}$$

The total surface force on the wire is

$$\widehat{\mathrm{F}}=2\pi\mathrm{R}_{w}\mathrm{L}\widehat{\mathrm{S}}_{rz}\Big|_{\mathrm{r}=\mathrm{R}_{w}}. \tag{13}$$

In view of Eq. (14), Eqs. (6)-(13) can be reduced to the following set of non-dimensional equations respectively:

$$r^* = \frac{r}{R_w}, \hat{w}^* = \frac{\hat{w}}{V}, \frac{R_d}{R_w} = \delta > 1, \lambda^* = \frac{\hat{b}}{\hat{a}}\left(\frac{V}{R_w}\right)^{n-1} \tag{14}$$

$$r\frac{d^2\hat{w}}{dr^2} + \frac{d\hat{w}}{dr} + \lambda\left(\left(\frac{d\hat{w}}{dr}\right)^n + nr\left(\frac{d\hat{w}}{dr}\right)^{n-1}\frac{d^2\hat{w}}{dr^2}\right) = 0, \tag{15}$$

$$\hat{w} = 1 \text{ at } r = 1 \text{ and } \hat{w} = 0 \quad r = \delta \tag{16}$$

$$\hat{w}_{ave} = \frac{\hat{w}_{ave}\left(R_d^2 - R_w^2\right)}{2R_w V} = \int_1^\delta r\hat{w}(r)\,dr, \tag{19}$$

$$Q = \frac{\hat{Q}}{2\pi R_w^2 V} = \int_1^\delta r\hat{w}(r)\,dr, \tag{20}$$

$$R_c = \frac{R_c}{R_w} = \left(\left[1 + 2\int_1^\delta r\hat{w}(r)\,dr\right]^{1/2}\right), \tag{21}$$

$$\hat{S}_{rz}\Big|_{r=1} = \frac{\hat{S}_{rz}R_w}{\mu V}\Bigg|_{r=1} = \frac{d\hat{w}}{dr} + \lambda\left(\frac{d\hat{w}}{dr}\right)^n\Bigg|_{r=1}, \tag{22}$$

$$F = \frac{F}{2\pi\mu L V} = \frac{d\hat{w}}{dr} + \lambda\left(\frac{d\hat{w}}{dr}\right)^n\Bigg|_{r=1}, \tag{23}$$

3. Analysis of AdomianDecomposition Method (ADM)

ADM is an analytical technique for decomposing an unknown function into infinitely many components. For more understanding,we take the followingequation:

$$\hat{w}(r,t) = \sum_{k=0}^{\infty}\hat{w}_k(r), \tag{24}$$

To find the components $\hat{w}_0, \hat{w}_1, \hat{w}_2$ ...,separately, decomposition method is used.
Consider the following nonlinear differential equation:

$$\hat{L}_r\hat{w}(r) + \hat{R}\hat{w}(r) + \hat{N}\hat{w}(r) = \hat{G}(r), \tag{25}$$

$$\hat{L}_r\hat{w}(r) = \hat{G}(r) - \hat{R}\hat{w}(r) - \hat{N}\hat{w}(r). \tag{26}$$

Here $\hat{L}_r = \frac{\partial^2}{\partial r^2}$ is the linear operator, $\hat{G}(r)$ the source term, $\hat{R}(r)$ the remainder linear operator while $\hat{N}\ \hat{w}(r)$ is a nonlinear term.

ApplyingL_r^{-1}on both side to the Eq. (26)

$$\hat{L}_r^{-1}\hat{L}_r\hat{w}(r) = \hat{L}_r^{-1}g(r) - L_r^{-1}\hat{R}\hat{w}(r) - L_r^{-1}\hat{N}\hat{w}(r), \tag{27}$$

$$\hat{w}(r) = f(r) - \hat{L}_r^{-1}\hat{R}\hat{w}(r) - \hat{L}_r^{-1}\hat{N}\hat{w}(r), \tag{28}$$

The function $f(r)$ arising from $\hat{L}_r^{-1}g(r)$ after using the conditions given in Eq. (16). The operator $L_r^{-1} = \iint(.)\,drdr$ is used for second order differential equations.

The series solution of $\widehat{w}\left(r\right)$ using ADM we have,

$$\sum_{k=0}^{\infty}\widehat{w}_k\left(r\right)=f\left(r\right)-\widehat{L}_r^{-1}\widehat{R}\sum_{k=0}^{\infty}\widehat{w}_k\left(r\right)-\widehat{L}_r^{-1}\widehat{N}\sum_{k=0}^{\infty}\widehat{w}_k\left(r\right), \tag{29}$$

In view of adomianpolynomials the nonlinear term $\widehat{N}\sum_{k=0}^{\infty}\widehat{w}_k\left(r\right)$ can be expressed as

$$\widehat{N}\sum_{k=0}^{\infty}\widehat{w}_k\left(r\right)=\sum_{k=0}^{\infty}\widehat{\mathbf{A}}_k, \tag{30}$$

where the components $\widehat{w}_0, \widehat{w}_1, \widehat{w}_2, \widehat{w}_3$ ..., are determined as

$$\widehat{w}_0+\widehat{w}_1+\widehat{w}_2+\widehat{w}_3+\widehat{w}_4\ldots\ldots=f\left(r\right)-\widehat{L}_r^{-1}R\left(\widehat{w}_0+\widehat{w}_1+\widehat{w}_2+\widehat{w}_3\ldots\right)-\widehat{L}_r^{-1}N\left(\widehat{\mathbf{A}}_0+\widehat{\mathbf{A}}_1+\ldots\right). \tag{31}$$

To determine the series components $\widehat{w}_0, \widehat{w}_1, \widehat{w}_2, \widehat{w}_3$ ..., it should be noted that ADM suggest that $f(r)$ in fact describe the zeroth component $\widehat{w}_0$.

The recursive relation is defined as:

$$\widehat{w}_0\left(r\right)=f_0\left(r\right), \tag{32}$$

$$\widehat{w}_1\left(r\right)=-\widehat{L}_r^{-1}R\left[\widehat{w}_0\left(r\right)\right]-\widehat{L}_r^{-1}\left(\widehat{\mathbf{A}}_0\right), \tag{33}$$

$$\widehat{w}_2\left(r\right)=-\widehat{L}_r^{-1}R\left[\widehat{w}_1\left(r\right)\right]-\widehat{L}_r^{-1}\left(\widehat{\mathbf{A}}_1\right), \tag{34}$$

$$\widehat{w}_3\left(r\right)=-\widehat{L}_r^{-1}R\left[\widehat{w}_2\left(r\right)\right]-\widehat{L}_r^{-1}\left(\widehat{\mathbf{A}}_2\right). \tag{35}$$

By following the same process we can find the other terms.

4. Analysis of Optimal Homotopy Asymptotic Method (OHAM)

The OHAM method has been widely used for the solution of non-linear differential equations, particularly to those arising in Fluid Mechanics.Such equations often arise in non-Newtonian fluids where OHAM can be easily applied. For better understanding we consider

$$\widehat{\mathrm{A}}(\widehat{w}(r))+\widehat{G}(r)=0,\quad r\in\widehat{\Lambda},\ \widehat{\mathrm{B}}(\widehat{w},\ \frac{d\widehat{w}}{dr})=0,\ r\in\Im, \tag{36}$$

With

$$\widehat{\mathrm{A}}=\widehat{L}+\widehat{N}, \tag{37}$$

where, $\widehat{\mathrm{A}}$, $\widehat{\mathrm{B}}$, $\widehat{w}$, $\Im$ and $\widehat{G}(r)$ are differential operator, boundary operator, the unknown function, boundary of the domain $\widehat{\Lambda}$ and analytical function respectively.

In Eq. (38) the linear and nonlinear operator is represented by $\widehat{L}$ and $\widehat{N}$ respectively.

We consider $\varnothing\left(r,p\right):\widehat{\Lambda}\times\left[0,1\right]\rightarrow\widehat{R}$ which satisfies

$$\left[1-p\right]\left[\widehat{L}(\varnothing\left(r,\widehat{p}\right))+\widehat{G}\left(r\right)\right]-\widehat{H}\left(\widehat{p}\right)\begin{bmatrix}\widehat{L}\left[\widehat{w}\left(r\right)\right]+\\ \widehat{N}\left[\widehat{w}\left(r\right)\right]+\widehat{G}\left(r\right)\end{bmatrix}=0,\widehat{\mathrm{B}}\left(\varnothing\left(r,\widehat{p}\right),\frac{\partial\varnothing\left(r,\widehat{p}\right)}{\partial r}\right)=0. \tag{38}$$

Where $\widehat{H}\left(\widehat{p}\right)$ represents the non-zero auxiliary function and $\varnothing\left(r,\widehat{p}\right)$ is the an unknown function. For $\widehat{p}=0$, Eq. (39) only recuperate the linear part of solution i.e., $\varnothing\left(r,\widehat{p}\right)=\widehat{w}_0\left(r\right)$,

$$\widehat{L}\left[\varnothing\left(r,0\right)\right]=0,\widehat{\mathrm{B}}\left(\widehat{w}_0,\frac{\partial\widehat{w}_0}{\partial r}\right), \tag{39}$$

For $\widehat{p}=1$, we recuperate the nonlinear boundary value problem and thissolution approachto the exact solution such as $\varnothing(r,1)=\widehat{w}(r)$. So we can say that the solution $\varphi(r,p)$approaches to exact solution as $\widehat{p}$ approaches 0 to 1.

$\widehat{H}(\widehat{p})$ isthe auxiliary function can be chosen as

$$\widehat{H}(\widehat{p})=\widehat{p}C_1+\widehat{p}^2C_2+\widehat{p}^3C_3+... \tag{40}$$

The auxiliary constants $C_1, C_2, C_3,...,$ are determined later to reduce solution inaccuracy.

For estimated solution, $\varnothing(r,\widehat{p})$ is expanding with respect to p by using Taylor series [21, 23, 24, 28, 33-35].

$$\varnothing(r,\widehat{p},C_i)=\widehat{w}_0(r)+\sum_{k=1}^{\infty}\widehat{w}_k(r,C_i)\widehat{p}^k, \tag{41}$$

By usingEqs. (40) and(41) into equation (38), and comparing the coefficients ofthe same powers, of $\widehat{p}$, we obtain several order problems. Eq. (39) gives the zeroth order problem.In the following, the first and second order problems are presented as:

$$\widehat{L}(\widehat{w}_1(r))+\widehat{G}(r)=C_1\widehat{N}_0(\widehat{w}_0(r)),\ \widehat{\mathrm{B}}(\widehat{w}_1,\frac{d\widehat{w}_1}{dr})=0, \tag{42}$$

$$\widehat{L}(\widehat{w}_2(r))-\widehat{L}(\widehat{w}_1(r))=C_2\widehat{N}_0(\widehat{w}_0(r))+C_1[\widehat{L}(\widehat{w}_1(r))+\widehat{N}_1(\widehat{w}_1(r))],\ \mathrm{B}(\widehat{w}_2,\frac{d\widehat{w}_2}{dr})=0. \tag{43}$$

Generally the equation takes the form as,

$$\begin{gathered}\widehat{L}(\widehat{w}_k(r))-\widehat{L}(\widehat{w}_{k-1}(r))=C_k\widehat{N}_0(\widehat{w}_0(r))+\sum_{i=1}^{k-1}C_i[\widehat{L}(\widehat{w}_{k-1}(r))+\widehat{N}_{k-1}(\widehat{w}_0(r),\widehat{w}_1(r),...,\widehat{w}_{k-1}(r))],\\ \widehat{\mathrm{B}}(\widehat{w}_k,\frac{d\widehat{w}_k}{dr})=0,\ k=2,3,...\end{gathered} \tag{44}$$

Here $\widehat{N}_{k-1}(\widehat{w}_0(r),\widehat{w}_1(r),...,\widehat{w}_{k-1}(r))$ is the coefficient of $\widehat{p}^{k-1}$ in extension of $\widehat{N}(\varnothing(r,\widehat{p}))$.

$$\widehat{N}(\varnothing(r,\widehat{p}))=\widehat{N}_0(\widehat{w}_0(r))+\sum_{k-i=1}^{\infty}\widehat{N}_{k-i}(\widehat{w}_0,\widehat{w}_1,...,\widehat{w}_{k-i}). \tag{45}$$

The junction of Eq. (41) depends upon the auxiliary constants and the order of the problem.

If it converges at $\widehat{p}=1$, one has:

$$\widehat{w}(r,C_i)=\widehat{w}_0(r)+\sum_{k=1}^{\infty}\widehat{w}_k((r,C_i)),\ ;i=1,2,3,...,m. \tag{46}$$

Using Eq. (46) into Eq. (36), expression for the residual in the following is obtained as:

$$\widehat{R}(r,C_i)=\widehat{L}(\widehat{w}(r,C_i)+\widehat{G}(r)+\widehat{N}(\widehat{w}(r,C_i)),i=1,2,\ldots,m, \tag{47}$$

Several methods likeRitz Method,Galerkin's Method, Method of Least Squares and the Collocation Method are used to find the auxiliary constants.

Here we use the least squares method to find the auxiliary constant:

$$\widehat{\mathrm{J}}(C_i)=\int_a^b\widehat{R}^2(r,C_i)dr,\ \ ;i=1,2,3,...,m \tag{48}$$

$$\frac{\partial\widehat{\mathrm{J}}}{\partial C_i}=0,\ ;i=1,2,3,...,m, \tag{49}$$

here$a$,b (taking from domain) are constant that locate auxiliary constants which minimize the residual. Many researchers [21, 23, 24, 28, 33-35]fruitfully implemented this methodforsolving highly non-linear boundary value problems of physicsand engineering and gained pleasingoutcome. As the number of the auxiliary constant increase the solution errors, reduce and a consequence the solution of the problem converges to the exact solution.

5. Solution to the Problem

The analytical solution of Eq. (15) and (16) can be found by applying ADM. Following the same process of ADM given in section 3; the zeroth, first and second order solutions of the problem with respect to various values of power index $n$isgiven as:

Zeroth, first and second order solution for $n = 2$respectively are:

$$\widehat{w}_0 = \frac{-r+\delta}{-1+\delta}, \tag{50}$$

$$\widehat{w}_1 = -\frac{\left(-1+\delta-\lambda\right)\left(r\left(-1+\delta\right)\ln r-\left(-1+r\right)\delta\ln\delta\right)}{\left(-1+\delta\right)^3}, \tag{51}$$

$$\widehat{w}_2 = \frac{1}{2\left(-1+\delta\right)^6}(r\ln r^2 - 5r\delta\ln\delta^2 + (10r\delta^2 - 10r\delta^3 + 5r\delta^4 - r\delta^5 + (5r\lambda - 20r\delta\lambda) + 30r\delta^2\lambda)\ln r^2$$
$$+(-20r\delta^3\lambda + 5r\delta^4\lambda + 6r\lambda^2 - 18r\delta\lambda^2 + 18r\delta^2\lambda^2 - 6r\delta^3\lambda^2 + 2r\lambda^3 - 4r\delta\lambda^3 2r\delta^2\lambda^3)\ln r^2 + (2r\delta -$$
$$8r\delta^2 + 12r\delta^3 - 8r\delta^4 + 2r\delta^5 + 10r\delta\lambda)\ln r\ln\delta - (30r\delta^2\lambda - 30r\delta^3\lambda + 10r\delta^4\lambda - 12r\delta\lambda^2 + 24r\delta^2\lambda^2$$
$$-12r\delta^3\lambda^2)\ln r\ln\delta + (4r\delta\lambda^3 - 4r\delta^2\lambda^3)\ln r\ln\delta - (-r\delta - \delta^2 + 2r\delta^2 + 2\delta^4 - 2r\delta^4 - \delta^5 + r\delta^5 + 5\delta\lambda$$
$$-5r\delta\lambda - 5\delta^2\lambda^2)\ln\delta^2 + 5r\delta^2\lambda + 5\delta^3\lambda - 5r\delta^3\lambda - 5\delta^4\lambda^2 + 5r\delta^4\lambda - 6\delta\lambda^2 + 6r\delta\lambda^2 + 6\delta^3\lambda^2 - 6r\delta^3\lambda^2 -$$
$$2\delta\lambda^3 + 2r\delta\lambda^3 - 2\delta^2\lambda^3 + 2r\delta^2\lambda^3)\ln\delta^2. \tag{52}$$

The total velocity for $n = 2$ is:

$$\widehat{w} = \frac{-r+\delta}{-1+\delta} - \frac{\left(-1+\delta-\lambda\right)\left(r\left(-1+\delta\right)\ln r-\left(-1+r\right)\delta\ln\delta\right)}{\left(-1+\delta\right)^3} + \frac{1}{2\left(-1+\delta\right)^6}(r\ln r^2 - 5r\delta\ln\delta^2 +$$
$$(10r\delta^2 - 10r\delta^3 + 5r\delta^4 - r\delta^5 + (5r\lambda - 20r\delta\lambda) + 30r\delta^2\lambda)\ln r^2 + (-20r\delta^3\lambda + 5r\delta^4\lambda + 6r\lambda^2 -$$
$$18r\delta\lambda^2 + 18r\delta^2\lambda^2 - 6r\delta^3\lambda^2 + 2r\lambda^3 - 4r\delta\lambda^3 2r\delta^2\lambda^3)\ln r^2 + (2r\delta - 8r\delta^2 + 12r\delta^3 - 8r\delta^4 + 2r\delta^5 +$$
$$10r\delta\lambda)\ln r\ln\delta - (30r\delta^2\lambda - 30r\delta^3\lambda + 10r\delta^4\lambda - 12r\delta\lambda^2 + 24r\delta^2\lambda^2 - 12r\delta^3\lambda^2)\ln r\ln\delta +$$
$$(4r\delta\lambda^3 - 4r\delta^2\lambda^3)\ln r\ln\delta - (-r\delta - \delta^2 + 2r\delta^2 + 2\delta^4 - 2r\delta^4 - \delta^5 + r\delta^5 + 5\delta\lambda - 5r\delta\lambda - 5\delta^2\lambda^2)\ln\delta^2$$
$$+5r\delta^2\lambda + 5\delta^3\lambda - 5r\delta^3\lambda - 5\delta^4\lambda^2 + 5r\delta^4\lambda - 6\delta\lambda^2 + 6r\delta\lambda^2 + 6\delta^3\lambda^2 - 6r\delta^3\lambda^2 - 2\delta\lambda^3 + 2r\delta\lambda^3 - 2\delta^2\lambda^3$$
$$+2r\delta^2\lambda^3)\ln\delta^2. \tag{53}$$

Zeroth, first and second order solution for $n = 3$respectively are:

$$\widehat{w}_0 = \frac{-r+\delta}{-1+\delta}, \tag{54}$$

$$\widehat{w}_1 = -\frac{\left(\left(-1+\delta\right)^2+\lambda\right)\left(r\left(-1+\delta\right)\ln r-\left(-1+r\right)\delta\ln\delta\right)}{\left(-1+\delta\right)^4}, \tag{55}$$

$$\widehat{w}_2 = \frac{1}{2\left(-1+\delta\right)^7}(-(6r\lambda - 24r\delta\lambda + 36r\delta^2\lambda - 24r\delta^3\lambda + 6r\delta^4\lambda + 6r\lambda^2 - 12r\delta\lambda^2 + 6r\delta^2\lambda^2)\ln r$$
$$-(24r\delta^2\lambda + 16r\delta^3\lambda - 4r\delta^4\lambda\ln - 3r\lambda^2 + 6r\delta\lambda^2 + 3r\delta^2\lambda^2)\ln r^2 - (6r\delta\lambda + 18\delta^2\lambda - 18r\delta^2$$
$$18\delta^3\lambda + 18r\delta^3\lambda + 6\delta^4\lambda - 6r\delta^4\lambda - 6\delta\lambda^2 + 6r\delta\lambda^2 + 6\delta^2\lambda^2 - 6r\delta^2\lambda^2 + 6\delta\lambda)\ln\delta - (2r\delta - 10r\delta^2$$
$$+20r\delta^3 - 20r\delta^4 + 10r\delta^5 - 2r\delta^6 + 8r\delta\lambda - 24r\delta^2\lambda + 24r\delta^3\lambda - 8r\delta^4\lambda + 6r\delta\lambda^2 - 6r\delta^2\lambda^2)\ln r\ln\delta$$
$$+(\delta - r\delta - 3\delta^2 + 3r\delta^2 + 2\delta^{32} - 2r\delta^{32} + 2\delta^4 - 2r\delta^4 - 3\delta^5 + 3r\delta^5 + \delta^{62} - r\delta^6 + 4\delta\lambda - 4r\delta\lambda^2 -$$

$$4\delta^2\lambda+4r\delta^2\lambda-4\delta^3\lambda+4r\delta^3\lambda+4\delta^4\lambda-4r\delta^4\lambda+3\delta\lambda^2-3r\delta\lambda^2+3\delta^2\lambda^2-3r\delta^2\lambda^2)\ln\delta^2. \quad (56)$$

The total velocity for $n = 3$ is:

$$\widehat{w}=\frac{\delta-r}{\delta-1}-\frac{\left(\left(-1+\delta\right)^2+\lambda\right)\left(r\left(-1+\delta\right)\ln r-\left(-1+r\right)\delta\ln\delta\right)}{\left(-1+\delta\right)^4}+\frac{1}{2\left(-1+\delta\right)^7}(-(6r\lambda-$$
$$24r\delta\lambda+36r\delta^2\lambda-24r\delta^3\lambda+6r\delta^4\lambda+6r\lambda^2-12r\delta\lambda^2+6r\delta^2\lambda^2)\ln r-(24r\delta^2\lambda+16r\delta^3\lambda$$
$$-4r\delta^4\lambda\ln-3r\lambda^2+6r\delta\lambda^2+3r\delta^2\lambda^2)\ln r^2-(6r\delta\lambda+18\delta^2\lambda-18r\delta^2\lambda-18\delta^3\lambda+18r\delta^3\lambda+$$
$$6\delta^4\lambda-6r\delta^4\lambda-6\delta\lambda^2+6r\delta\lambda^2+6\delta^2\lambda^2-6r\delta^2\lambda^2+6\delta\lambda)\ln\delta-(2r\delta-10r\delta^2+20r\delta^3-$$
$$20r\delta^4+10r\delta^5-2r\delta^6+8r\delta\lambda-24r\delta^2\lambda+24r\delta^3\lambda-8r\delta^4\lambda+6r\delta\lambda^2-6r\delta^2\lambda^2)\ln r\ln\delta+(\delta$$
$$-r\delta-3\delta^2+3r\delta^2+2\delta^{32}-2r\delta^{32}+2\delta^4-2r\delta^4-3\delta^5+3r\delta^5+\delta^{62}-r\delta^6+4\delta\lambda-4r\delta\lambda^2-$$
$$4\delta^2\lambda+4r\delta^2\lambda-4\delta^3\lambda+4r\delta^3\lambda+4\delta^4\lambda-4r\delta^4\lambda+3\delta\lambda^2-3r\delta\lambda^2+3\delta^2\lambda^2-3r\delta^2\lambda^2)\ln\delta^2. \quad (57)$$

Zeroth, first and second order solution for $n = 4$respectivelyare:

$$\widehat{w}_0=\frac{\delta-r}{\delta-1}, \quad (58)$$

$$\widehat{w}_1=\frac{\left(\left(-1+\delta\right)^3-\lambda\right)\left(r\left(-1+\delta\right)\ln r-\left(-1+r\right)\delta\ln\delta\right)}{\left(-1+\delta\right)^5}, \quad (59)$$

$$\widehat{w}_2=\frac{1}{2\left(-1+\delta\right)^9}((-8r\lambda+40r\delta\lambda-80r\delta^2\lambda+80r\delta^3\lambda-40r\delta^4\lambda+8r\delta^5\lambda-8r\lambda^2+16r\delta\lambda^2$$
$$-8r\delta^2\lambda^2)\ln r-(r-8r\delta+28r\delta^2-56r\delta^3+-70r\delta^4-56r\delta^5+28r\delta^6-8r\delta^7+r\delta^8+5r\lambda$$
$$-25r\delta\lambda+50r\delta^2\lambda-50r\delta^3\lambda+25r\delta^4\lambda-5r\delta^5\lambda+4r\lambda^2-8r\delta\lambda^2+4r\delta^2\lambda^2)\ln r^2+(8\delta\lambda-$$
$$8r\delta\lambda-32\delta^2\lambda+32r\delta^2\lambda+48\delta^3\lambda-48r\delta^3\lambda-32\delta^4\lambda+32r\delta^4\lambda+8\delta^5\lambda-8r\delta^5\lambda+8\delta\lambda^2-$$
$$8r\delta\lambda^2-8\delta^2\lambda^2+8r\delta^2\lambda^2)\ln\delta-(2r\delta-14r\delta^2+42r\delta^3-70r\delta^4+70r\delta^5-42r\delta^6+14r\delta^7$$
$$-2r\delta^8+10r\delta\lambda-40r\delta^2\lambda+60r\delta^3\lambda-40r\delta^4\lambda+10r\delta^5\lambda+8r\delta\lambda^2-8r\delta^2\lambda^2)\ln r\ln\delta+(\delta-$$
$$r\delta-5\delta^2+5r\delta^2+9\delta^3-9r\delta^3-5\delta^4+5r\delta^4-5\delta^5+5r\delta^5+9\delta^6-9r\delta^6-5\delta^7+5r\delta^7+\delta^8$$
$$-r\delta^8+5\delta\lambda-5r\delta\lambda-10\delta^2\lambda+10r\delta^2\lambda+10\delta^4\lambda-10r\delta^4\lambda-5\delta^5\lambda+5r\delta^5\lambda+4\delta\lambda^2-4r\delta\lambda^2+$$
$$4\delta^2\lambda^2-4r\delta^2\lambda^2)\ln\delta^2. \quad (60)$$

The total velocity for $n = 4$.

$$\widehat{w}=\frac{\delta-r}{\delta-1}-\frac{\left(\left(-1+\delta\right)^3-\lambda\right)\left(r\left(\delta-1\right)\ln r-\left(r-1\right)\delta\ln\delta\right)}{\left(-1+\delta\right)^5}+\frac{1}{2\left(-1+\delta\right)^9}((-8r\lambda+$$
$$40r\delta\lambda-80r\delta^2\lambda+80r\delta^3\lambda-40r\delta^4\lambda+8r\delta^5\lambda-8r\lambda^2+16r\delta\lambda^2-8r\delta^2\lambda^2)\ln r-(r-$$
$$8r\delta+28r\delta^2-56r\delta^3+-70r\delta^4-56r\delta^5+28r\delta^6-8r\delta^7+r\delta^8+5r\lambda-25r\delta\lambda+50r\delta^2\lambda$$
$$-50r\delta^3\lambda+25r\delta^4\lambda-5r\delta^5\lambda+4r\lambda^2-8r\delta\lambda^2+4r\delta^2\lambda^2)\ln r^2+(8\delta\lambda-8r\delta\lambda-32\delta^2\lambda+$$
$$32r\delta^2\lambda+48\delta^3\lambda-48r\delta^3\lambda-32\delta^4\lambda+32r\delta^4\lambda+8\delta^5\lambda-8r\delta^5\lambda+8\delta\lambda^2-8r\delta\lambda^2-8\delta^2\lambda^2$$
$$+8r\delta^2\lambda^2)\ln\delta-(2r\delta-14r\delta^2+42r\delta^3-70r\delta^4+70r\delta^5-42r\delta^6+14r\delta^7-2r\delta^8+10r\delta\lambda$$
$$-40r\delta^2\lambda+60r\delta^3\lambda-40r\delta^4\lambda+10r\delta^5\lambda+8r\delta\lambda^2-8r\delta^2\lambda^2)\ln r\ln\delta+(\delta-r\delta-5\delta^2+5r\delta^2+$$
$$9\delta^3-9r\delta^3-5\delta^4+5r\delta^4-5\delta^5+5r\delta^5+9\delta^6-9r\delta^6-5\delta^7+5r\delta^7+\delta^{82}-r\delta^8+5\delta\lambda-5r\delta\lambda$$
$$-10\delta^2\lambda+10r\delta^2\lambda+10\delta^4\lambda-10r\delta^4\lambda-5\delta^5\lambda+5r\delta^5\lambda+4\delta\lambda^2-4r\delta\lambda^2+4\delta^2\lambda^2-4r\delta^2\lambda^2)\ln\delta^2. \quad (61)$$

6. RESULTS AND DISCUSSION

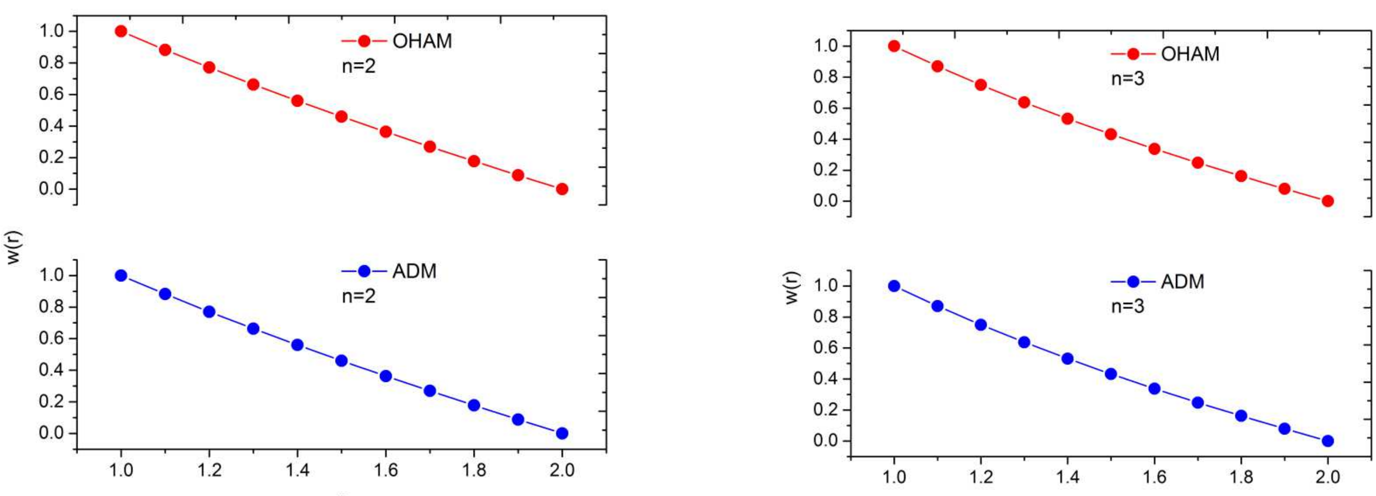
The nonlinear equation () corresponding to the boundary conditions () and () has been solved analytically by utilizing the Adomian Decomposition Method for various numerical values of the physical parameters. Here, we include the discussion on an analytical basis obtained by ADM. Velocity profiles, shear stress, force on the total wire and thickness of the coated wire are displayed graphically. The effect of emerging parameters such as power law index, material parameter of Sisko fluid, radii ratio and the speed of the wire will be discussed in detail.Additionally, Newtonian ($\lambda = 0$) and sisko ($\lambda \neq 0$) fluids are also compared.

The convergence of the series solution is established in table 1. For the accuracy of ADM, a comparison of the present result is made with OHAM and the published work of Sajid et al. [25] as shown in table 2. The results are found in a very good agreement.

Graphical comparisonof ADM and OHAM is shown in figure 3. The effects of material parameter λandthe power index(n) on the velocity profile is depicted in figure4. In figure4, the velocity profile decreases as either of the power index or material parametervalue increases. The effect of material parameter λonvelocityprofileis shown in figures 5-7 by taking three different values of n. These figures show the comparison between the Newtonian fluid (when $\lambda = 0$) and the Sisko fluid (when $\lambda \neq 0$). It is also observed from these figures, that velocity profiles decreasessignificantly by increasingthe power index and material parameter values. The reduction in velocity when $n = 4$ is less than that of when$n = 2$ and $n = 3$. This shows the shear-thickening occurrence of the underlying non-Newtonian fluid.

Thickness of the coated wire is a function of material parameterλ, the power index n and the radii ratio δ. Figures8-10 illustrate the effect of these parameters on the thickness of the coated wire. Figure 8 shows the effects of the power index and material parameter on the thickness of coated wire. In this analysis, it is clear that the thickness of the coated wire increases as either the power index or material parameter increases. Figure 9 shows the effects of enlarging the material parameter and the power index by increasing the radii ratio on the thickness of coated wire. From this simulation, it is observed that the material parameter, power index and radii ratio significantly affects the thickness of coated wire. Figure 10 is drawn to see the impact of the radii ratio and the wire drawing speed on thickness of the coated wire. It is observed that, the increase in radii ratio significantly affects the thickness of coated wire. Also, it is investigated that by changing the wire drawing speed, less sensitivity in the change of wirecoating occurs.In this case when the radii ratio (especially when the diameter of the coating die) is small.Figures 11 and 12 display the impact of the power index and material parameter on the shear stress and the total force on the surface of the coated wire respectively. It is observed that the shear stress and the total force on the surface of coated wire exhibits a linear increase with increasing power index and material parameter.

Figures 11 and 12 show the linear effect of power index and material parameter on the shear stress and total force on the surface of the wire with increasing power index and material parameter.



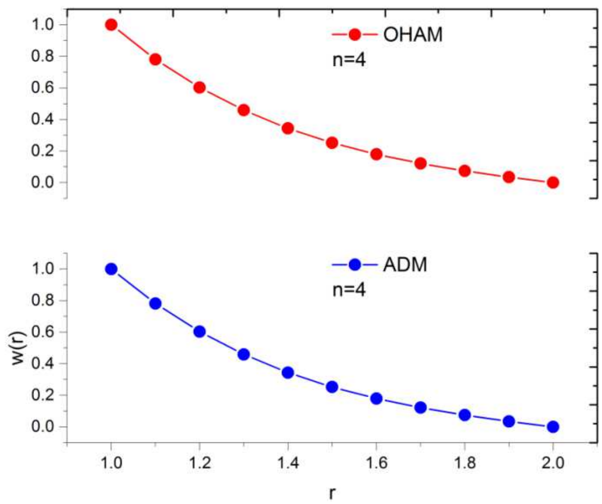


Figure 3. Velocity comparison of ADM and OHAM.

Table 1. ADM error for $\lambda = 0.4, n = 0.2, \delta = 2.$

r	**1st Order**	**2nd Order**
1	**0**	**0**
1.1	5.2413E-13	3.242E-14
1.2	0.1602E-10	1.028E-13
1.3	1.2012E-10	0.102E-12
1.4	4.1202E-11	1.246E-12
1.5	3.0234E-10	2.102E-12
1.6	2.1028E-11	1.812E-12
1.7	5.2139E-10	0.211E-12
1.8	1.0123E-11	0.224E-13
1.9	3.2450E-12	0.724E-13
2	0.4535E-13	0.317E-14

Table 2: Numerical comparison of velocity distribution between OHAM, ADM and Sajid et al. [25] when $\lambda = 0.4, n = 0.2, \delta = 2.$

r	**OHAM**	**ADM**	**Sajid et al. [25]**
1	**1**	**1**	**1**
1.1	0.01263242	0.01263242	0.01263242
1.2	0.03647282	0.03647282	0.03647280
1.3	0.02894526	0.02894525	0.02894514
1.4	0.011607241	0.011607240	0.011607220
1.5	0.010442045	0.010442035	0.010442012
1.6	0.001252401	0.001252400	0.001252401
1.7	0.006014981	0.006014981	0.006014981
1.8	0.004101612	0.004100632	0.004100630
1.9	0.000213520	0.000213520	0.000213521
2.0	0.000012421	0.000012421	0.000012420

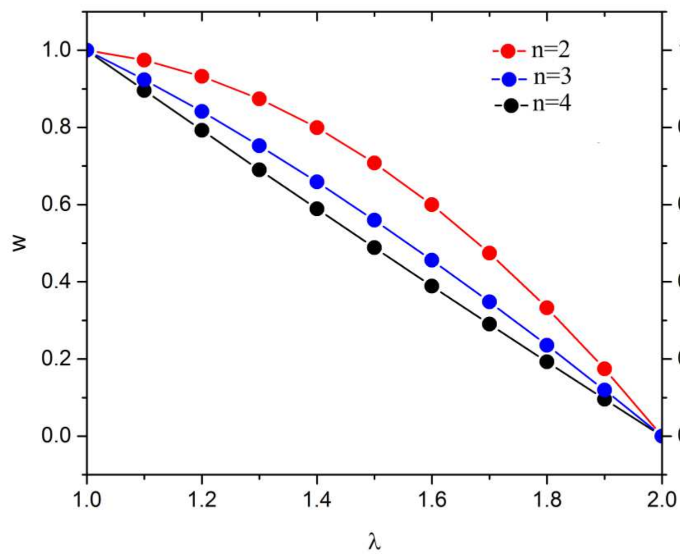


Fig. 4. Effect on dimensionless velocity profiles for different values of power index n when $\delta = 2, \lambda = 0.2$.

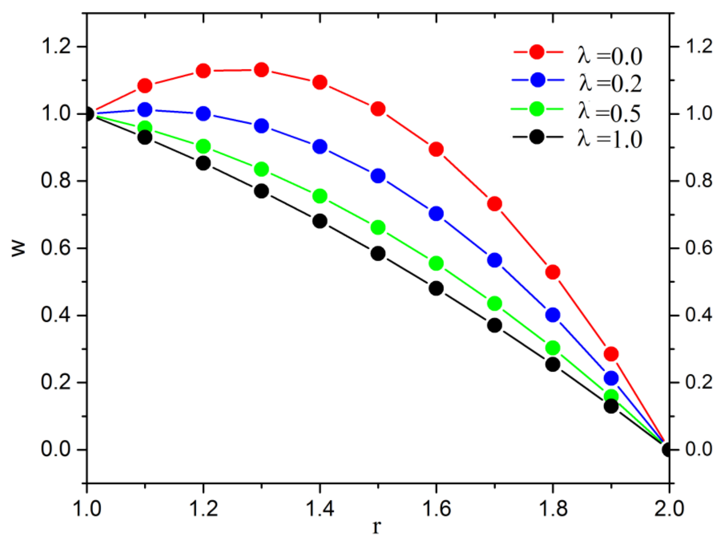


Fig. 5. Dimensionless velocity profiles by taking different values of material parameter λ when $\delta = 2, n = 2$.

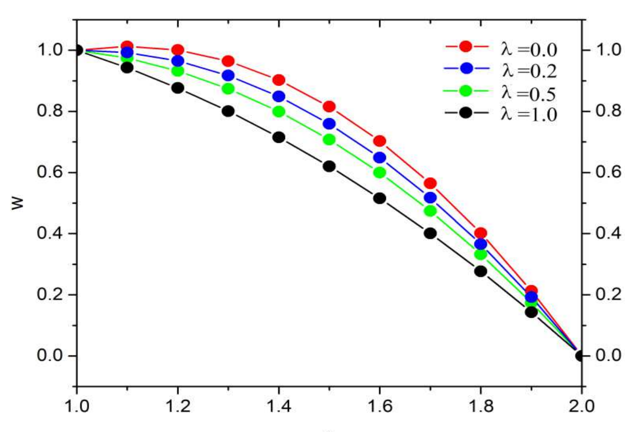


Fig. 6. Dimensionless velocity profiles by taking different values of material parameter λ when $\delta = 2, n = 3$.

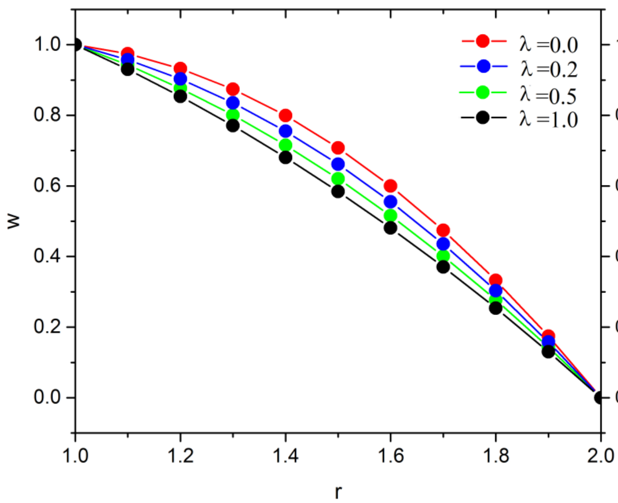


Fig. 7. Effect on dimensionless velocity profiles by taking various values of material parameter λ when $\delta = 2, n = 4$.

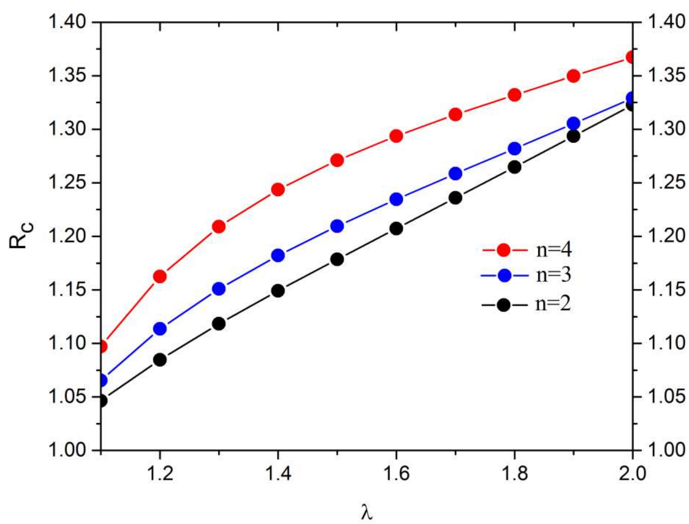


Fig. 8. Thickness of the coated wire for the different values of the power index n verses λ when $\delta = 2$.

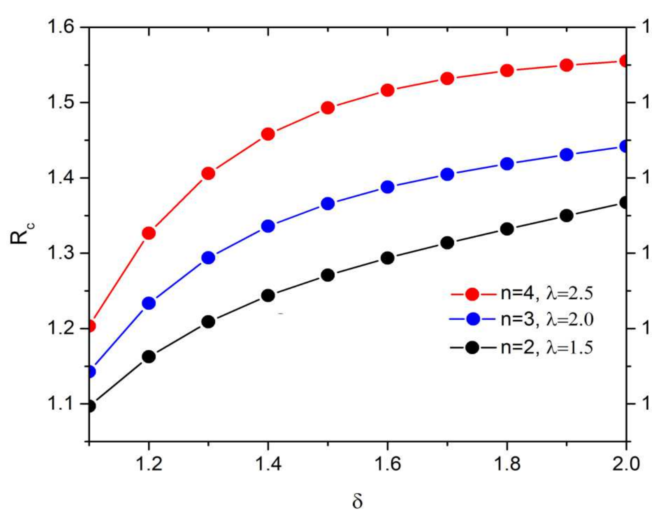


Fig. 9. Effect on the thickness of the coated wire for different values of the power index n and material parameter λ when $\delta = 2$.

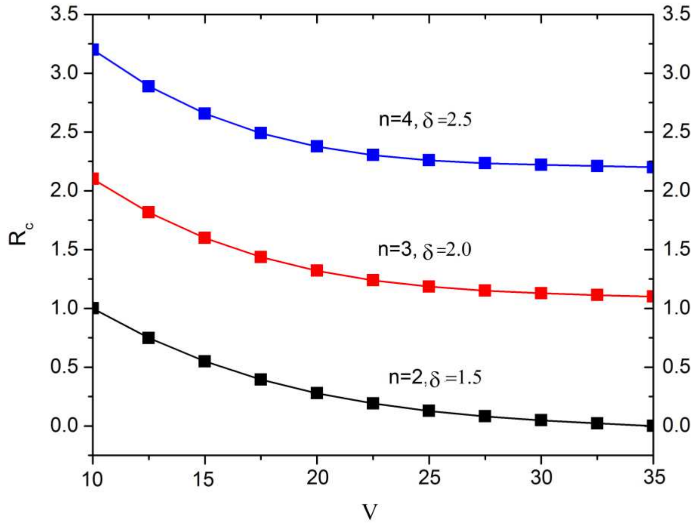


Fig. 10. Effect on the thickness of the coated wire for the different values of the radii ratio δ verses speed of wire V when $\lambda = 0.2$.

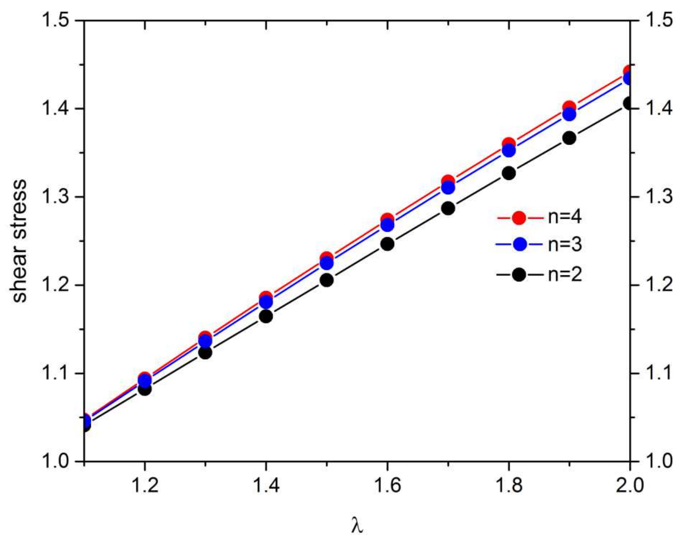


Fig. 11. Effect of the power index n and material parameter λ on the shear stress on the bare wire surface when $\delta = 2$.

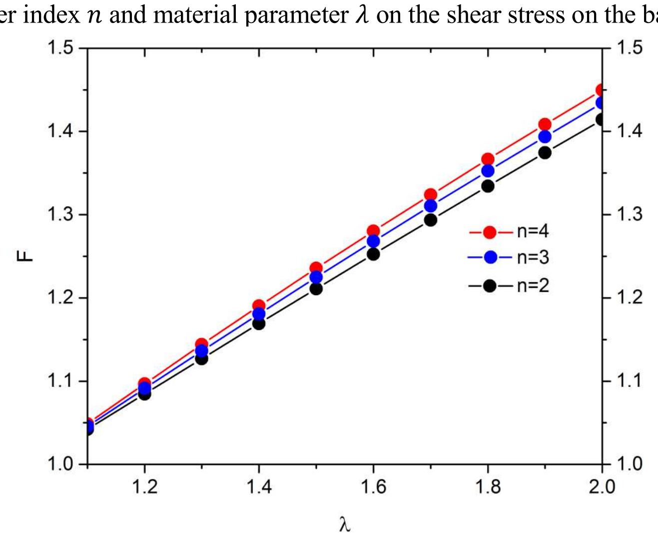


Fig. 12. Effects of power index n and material parameter λ on the total force on the surface of coated wire when $\delta = 2$.

7. Conclusion

The wire coating analysis in pressure type coating die is investigated using Sisko fluid as a coating material. The nonlinear differential equation is solved by ADM. The consequences are also verified by OHAM. The effect of emerging parameters is discussed and sketched. Velocity profile decreases with increasing power index n and material parameter λ. The velocity profile for Newtonian fluid ($\lambda = 0$) is much greater than the Sisko fluid ($\lambda \neq 0$). It is also observed that the thickness of the coated wire significantly depends on the power index n, die radius, material parameter λ, radii ratio and the wire drawing speed V. Also the shear stress and total force on the surface of coated wire increases with power index and material parameter.

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