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A Review on the Default Boundary and Approaches Used in the Credit Risk Models

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Received: May 19, 2017 Accepted: July 18,2017

ABSTRACT

Financial risk is the term for the risk related to financing, including default risk in a company's loan. Risk is a term used to reflect the potential for financial loss and the uncertainty of a return. There are many types of financial risk such as credit risk, operational risk, market risk, liquidity risk and so on. Thus, this paper reviews the evolution of credit risk in the academic literature especially on credit default risk. Here, credit default risk is when an issuer does not make payment on the loan when the time comes. Due to that, there are three main approaches to modeling credit risk which are structural approach, incomplete information approach and reduced form approach in order to curb the credit default risk situation. However, the objective of this review paper is to discuss on the parameters used in the credit risk models especially on the default boundary for structural approach only. This is because the default boundary is the main critical input in order to determine the level of the firm's asset that causes the firm to default when this value of the firm's asset dropped to this default boundary level. Besides that, this paper also provides the progress within the framework of structural approach especially on the Black-Scholes model, Merton model and the KMV-Merton model. This also concludes that these credit risk models can estimate and monitor the credit default risk for any company so that the banking industry may reduce bad investments and losses.

KEYWORDS: Credit Risk, Default Boundary, Financial Risk, Structural Approach, Credit Default Risk.

INTRODUCTION

The existing literature on credit risk assessment starts when risky loans occur in the application of financial derivatives such as loans at risk of default by the firm.

Therefore, many scholars participate actively in here to resolve the credit risk by developing the credit risk models in term of quantitative analysis. There are three main approaches to modelling credit risk which are structural approach, reduced form approach and incomplete information approach that attempt to combine the structural approach and reduced form approach.

Generally, a good firm's financial position is when the firm's asset is above a certain threshold, which is referred to the default boundary. There are numerous approaches used by the previous research to estimate the parameters used in the credit risk models especially for default boundary conditions. Here, the default boundary is when the value of the firm's asset dropped to this boundary, and then the firm will default. So, the default boundary needs to be studied empirically in order to understand its properties. This is because even the firm's asset value dropped to this default boundary, but the firm is still continuing to trade transactions.

Thus, the objective of this paper is to review on the default boundary used by the previous authors in the credit risk models. The outlines of this review paper are as follows: section 2 discusses the default boundary used in the credit risk models, section 3 presents the contributions done on the Black-Scholes model and in section 4, this review conclude.

METHODOLOGY

In terms of structural approach, the most widely used model in credit default risk is the Merton model initiated by Robert C. Merton, which is modelled the behavior of the firm's value. In [8, 27] have developed an approach based on the structure of the contingent claims approach which is a generalization of the theory of option pricing model.

In the academic literature, there are two types of default boundary which are an endogenous default boundary and an exogenous default boundary. An endogenous is where the default is chosen by managers to maximize the value of equity [22,18]. Subsequently, Merton model is followed by [7, 22] which are the two major contributions within the framework of Merton. In [7] have introduced the safety covenants where the default

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time is uncertain or infinite maturity when the firm's assets below certain default point. This is an example of an endogenous default boundary. As mentioned before, this kind of default boundary is when the default is chosen by managers to maximize the value of equity. Therefore, there are some variables that need to be considered such as the maturity date, default costs and riskiness of firm's operation so that the default boundary is not determined based on the principal of debt only. Next, an arbitrary maturity is considered [24] by extending the investigation for debt issued by the corporate firm. So, the default boundary for an endogenous is as shown below:

$$V = \frac{\left(\left(\frac{c}{r}\right)\left(\frac{A}{rT}\right) - B\right) - \frac{AP}{(rT)} - \varpi \frac{x}{r}}{1 + \alpha x - (1 - \alpha)B}$$
(1)

where variable A is equal to

$$2ae^{-rT}N[a\sigma\sqrt{T}] - 2zN[z\sigma\sqrt{T}] - \frac{2}{\sigma\sqrt{T}}n[z\sigma\sqrt{T}] + \frac{2e^{-rT}}{\sigma\sqrt{T}}n[a\sigma\sqrt{T}] + (z-a)$$

and B equal to

$$-\left(2z+\frac{2}{z\sigma^2T}\right)N[z\sigma\sqrt{T}]-\frac{2}{\sigma\sqrt{T}}n[z\sigma\sqrt{T}]+$$
$$(z-a)+\frac{1}{z\sigma^2T}.$$

Furthermore,

$$a = \frac{\left(r - \delta - \left(\frac{\sigma^2}{2}\right)\right)}{\sigma^2}$$

where as

$$z = \frac{\left(\left(a\sigma^2\right)^2 + 2r\sigma^2\right)^{\frac{1}{2}}}{\sigma^2}$$

and meaning of the other variables involved as mentioned above are referred to the list of symbols.

Apart from that, an exogenous is referring to the par value of debt obligation. For an example, in [8, 27] use an exogenous default boundary. Here, both models are limited to zero coupon bond so that there is no default boundary until the maturity date comes. So, if the value of the firm's asset drops to the default boundary or the par value of debt obligation at the maturity date, then the firm will immediately default.

Later on, default may happen before the maturity date where both default boundaries include the coupon payment of debt so that the risk of default can be monitored continuously rather than estimating the risk only at the maturity date. For an example, in [26] use an exogenous default boundary with the coupon payment and the default boundary through time refers to the par value of debt obligation. Same goes here; the firm will default when the value of firm drops to this barrier. However, default barrier is more precise if a constant proportion of the level for debt principal is included as mentioned by [19] and the default boundary V is assumed below

$$V = kP \tag{2}$$

where k is the specified fraction of the debt obligation, P. Furthermore, the variable k is the best way to describe the observe default rate for the corporate debt.

Later on, the Moody's-KMV as mentioned by [10] calculating the default boundary as shown below:

$$B = ST + \frac{1}{2}(LT) \tag{3}$$

where ST is the par value of short term liabilities and LT is the par value of long term liabilities instead of default boundary is the principal debt at maturity date by [27].

In terms of structural approach, the study of credit risk is done on the model developed by [8]. In fact, the Black-Scholes model in 1973 was the successful original model in the finance field. The starting point to derive the Black-Scholes model is a stochastic process, the random behavior of assets and basic stochastic calculus, as discussed by [36]. In [8] model, which is refers to a European call option is shown as below:

$$C = S(0)N(d_{1}) - Xe^{-rT}N(d_{2}),$$

$$d_{1} = -(a - \sigma\sqrt{T}),$$

$$d_{2} = -a,$$
(5)
$$a = \frac{\ln\left(\frac{X}{S(0)}\right) - (r - \frac{1}{2}\sigma^{2})T}{\sqrt{T}}.$$
(6)

and

The most widely used model in credit default risk is the Merton model initiated by [27], extended the [8] model to be used in the simplest form of corporate debt where [27] model is shown by [8] as stated in the Equation (4), (5) and (6) by describing the company's equity(E) as a European call option \bigcirc , the face value or the principal amount of debt(D) as the exercise price(X) and the company's value(V) as the stock price(S) which is

 $\sigma \sqrt{T}$

$$E = V(0)N(d_1) - De^{-rT}N(d_2)$$
$$d_1 = -(a - \sigma\sqrt{T})$$
$$d_2 = -a.$$

and

There are many credit risk models that have been developed to reduce credit default in the world, but this section includes only the major contributions that extend the framework of the [8, 27].

There are several models of industrial credit measurement such as Moody's KMV, Credit Metrics, Credit Portfolio View, Credit Risk + and so on. Thus, this review also undertakes to focus on the industrial model of corporate debt, which is the KMV model as it is believed that this type of credit risk model can address the issue of insufficient statistics for the default probability as a result of the failure to come up with the better credit risk model. While others simply refer to this model as Merton model, in [6] prefer to call it as KMV-Merton model because for obtaining KMV-Merton default probabilities from equity data observed are nontrivial extension of the ideas in the classic Merton model. The founders of KMV have developed this extension from the classic Merton model and they deserve some credit for the development.

In terms of credit risk management of default credit risk, the KMV-Merton model needs to be considered in estimating the probability of default. Application framework of Merton model for valuation of corporate debt has contributed to the progress in the field of risk management in order to reduce the default credit risk. In order to determine the default probability for companies, researches and models are studied by [11].

In 2002, KMV-Merton model has been purchased by Moody's, the KMV Corporation. KMV-Merton model developed by KMV Corporation based on the Merton model bond pricing. KMV-Merton model has been used widely by many parties in many countries in predicting the default probability for corporate liabilities. There are a lot of authors who have been interested in KMV-Merton model [5, 11]. The most critical inputs to the KMV-Merton model are clearly the asset value, default boundary or leverage and asset volatility as indicated by [11]. As the market value of impaired assets decreases, the probability of default increases and vice versa. The KMV-Merton model is shown as below:

$$P_t = N \left[-\frac{\ln \frac{V_t}{B} + \left(\mu - \frac{1}{2}\sigma^2\right)t}{\sigma\sqrt{t}} \right] (7)$$

and the distance to default (DD) is

$$DD = \frac{\ln \frac{V_t}{B} + \left(\mu - \frac{1}{2}\sigma^2\right)t}{\sigma\sqrt{t}}.$$
(8)

Here, the Equation in (7) and (8) will be used to calculate the credit default risk probability of a company for each end of a year continuously.

RESULTS AND DISCUSSION

According to [12], it found that the observed boundary levels significantly depend on the volatility of asset as well as the costs of default. In addition, the volatility of the asset at default time that describes quantitatively is mediocre and the cross-section of default still has not precisely explained. Then, he proposed a model for default boundary. As a result, the model gives more precise predictions or there is an improvement regarding to the cross-sectional variation. However, there are some cross-sectional errors happen here due to the dependence of the model on the market value only. So, the precise default boundary needs to be considered in order to get more precise of default probability.

Subsequently, in [1] find that the measurement of expected return of asset and its volatility gives more impact on the ability of the model to predict default probabilities of firms rather than the default boundary. Furthermore, the firm's asset of structural approaches tends to reflect the market condition and then leads to capture the default risk of that firm.

Some researchers state that the KMV-Merton model produces insufficient statistics in determining credit risk, can only do better for the high number of companies that are good, has problems in distinguishing the probability of default of companies with good performance and normal, and violate the basic theory of risk return. According to [6], KMV-Merton model does not generate sufficient statistics to predict the probability of default. Therefore, there is a reduced form model, which is a better predictor of default predictions rather than the structural model to address this issue in the KMV-Merton model. Furthermore, the lack of knowledge of the statistical properties of the forecast default in KMV-Merton model, can contribute to insufficient statistics. In [5] have developed a model for forecasting default and distance to default as covariate to check the accuracy of the Merton model. Here, there is a weakness in this model which is a model that does not have sufficient statistical probability of default, but it is possible to improve the probability of default in the classic Merton [27] model by conditioning on other independent variables. So, this model is tested continuously up till now to ensure the ability of this model to carry out in any environment and also to identify the variables that contribute to the probability of credit default.

KMV-Merton model has been used widely by many parties in many countries in predicting the probability of default for corporate liability. In [25] predict default of small and medium sized businesses in the United Kingdom. In [4] estimates the default probability for KPN Company in the Netherlands. However, there are still some weaknesses in this model as it cannot be used for any type of case. The study conducted by [31] has extended the KMV-Merton model to the long-term debt as well as short-term debt. It shows that this model can be used to rate companies rather than in estimating the probability of default among companies.

In [29] have analysed the credit risk of Malaysian companies using the Merton model and found that there are significant flaws in the model in which some of the results do not represent the real situation of the credit risk of companies. Apart from that, in [30] have predicted probability of default using KMV-Merton and their study shows that KMV-Merton cannot be used for several companies.

In [28] has investigated the probability of credit default of state bank in Indonesia from the year of 2002 up till the year 2010. The credit default probabilities are estimated using the Merton model, and there are inherent limitations associated with the impact of the financial crisis, oil prices and the volatility index affects the probability of default. Thus, any researcher should consider this proposal to investigate these effects empirically.

Instead, in [35] have shown that the probability of default in KMV-Merton model can authorize significant prediction in a longer period. Based on empirical research, KMV-Merton model performed by other scholars and measuring accuracy and reliability are not proven without any doubt. Therefore, modifications should be done on the KMV-Merton model to overcome the weaknesses discovered in this review paper.

However, during the investigation, the review also considers modifications done by the researchers on KMV-Merton model. In [34]use Merton [27] model to calculate the monthly Default Likelihood Indicator, the likelihood of default for individual firms and examine the impact of the risk of default may have on equity returns. As a result, the default is worth considering in variable-rate asset test, above and beyond the size and book-to-market (BM). Subsequently, in [15] have shown that KMV is approximately equal to the maximum likelihood estimates Maximum Likelihood Estimation (MLE) developed by [14]. However, the KMV algorithm is silent about the nature of the distribution properties compared with MLE method. Instead, the MLE approach is flexible and can be easily applied to different credit risk model structure.

After that, in [31] explains the basic idea of the KMV under two different models, and extends Merton's KMV to fit into the firm, which has two classes of the debts. In conclusion, KMV has the ability to forecast the firm's default. [21]has proposed a new method based on genetic algorithms to solve the problem of optimal default point for when the KMV model may vary from country to country. Here, the percentage of accuracy of genetic algorithm-based KMV model (GA-KMV model) is higher than those for the other two models, namely quantile regression-based KMV model (QR-KMV) and KMV model.

In [37]have amended the model by using the interest rate volatility rather than a fixed rate on the basis of the existing research. Comparing the uncertainty KMV model to the traditional model with listed companies are regarded as default samples and non-listed companies are regarded as normal samples in Shanghai and Shenzhen stock exchanges. It find that it performs slightly better as a predictor in the uncertainty KMV model and in samples of forecasts. In this way, these scholars still continue researching on uncertainty interest rate.

In fact, KMV Corporation or Moody's KMV Corporation has developed the KMV Merton model based on a model, which is made by [27] approach. In [11] find that the credit default risk probability by the debtor is too small to be produced by [27] approach for use in industry. Apart from that, KMV calculates distance to default (DD) using database around the world which contains much information of corporate default to determine the probability of default (PD) associated with each level of default compared with [27] model that only uses the normal distribution function to calculate PD.

Unlike other models, structure approach that combines market data as an essential component and makes the structure model responsive to environment factor. This is a great strength on most models because it allows any financial institution to act more quickly in identifying problems.

In [19] find that the structure models produce low default probabilities for investment-grade bonds. In [13] state that DD is insufficient decisive to determine the credit quality of a company, but by adding the additional independent variables will result in a better outcome.

In [32] finds that DD is not decisive enough of default determinant due to many companies that are still continuing to trade while technically almost bankrupt as liabilities exceed assets. In this case, by calibrating DD to a more authentic source, then maybe the accuracy of the structural model can be modified.

In [2] find that the probability of default generated by the structural model decreases during the pre-global financial crisis (GFC), but the default probability by that model increases during the GFC. So, the structural model still cannot produce accurate default probabilities due to the insufficient causality. Apart from that, in [3] also find that the accuracy of the structural model is medium because it does fluctuate with market, but can overstate or understate default risk depending on market volatility.

CONCLUSION AND RECOMMENDATIONS

The studies on the default boundary need to specify a more precise boundary condition in order to reduce the cross-sectional errors for predicting the default probability of the company, which is slightly better than the existing model. Apart from that, a review of the contributions within the framework of structural approach gives an insight for researchers to enhance the existing models in measuring the credit risk especially for banks to reduce losses and bad investments. This is why the default probability models can be quite critical in providing the status of firm.

However, most researches do not intend to eliminate the risk altogether by using any financial model, but only a better picture for banks in credit risk monitoring, particularly in cases of loans approval.

ACKNOWLEDGEMENT

Our sincere appreciation is directed to the Ministry of Education, Malaysia and Universiti Teknologi MARA, Shah Alam, Selangor, Malaysia. This research is funded by Fundamental Research Grant Scheme 600-RMI/FRGS 5/3 (65/2013).

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