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Soft Ultra-Fılters and Soft G-Fılter of MTL-Algebras

M. Uzair Khan[*,1], Raees Khan[1], Imran Khan[1], Zahoor Ahmad[2], Zia Ur Rahman[3], Shahid Ali[3]
[1]Department of Mathematics &Statistics, Bacha Khan University, Charsadda, KPK, Pakistan[1]
[2]Department of Geology & Geophysics, Bacha Khan University, Charsadda, KPK, Pakistan
[3]Department of Computer Science, Bacha Khan University, Charsadda, KPK, Pakistan[1]

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ABSTRACT

The aim of this paper is to present the ideas of soft ultra filters & soft G filters in MTL Algebras, some examples are given and some results are proved using these concepts.
KEYWORDS:MTL-algebras, Soft sets, Soft filters, Soft Ultra Filters,Soft Prime filters, Soft G-filters.

1. INTRODUCTION

The logic MTL, Monoidal t-norm based logic was presented by F. Estva and L. Godo and discuss several properties of MTL-algebra in [6] The Boolean logic (BL) was introduced by P. Hajek and discuss the properties in [1].The fuzzy reasoning and implication operators was discussed by Ying in ([2],[3]). A proper logical system for fuzzy propositional calculus and a innovative arithmetical configurations (R_0-algebras) is proposed by Wang (see [4] and [5]).In ([7] [8]) Jun and Zhang deliberate the fuzzy configuration of filters and further studied the classifications of fuzzy filters in MTL-algebras. Jun and Zhang also explored the fuzzi-fication of Boolean & MV-filters. In paper [10] Moldtsov gives the idea of soft set theory. Maji-et-al further work on soft set and defines some operations on soft set (see [11]). These operations were corrected by Ali et al [12].X.H. Zhang et al gives the concept of fuzzy ultra filters and fuzzy G-filters in [9] as a continuation of this research paper, we additional study Soft ultra filters, soft prime filters and soft G-filters in a monoidal t-norm logic (MTL) algebras. We gives some examples and prove some results.

2. PREMİLİNRİES

We reminiscence some concepts and their significant properties, by a *lattice* we mean a moderatelyorderly set in which every two components has a supremum& infimum (this may be alsoidentified assmallest upper join &highest lower meet, respectively)

By a *residuted framework* by mean a lattice $E=(E,\le,\wedge,\vee,\otimes,\to,0,1)$ having the smallest component 0 and the biggestcomponent 1, &capable with the two binary processes $\otimes$ (called *product*) and $\to$ (called *residuum*) like,

(i) $\otimes$ is an isotone, commutative & associative.

(ii) $x\otimes 1=x$ for all $x\in E$.

(iii) The Galois correspondence grips, which is

$$x\otimes y\le z\Rightarrow x\le y\to z$$

for all z, y, x $\in E$.

2.1 Definition [6]

A residuated framework $E=(E,\le,\wedge,\vee,\otimes,\to,0,1)$ is entitled an MTL-algebra if it fulfills the pre-linearity equation

$$(x\to y)\vee(y\to x)=1$$

for all $x,y\in E$.

2.2 Proposition [6, 9]

The subsequent possessions grip in any resituated lattice $E=(E,\le,\wedge,\vee,\otimes,\to,0,1)$

Corresponding author: M. Uzair Khan, Department of Mathematics &Statistics, Bacha Khan University, Charsadda, KPK, Pakistan. uzairqau@gmail.com

(u_1) $x \le y \Leftrightarrow x \to y = 1.$

(u_2) $0 \to x = 1, 1 \to x = x, x \to (y \to x) = 1.$

(u_3) $y \le (y \to x) \to x.$

(u_4) $x \to (y \to z) = (x \otimes y) \to z = y \to (x \to z).$

(u_5) $x \to y \le (z \to x) \to (z \to y), (x \to y) \le (y \to z) \to (x \to z).$

(u_6) $y \le x \Rightarrow x \to z \le y \to z, z \to y \le z \to x.$

(u_7) $(\vee_{i\in\Gamma}\, y_i) \to x = \wedge_{i\in\Gamma}(y_i \to x).$

We describe

$x^* = \vee\{y \in E \mid x \otimes y = 0\}$, Equivalently, $x^* = x \to 0$. Then

(u_8) $0^* = 1, 1^* = 0, x \le x^{**}, x^* = x^{***}.$

In MTL-algebra, the subsequent are accurate

(u_9) $x \to (y \vee z) = (x \to y) \vee (x \to z).$

(u_{10}) $x \otimes y \le x \wedge y.$

$\forall\ x, y, z \in E.$

2.3 Definition [6]

Let E is MTL-algebra. A not empty subset F of E is named a filter of E if it fulfills

(a_1) $x \otimes y \in F$ For all $x, y \in F$.

(a_2) If $x \in F$, $x \le y$ then $y \in F$ for all $y \in E$.

2.4 Proposition [6]

A not empty subset F of the MTL-algebra E is the filter of E if &only if it fulfills the following

(F_1) $1 \in F$

(F_2) $x \to y \in F \ \Rightarrow\ y \in F.\ \forall x, y \in E.$

A not empty subset F of an MTL algebra E is also called a deductive system if the above two conditions are true.

2.5 Definition [5]

Let F be a not empty subset of an MTL algebra E. Then F is called a prime filter of E if, F is a proper filter and $x \vee y \in F$ implies $x \in F$ or $y \in F$ $\forall x, y \in E$.

2.6 Definition [5]

Let F be a not empty subset of an MTL-algebra E. Then F is called an ultrafilter of E if F is a proper filter and $x \in F$ or $x^* \in F$ for all $x \in E$.

2.7 Definition [5]

Let F is a not empty subset of an MTL-algebra E. So F is called a G-filter if

$$(x \otimes x \to y \in F) \text{ implies } (x \to y) \in F$$

$\forall x, y \in E.$

2.8 Theorem

A softset (F, E) above U is a softfilter in an MTL-algebra E if it fulfills the subsequentcircumstances

$$(1)\ F(1) \supseteq F(x)$$
$$(2)\ F(y) \supseteq F(x) \cap F(x \to y)$$

$\forall x, y \in E.$

2.9 Definition [1, 7, 8]

Suppose that U is a preliminary universal set and E is all likely considerations set below consideration with reverence to U. The power set of U $(i.e. \text{ the set of all sub set of } U)$ is symbolized by $P(U)$ & A is subset of E. Typically, limitations are characteristics, qualities or possessions of an substances in U.

A couple (F & A) is named a soft set above U, here F is a plotting given by

$$F : A \to P(U).$$

Additionally, a softset above U is a parameterized intimate of sub-sets of the universal set U. For $\varepsilon \in A$, $F(\varepsilon)$ might be measured as a set of $\varepsilon -$estimated elements of a soft-set (F, A).

3. SOFT FILTERS

No, wedescribe soft filter of a MTL-algebra. Some characterizations of a soft filter are investigated. Throughout this paper E is an MTL-algebra and U is notblank set.

3.1 Definition

The softset (F, E) above U is entitled a soft filter of an MTL-algebra E if F satisfies

(1) $F(x \otimes y) \supseteq F(x) \cap F(y)$ For all $x, y \in E$

(2) F is order-preserving, that is, $x \le y \Rightarrow F(x) \subseteq F(y)$, for all $x, y \in E$.

3.2 Example

Let $E = \{0, a, b, c, d, 1\}$, where $0 < b < a < 1$, $0 < d < a < 1$, and $0 < d < c < 1$.

Define $\otimes$ and $\to$ as follows

$\otimes$	0	a	b	c	d	1
0	0	0	0	0	0	0
a	0	b	b	d	0	a
b	0	b	b	0	0	b
c	0	d	0	c	d	c
d	0	0	0	d	0	d
1	0	a	b	c	d	1

$\to$	0	a	b	c	d	1
0	1	1	1	1	1	1
a	d	1	a	c	c	1
b	c	1	1	c	c	1
c	b	a	b	1	a	1
d	a	1	a	1	1	1
1	0	a	b	c	d	1

Then E is an MTL-algebra. Let'sdescribe the soft sets (F_1, E), (F_2, E) and (F_3, E) over E by

$$F_1 : E \to P(U)$$

$$F_1(x) = \begin{cases} A \text{ if } x \in \{0, c, d\}, \\ U \text{ if } x \in \{a, b, 1\}, \end{cases}$$

$$F_2 : E \to P(U)$$

$$F_2(x) = \begin{cases} A \text{ if } x \in \{0, a, b, c, d\}, \\ U \quad \text{ if } x = 1 \end{cases}$$

and

$$F_3 : E \to P(U)$$

$$F_3(x) = \begin{cases} A \quad \text{ if } x = 0, \\ U \text{ if } x \in \{a, b, c, d, 1\}, \end{cases}$$

where $A = \{y, z\} \subset U = \{x, y, z\}$. Then F_1, F_2 and F_3 are soft filters of E.

3.3 Theorem

The softset (F, E) in E is soft-filter in E only if it fulfills

$(1)\ (\forall x \in E)\ (F(1) \supseteq F(x))$.

$(2)\ (\forall x, y \in E)(F(y) \supseteq F(x) \cap F(x \to y))$.

3.4 Boolean soft filter

A softfilter F of E is supposed to be a Boolean if the subsequent equality fulfills
$F(x \vee x^*) = F(1)\ \ (x \in E)$ Where $x^* = x \to 0$

3.5 Proposition

Each Boolean soft-filter F of E fulfills the subsequentsituation
$(\forall x, y, z \in E) \qquad F(x \to z) \supseteq F(x \to (z^* \to y)) \cap F(y \to z)$.

3.6 Theorem

Let F be a softfilter of E then subsequent declarations are equal

(1) F Is Boolean.

$(2)(\forall x, y, z \in E)\ F(x \to z) \supseteq F(x \to (z^* \to y)) \cap F(y \to z)$.

$(3)(\forall x, y \in E) F(x) \supseteq F((x \to y) \to x)$.

3.7 MV-soft filter

A softset (F, E) in E is said to be MV-soft filter if it is a softfilter of E that fulfills the subsequentsituation

$$(\forall x, y \in E) F(x \to y) \subseteq F(((y \to x) \to x) \to y)).$$

3.8 Soft Ultra Filter

A soft filter F of a MTL-algebra E is said to be soft ultra filter of E if F fulfills the following condition
$F(x) = F(1)$ or $F(x^*) = F(1)$ $(\forall x \in E)$.

3.9 Example

Let E={0,a,b,c,d,1} the residuum $\to$& product $\otimes$ are well clear as

$\otimes$	0	a	b	c	d	1
0	0	0	0	0	0	0
a	0	d	c	0	d	a
b	0	b	b	0	0	b
c	0	0	c	0	0	c
d	0	d	0	0	d	d
1	0	a	b	c	d	1

$\to$	0	a	b	c	d	1
0	1	1	1	1	1	1
a	c	1	b	b	a	1
b	d	a	1	a	d	1
c	a	1	1	1	a	1
d	b	1	b	b	1	1
1	0	a	b	c	d	1

So $E(\wedge, \vee, \otimes, \to 0, 1)$ is a MTL-algebra.

Let softset (F, E) in E is define as

$$F(x) = \begin{cases} E & \text{if } x \in \{1, a, d\}, \\ \{0\} & \text{otherwise.} \end{cases}$$

F is soft filter of E and satisfies
$F(x) = F(1)$ Or $F(x^*) = F(1)$ $(\forall x \in E)$.

So F is a soft ultra filter of E.

3.10 Soft prime filter

A soft filter F of a MTL-algebra E is said to be a soft prime filter of E when F fulfills the following condition
$(\forall x, y \in E) F(x \vee y) \subseteq F(x) \cup F(y)$.

3.11 Theorem

Let's (F, E) be a soft set of a MTL Algebra E. Then F is a soft ultra filter of E if and only if it satisfies:

(1) F is the soft Boolean filter of E,

(2) F is the soft prime filter of E.

Proof Let us assume that the softset F in E is a soft Boolean & soft prime filter of E.
We know that
$\forall\, x \in E$ We have

$$F(1) = F(x \vee x^*) \quad \text{Definition 3.4}$$
$$\subseteq F(x) \cup F(x^*) \quad \text{Definition 3.8}$$
$$\Rightarrow F(1) \subseteq F(x) \cup F(x^*)$$
$$\Rightarrow F(1) \subseteq F(x) \text{ or } F(1) \subseteq F(x^*)$$

We know from Theorem 3.3
$F(x) \subseteq F(1)$ & $F(x^*) \subseteq F(1)$
We have
$F(x^*) = F(1)$ Or $F(x^*) = F(1)$.
So F is a soft ultra filter of E.
Conversely Let F be a soft ultra filter of E. Since we know that $\forall\ x \in E$

$$x \leq x \vee x^*,\ x^* \leq x \vee x^*$$

by Definition 3.1,

$$F(x) \subseteq F(x \vee x^*) \text{ and } F(x^*) \subseteq F(x \vee x^*).$$

By Definition of soft ultra filter we have $(\forall\, x \in E)$.

$$F(x) = F(1) \text{ or } F(x^*) \subseteq F(1).$$

Thus,

$$F(1) \subseteq F(x \vee x^*) \ldots\ldots\ldots (A)$$

By (A) and Theorem:3.3, we get

$$F(x \vee x^*) = F(1).$$

Thus F is a soft Boolean-filter of E.
Now by Proposition 2.2 (u_2)

$$x \vee y = ((x \to y) \to y) \wedge ((y \to x) \to x).$$
$$\leq (x \to y) \to y \qquad \because a \wedge b \leq a$$
$$x \vee y \leq (x \to y) \to y.$$

By Definition d3

$$F(x \vee y) \subseteq F((x \to y) \to y) \ldots\ldots\ldots (B)$$

Since $0 \leq y$ and By Suggestion 2.2 (u_6)

$$(x \to y) \to y \leq x^* \to y$$

by Definition 3.1,

$$F((x \to y) \to y) \subseteq F(x^* \to y).$$

From (B) we have

$$F(x \vee y) \subseteq F(x^* \to y) \ldots\ldots (1)$$

For several $x, y \in E$ if

$$F(x) = F(1) \ldots\ldots (C)$$

Then By Theorem 3.3,

$$F(x \vee y) \subseteq F(1)$$
$$= F(x) \qquad \text{By } (C)$$
$$\subseteq F(x) \cup F(y) \quad \because a \leq a \vee b$$
$$\Rightarrow F(x \vee y) \subseteq F(x) \cup F(y).$$

By Definition 3.8,If

$$F(x) \neq F(1)(\forall x \in E).$$

Then by Definition 3.8,

$$F(x^*) = F(1)\ldots\ldots(D),$$

thus by Theorem 3.3,

$$\begin{aligned} F(y) &\supseteq F(x^*) \cap F(x^* \to y) \\ &= F(1) \cap F(x^* \to y) \quad \text{By } (D) \\ &= F(x^* \to y) \quad \therefore F(x) \subseteq F(1) \\ &\Rightarrow F(y) \supseteq F(x^* \to y)\ldots\ldots(2) \end{aligned}$$

Combining (1) and (2) we have

$$\begin{aligned} F(x \vee y) &\subseteq F(x^* \to y) \\ &\subseteq F(y) \\ &\subseteq F(x) \cup F(y) \\ &\Rightarrow F(x \vee y) \subseteq F(x) \cup F(y). \end{aligned}$$

Thus F is a soft prime filter of E.

This complete the proof.

3.12 Soft G-Filter

A soft filter F of MTL algebra E is said to be the soft G-filter of E if it fulfills the subsequentsituation

$$(\forall x, y \in E), F(x \otimes x \to y) \subseteq F(x \to y).$$

3.13 Example

Let E={0,a,b,c,d,1 in which $\to, \otimes$ is clear as

$\otimes$	0	a	b	c	d	1
0	0	0	0	0	0	0
a	0	a	c	c	0	a
b	0	c	b	c	d	b
c	0	c	c	c	0	c
d	0	0	d	0	0	d
1	0	a	b	c	d	1

$\to$	0	a	b	c	d	1
0	1	1	1	1	1	1
a	d	1	b	b	d	1
b	0	a	1	a	d	1
c	d	1	1	1	d	1
d	a	1	1	1	1	1
1	0	a	b	c	d	1

So $E(\wedge, \vee, \otimes, \to 0, 1)$ is a MTL algebra. Let the soft-set (F, E) in E is define as

$$F(x) = \begin{cases} E & \text{if } x \in \{1, a\}, \\ \{1\} & \text{otherwise.} \end{cases}$$

F is soft-filter of E and fulfills.

$(\forall x, y \in E), F(x \otimes x \to y) \subseteq F(x \to y)$

So F is a soft G-filter of E.

3.14 Theorem

A soft set (F, E) in E is Boolean soft-filter of E if &only if it fulfills the following

a. F is soft G filter of E.

b. F is soft MVfilter of E.

Proof. Let us assume that the F is soft Boolean filter of E.

Meanwhile

$$y \le ((y \to x) \to x) \to y$$

We get

$$(((y \to x) \to x) \to y) \le y \to x \quad \because y \le x \Rightarrow x \to z \le y \to z.$$

By means of

$$(1)\ \ x \to y \le (y \to z) \to (x \to z)$$

$$(2)\ \ x \to (y \to z) = y \to (x \to z)$$

$$(3)\ \ y \le x \Rightarrow x \to z \le y \to z$$

$$(4)(((y \to x) \to x) \to y) \le y \to x.$$

We get

$$x \to y \le ((y \to x) \to x) \to ((y \to x) \to y)$$

$$= (y \to x) \to (((y \to x) \to x) \to y)$$

$$\le ((((y \to x) \to x) \to y) \to x) \to (((y \to x) \to x) \to y)$$

$$x \to y \le ((((y \to x) \to x) \to y) \to x) \to (((y \to x) \to x).$$

By Theorem 3.6 and Definition 3.1,

$$F(((y \to x) \to x) \to y) \supseteq F(((((y \to x) \to x) \to y) \to x) \to (((y \to x) \to x) \to y))$$

$$\supseteq F(x \to y)$$

$$\Rightarrow F(x \to y) \subseteq F(((y \to x) \to x) \to y).$$

Hence verify that F is a MV-softfilter of E. Let $x, y \in E$ and by Proposition 3.5 we have,

$$F(x \to y) \supseteq F(x \to (x \to y)) \cap F(x \to x)$$

$$= F(x \to (x \to y)) \cap F(1) \qquad \because (x \to x = 1)$$

$$= F(x \to (x \to y)) \qquad \because (F(x) \subseteq F(1))$$

$$= F(x \otimes x \to y) \quad \text{By Proposition } (u_5)$$

$$\Rightarrow F(x \to y) \supseteq F(x \otimes x \to y).$$

Thus F is a soft G-filter of E.

Conversely

By supposition that F is a soft G-filter and soft MV-filter of E.

Soby Proposition 2.2 (u_3)

$$x \le (x \to y) \to y$$

and by proposition 2.2 (u_6)

$$\Rightarrow (x \to y) \to x \le (x \to y) \to (x \to y) \to y.$$

By suggestion 2.2 (u_5)

$$\Rightarrow (x \to y) \to x \le ((x \to y) \otimes (x \to y)) \to y,$$

and by Definition 3.1 we have

$$F((x \to y) \to x) \subseteq F(((x \to y) \otimes (x \to y)) \to y)$$

$$\subseteq F((x \to y) \to y) \qquad \text{By Definition 3.12}$$

$$F((x \to y) \to x) \subseteq F((x \to y) \to y) \ldots\ldots (1)$$

Now by proposition 2.2 (u_7)

$$(x \to y) \to y \le (x \to (x \to y)) \to (x \to y).$$

By proposition 2.2 (u_6)

$$((x \to (x \to y)) \to (x \to y)) \to x \le ((x \to y) \to y) \to x$$

By Definition 3.1,

$$F(((x \to (x \to y)) \to (x \to y)) \to x) \subseteq F(((x \to y) \to y) \to x)\ldots\ldots(A)$$

By Explanation 3.7 and Explanation 3.1 we consume

$$F((x \to y) \to x) \subseteq F\left(((x \to (x \to y)) \to (x \to y)) \to x\right)$$

$$\subseteq F\left(((x \to y) \to y) \to x\right) \quad \text{by } (A)$$

$$\Rightarrow F((x \to y) \to x) \subseteq F\left(((x \to y) \to y) \to x\right)\ldots\ldots(2)$$

From (1) and (2) we get

$$F((x \to y) \to x) \subseteq F((x \to y) \to y) \cap F\left(((x \to y) \to y) \to x\right).$$

By Theorem 3.3,

$$F((x \to y) \to x) \subseteq F(x).$$

So by Theorem 3.6 F is soft Boolean filter of E.
This complete the proof.

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