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A New Analytical Solution for Creep Stresses in Thick-Walled Spherical Pressure Vessels

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ABSTRACT

In this paper, based on basic equations of steady-state creep of spherically symmetric problems, a new exact closed form solution for creep stresses in isotropic and homogeneous thick spherical pressure vessels are presented. The creep response of the material is governed by Norton's law. All results have been obtained in nondimensional form. Effect of changes in material properties on stresses and displacement is discussed.
KEYWORDS: Sphere, Pressure Vessel, Thick-Walled, Creep, Analytical Solution.

1. INTRODUCTION

Creep analyses of thick-walled spherical pressure vessels subjected to internal and or external pressure are important in solid mechanics and engineering applications. Assuming the infinitesimal strain theory, creep problems in engineering material and steady-state creep solution for a spherical vessel under internal pressure are studied by Finnie and Heller [1]. Use of the finite strain theory, and with considering large strains, Bhatnagar and Arya [2], obtained the creep analysis of a pressurized thick-walled spherical vessel made of a homogeneous and isotropic material. Using internal stress arising from a spherically symmetric, finite plastic strain, creep of a hollow sphere subjected to inner and outer pressures, and also thermal stress, is discussed by Sakaki *et al.* [3]. Assuming the elastic behavior of the material is undergoing both creep and dimensional changes, Miller [4] obtained stresses and displacements in a thick spherical shell subjected to internal and external pressure loads. Liu and Chen [5] investigated the creep rupture analysis of a pressurized sphere in details based on the approach of continuum damage mechanics in conjunction with the finite element technique. Based on the generalized damage equation, a rupture analysis of a thick sphere subjected to a sinusoidal pulsating internal pressure is performed by Liu [6]. A numerical model developed for the computation of creep damages in a thick-walled sphere subjected to an internal pressure and a thermal gradient by Loghman and Shokouhi [7]. Hoseini *et al.* [8] obtained a new analytical solution for the steady state creep in pressurized rotating thick cylindrical shells. Marcadon [9] explored the main mechanisms that govern the viscous behaviour of hollow-sphere structures in view of their potential high-temperature applications, and more specifically under creep loading.

2. GOVERNING EQUATIONS AND SOLUTIONS

A thick-walled sphere with the inner and outer radii r_i and r_o is shown in Fig.1. The sphere is assumed under the action of inner and outer constant pressures P_i and P_o, respectively. Spherical coordinates (r,θ,ϕ) for this problem is used.

In this coordinates, the relations between radial and circumferential strains rate $(\dot{\varepsilon}_r, \dot{\varepsilon}_\theta = \dot{\varepsilon}_\phi)$ and radial, circumferential stresses $(\sigma_r, \sigma_\theta = \sigma_\phi)$ for an incompressible, isotropic material can be described with

$$\begin{bmatrix} \dot{\varepsilon}_r \\ \dot{\varepsilon}_\phi \end{bmatrix} = \frac{\dot{\varepsilon}_e}{2\sigma_e} \begin{bmatrix} 2 & -2 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} \sigma_r \\ \sigma_\phi \end{bmatrix} \qquad (1)$$

where $\dot{\varepsilon}_e$ and σ_e are the equivalent strain rate and equivalent stress, respectively.

The Norton equation gives a relation between the equivalent strain rate and equivalent stress that is suggested for steady state creep in the form

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$$\dot{\varepsilon}_e = B\sigma_e{}^n \tag{2}$$

where B and n are material parameters describing the creep performance.
Substituting Eq. (2) into Eqs. (1) as follows

$$\dot{\varepsilon}_r = -B\sigma_e{}^{n-1}\left(\sigma_\phi - \sigma_r\right) \tag{3}$$

$$\dot{\varepsilon}_\phi = \frac{1}{2}B\sigma_e{}^{n-1}\left(\sigma_\phi - \sigma_r\right) \tag{4}$$

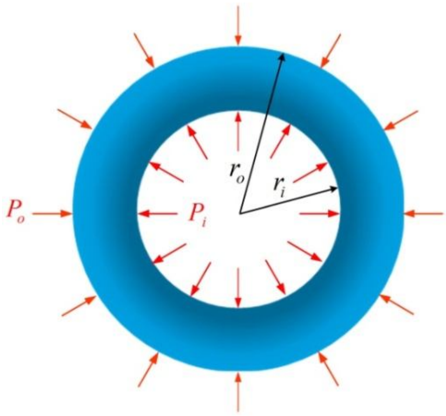

Fig. 1. Thick-walled spherical pressure vessel.

The von Mises effective stress is as

$$\sigma_e = \sigma_\phi - \sigma_r \tag{5}$$

Substituting Eq. (5) into Eqs. (3) and (4),

$$\dot{\varepsilon}_r = -B\sigma_e{}^n \tag{6}$$

$$\dot{\varepsilon}_\phi = \frac{1}{2}B\sigma_e{}^n \tag{7}$$

The strain rate $\left(\dot{\varepsilon}_r, \dot{\varepsilon}_\phi\right)$ and radial displacement rate $\left(\dot{u}\right)$ relations, can be written as

$$\dot{\varepsilon}_r = \frac{d\dot{u}}{dr} \tag{8}$$

$$\dot{\varepsilon}_\phi = \frac{\dot{u}}{r} \tag{9}$$

By eliminating $\dot{u}$ from Eqs. (8) and (9), the equation of compatibility can be obtained as

$$r\frac{d\dot{\varepsilon}_\phi}{dr} + \dot{\varepsilon}_\phi - \dot{\varepsilon}_r = 0 \tag{10}$$

Substituting Eqs. (6) and (7) into Eq. (10), equation of compatibility can be rewritten as

$$\frac{d\sigma_e^n}{\sigma_e^n} + 3\frac{dr}{r} = 0 \tag{11}$$

Solution of differential equation (11) is as follows

$$\sigma_e = \left(\frac{C_1}{Br^3}\right)^{\frac{1}{n}} \tag{12}$$

Substituting Eqs. (12) into Eq. (5), we have

$$\sigma_\phi - \sigma_r = \left(\frac{C_1}{Br^3}\right)^{\frac{1}{n}} \tag{13}$$

In this problem, equilibrium equation in the spherical coordinates is as

$$\frac{d\sigma_r}{dr} + 2\frac{\sigma_r - \sigma_\phi}{r} = 0 \tag{14}$$

With substituting Eq. (13) into (14) and solving it, radial and circumferential stresses are obtained as follows

$$\sigma_r = 2\left(\frac{C_1}{B}\right)^{\frac{1}{n}} \int_{r_i}^{r} r^{-\left(\frac{3}{n}+1\right)} dr + C_2 \tag{15}$$

$$\sigma_\phi = 2\left(\frac{C_1}{B}\right)^{\frac{1}{n}} \left(\int_{r_i}^{r} r^{-\left(\frac{3}{n}+1\right)} dr + r^{-\frac{3}{n}}\right) + C_2 \tag{16}$$

The boundary conditions for stresses are as follows

$$\begin{cases} \sigma_r\left(r = r_i\right) = -P_i \\ \sigma_r\left(r = r_o\right) = -P_o \end{cases} \tag{17}$$

Using the boundary conditions (17), the constants C_1 and C_2 are obtained

$$\begin{cases} C_1 = \left(\dfrac{P_i - P_o}{2B^{-\frac{1}{n}} \int_{r_i}^{r_o} r_i^{-\left(\frac{3}{n}+1\right)} dr}\right)^n \\ C_2 = -P_i \end{cases} \tag{18}$$

Hence, the radial, circumferential, and equivalent stresses, and effective strain rate are as follows

$$\sigma_r = \frac{P_i - P_o}{k^{-\frac{3}{n}} - 1} R^{-\frac{3}{n}} - \frac{P_i k^{-\frac{3}{n}} - P_o}{k^{-\frac{3}{n}} - 1} \tag{19}$$

$$\sigma_\phi = \frac{\left(P_i - P_o\right)\left(2n - 3\right)}{2n\left(k^{-\frac{3}{n}} - 1\right)} R^{-\frac{3}{n}} - \frac{P_i k^{-\frac{3}{n}} - P_o}{k^{-\frac{3}{n}} - 1} \tag{20}$$

$$\sigma_e = \left(\frac{-3}{2n}\right) \frac{P_i - P_o}{k^{-\frac{3}{n}} - 1} R^{-\frac{3}{n}} \tag{21}$$

$$\dot{\varepsilon}_e = B\left(\frac{-3}{2n}\right)^n \left(\frac{P_i - P_o}{k^{-\frac{3}{n}} - 1}\right)^n R^{-3} \tag{22}$$

where $R = r/r_i$ and $k = r_o/r_i$.

3. RESULTS AND DISCUSSION

Consider a sphere with inner and outer radii of 0.5 m and 0.8 m, respectively. The sphere subjected to internal pressure of 70 MPa. In this section, all results have been obtained in nondimensional form.

For values of $n = 1,2,4,10$, radial, circumferential, and von Mises equivalent stresses along the radial direction are plotted in Figs. 2–4.

Radial stresses in the radial direction are shown in Fig. 2. In this figure, radial stress increases as n increases, and radial stress for different values of n , is compressive.

Figures 3 and 4 are the plots of circumferential and von Mises equivalent stresses along the radial direction for different values of n .

It must be noted from Figs. 3 and 4 that at the same position, almost for $R < 1.3$ (Fig. 3) and $R < 1.24$ (Fig. 4), there are an decreases in the values of the circumferential and equivalent stresses as n increase, whereas for $R > 1.3$ (Fig. 3) and $R > 1.24$ (Fig. 4) this situation was reversed, respectively. According to these figures, for all values of n , the circumferential and equivalent stresses are tensile.

4. CONCLUSION

In this work, a new analytical solution has been developed for the creep analysis of isotropic and homogeneous thick-walled spherical pressure vessels. Norton's power law of creep is employed to derive general expressions for stresses and strain rates in the thick sphere. It is seen that the material parameter n has significant influence on the distributions of the creep stresses and radial displacement.

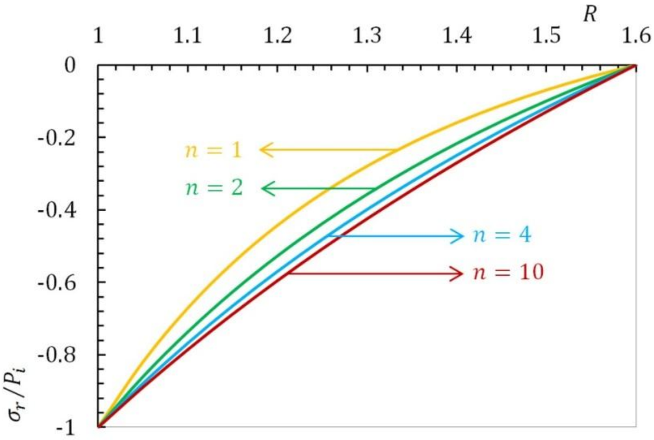

Fig. 2. Radial stress distribution along radial direction.

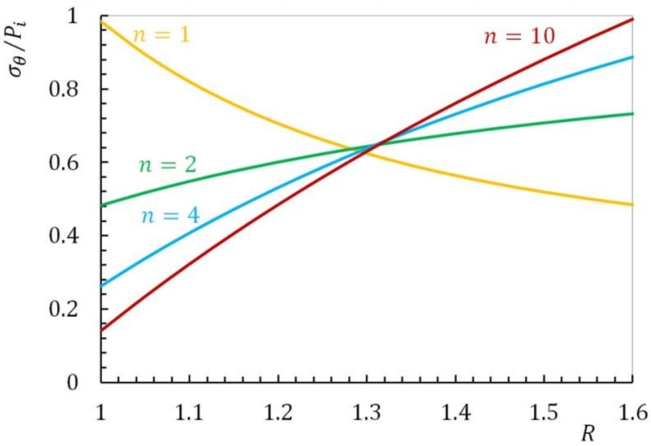

Fig. 3. Circumferential stress distribution along radial direction.

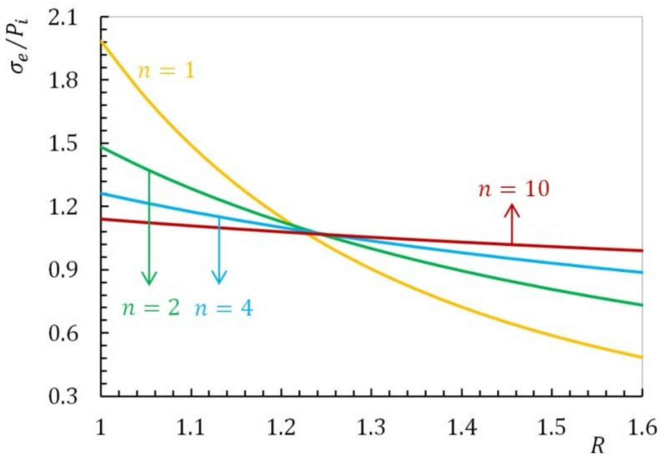

Fig. 4. Equivalent stress distribution along radial direction.

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