Optimization of Sliding Mode Control for a Vehicle Suspension System via Multi-objective Genetic Algorithm with Uncertainty

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ABSTRACT

In this paper, Sliding Mode Controller (SMC) is designed for a quarter-car active suspension system with parametric uncertainty in the mass of vehicle body and also Genetic Algorithms are employed to find optimal parameters of controller. An ideal suspension must be able to provide passengers comfort and improve safety performance. Furthermore, the necessity of trading off among the conflicting requirements of the suspensions in terms of comfort and road holding capability led to the use of multi-objective optimization techniques. The important conflicting objective functions that have been considered in this work are, namely, sprung mass acceleration, suspension deflection and energy consumption. Moreover, this approach returns the optimum answers in Pareto form that designer can, by making trade-offs, select desired answer. Finally, the obtained results demonstrate that use of the proposed controller provides good performance in improving and enhancing the road holding ability and riding quality compared with passive suspension system.

KEY WORDS: Sliding Mode Control, Vehicle Suspension, Multi-objective Optimization, Genetic Algorithm.

1. INTRODUCTION

Several researchers and engineers have extensively discussed the problem of vehicle suspension control in the automotive industry. A literature survey shows that a variety of solutions has been developed to improve the road holding ability and riding quality of cars [1]. An early design for automobile suspension systems focused on unconstrained optimizations for passive suspension system which indicate the desirability of low suspension stiffness, reduced unsprung mass, and an optimum damping ratio for the best controllability.

However, the suspension spring and damper do not provide energy to the suspension system and control only the motion of the car body and wheel by limiting the suspension velocity according to the rate determined by the designer [2]. Therefore, to overcome the above problem, Active and semi-active suspension systems are developed to achieve better performance than the conventional passive suspension systems [3]. An ideal active vehicle suspension system should have the capability of reducing the sprung mass displacement and acceleration and isolating driver and passengers from vibrations arising from road roughness [4]. A vast amount of work on controlled suspension systems is present in the technical and scientific literature. The first paper dealing with active suspensions dates back to the 1954 by Federpsiel and Labrosse. One of the first reviews of the state of the art of controlled suspensions was carried out by Hedrick and Wormely in 1975 [5]. The sliding mode control theory was proposed by Utkin in 1977. Thereafter, Slotine and Edwards and Spurgeon well developed the theoretical works of the sliding mode controller and expanded its applications [6].

In this paper a SMC is designed for a quarter vehicle active suspension system. Moreover, a multi-objective genetic algorithm (MOGA) is used for multi-objective optimization of sliding mode controller for quarter car vibration model. The conflicting objective functions that have been considered for minimization are, namely, acceleration sprung mass ($\ddot{z}_s$), suspension deflection ($z_s - z_u$) and actuator force. Furthermore, the design variables used in the optimization of vibration are, namely, vehicle suspension damping coefficient ($b_s$), vehicle suspension stiffness coefficient ($k_s$) and controller gains in sliding mode control. Prominently, it is shown that a trade-off optimum design can be verified from those Pareto fronts obtained by multi-objective optimization process. Finally, the superiority of time domain vibration performance of such design point is shown in comparison with those given in the literature [7].

2. Dynamic model of suspension

In this study, a two-degree-of-freedom quarter-car suspension model shown in "Figure 1" is considered for Controller design [8]. From first principle the equations of motion for this system can be derived as follows:
\begin{align*}
    m_s \ddot{z}_s &= -k_s (z_s - z_u) - b_s (\dot{z}_s - \dot{z}_u) + f_u \\
    m_u \ddot{z}_u &= k_s (z_s - z_u) + b_s (\dot{z}_s - \dot{z}_u) - k_t (z_u - z_d) - f_u
\end{align*}

Where \( m_s, m_u, b_s, k_s, k_t, f_u, z_s, z_u, z_d, \) and \( \dot{z}_s, \dot{z}_u \) are the sprung mass, unsprung mass, damping coefficient, spring stiffness, tire stiffness, actuator force, displacement of the sprung mass, displacement of unsprung mass, road profile, suspension deflection, tire deflection, the velocity of sprung mass and the velocity of unsprung mass, respectively. Note that only the vertical displacements of the system are considered in the study and that it is assumed that the tire is constantly in contact with the road surface [9].

![Quarter car active suspension system model](image)

**FIGURE. 1.** Quarter car active suspension system model

### 3. SLIDING MODE CONTROL

A sliding mode controller is a variable structure controller that can be effectively applied to nonlinear systems in spite of parameter uncertainties and external disturbances.[10] Suppose a non-linear system is defined by the general state-space equation:

\[ x = f(x, u, t) \]  

(1)

Where \( x \in \mathbb{R}^n \), is the state vector, \( u \in \mathbb{R}^m \) the input vector and \( n \) is the order of system and \( m \) the number of inputs. Then the sliding surface \( s(e, t) \), is given by:

\[ s(e, t) = \{ e : h^T e = 0 \} \]  

(2)

That \( h \in \mathbb{R}^n \) represents the coefficients, or slope of the sliding surface. Here, \( e = x - x_d = \left[ e \ \dot{e} \ .. e^{n-1} \right]^T \) is the negative tracking error vector.

Usually a time-varying sliding surface, \( s(t) \) is simply defined in the state-space \( \mathbb{R}^n \) by the scalar equation, given by

\[ s(e, t) = \left( \frac{d}{dt} + h \right)^{-1} e = 0 \]  

(4)

Where \( h \) is a strictly positive constant that can also be explained as the slope of the sliding surface. For instant, if \( n=2 \) (for a second ordered system)

\[ s = \dot{e} + he \]  

(5)

And hence, \( s \) is simply a weighted sum of the position and velocity error from (4). The \( n \)-order tracking problem is now being replaced by a \( 1^{st} \)-order stabilization problem in which the scalar \( s \) is to be kept at zero by governing a reaching condition.

By choosing Lyapunov function \( v(x) = \frac{1}{2} s^2 \), then, the following equation can guarantee that reaching condition be satisfied,

\[ v(x) = \frac{1}{2} s^2 < 0 \]  

(6)

It can be seen that, when operating in the sliding mode, system response is chattering along \( s=0 \). From (6), existence and convergence condition can thus be re-written as:

\[ ss \leq \eta s \]  

(7)

It can be shown that the sliding condition of (6) is always satisfied by:

\[ u = u_{eq} - k \cdot \text{sgn}(s) \]  

(8)

Where \( u_{eq} \) is called equivalent control input which is obtained by \( \dot{s} = 0 \) and \( k \) is a strictly positive constant [11]. To eliminate the chattering effect produced by the discontinuous function sign, a saturation function sat can be used in the place of sign. This saturation function is defined as follows:
\[ \text{sat}(s) = \begin{cases} \text{sgn}(s) & \text{if } |s| \geq \varphi \\ s & \text{if } |s| < \varphi \end{cases} \]  
(9)

With \( \varphi \) is a boundary layer around the sliding surface \( s \). [12].

4. Multi-objective optimization

Multi-objective optimization which is also called vector optimization has been defined as finding a vector of decision variables satisfying constraints to give acceptable values to all objective functions. The Pareto-based approach of NSGA-II has been recently used in a wide area of engineering MOPs because of its simple yet efficient non-dominance ranking procedure in yielding different level of Pareto frontiers. In this paper Modified NSGA-II algorithm as a MO tool searches the definition space of decision variables and returns the optimum answers in Pareto form [13].

In general, it can be mathematically defined as:

Find the vector

\[ X^* = \{x_1^*, x_2^*, ..., x_n^*\}^T \]  
(10)

To optimize

\[ F(X) = [f_1(X), f_2(X), ..., f_k(X)]^T \]  
(11)

Subject to \( m \) inequality constraints

\[ g_i(X) \leq 0 \quad t = 1, 2, ..., m \]  
(12)

And \( p \) equality constraints

\[ h_j(X) = 0 \quad j = 1, 2, ..., p \]  
(13)

Where \( X^* \in R^n \) is the vector of decision or design variables, and \( F(X) \in R^k \) is the vector of objective functions which each of them be either minimized or maximized [14].

5. RESULTS AND DISCUSSION

In this section, the performance of the quarter model with the sliding-mode controller is evaluated in time domain. Furthermore, the multi-objective genetic algorithm is applied for searching optimal parameters of controller. The numerical parameters of the vehicle model are taken from Haiping D. et al. [8], and are as follows:

- \( m_s = 320 \pm 0.2 m \)
- \( m_w = 40 kg \)
- \( b_r = 1 KN/m \)
- \( k_s = 18 KN/m \)
- \( k_r = 200 KN/m \)

The corresponding ground displacement for the wheel is given by

\[ z_j(\theta) = \frac{a}{2}(1-\cos(8\pi t)), \quad \text{if } .5 \leq t \leq .75 \quad \text{and } 3 \leq t \leq 3.25 \]

\[ 0, \quad \text{otherwise} \]

FIGURE 2. Typical road disturbance

Where \( a \) denotes bump amplitude. This type of road disturbance has been used by Sam Y. et al. [15] in their studies. The road disturbance is shown in "Figure 2".

In order to fulfill the objective of designing an active suspension system, i.e. to increase the ride comfort and road handling and decrease the total energy consumption there are three parameters to be observed in the simulations. The three parameters are the car body acceleration, the suspension deflection and force actuator.

The objective functions are considered defined as follow:
Now these objective functions are considered in a Pareto optimization process to obtain some important trade-offs among the conflicting objectives, simultaneously. Design variables in this simulation are $h$, $\Phi$, $k_s$, $s_k$, and $s_b$ and that are slope of sliding surface, boundary layer, control gain, spring stiffness and damping coefficient, respectively. For simulating the uncertainty the mass profile of car body are considered as shown "Figure 3". The evolutionary process of the multi-objective optimization is accomplished with a population size of 150 which has been chosen with crossover probability $P_c$ and mutation probability $P_m$ as 0.95 and 0.05, respectively. A total number of 147 non-dominated optimum design points have been obtained. It is widely accepted that visualization tools are valuable to provide the decision maker a meaningful way to analyze Pareto set and select good solutions. For a 2-dimensional problem it is normally easy to make an accurate graphical analysis of the Pareto set points, but for higher dimensions it becomes more difficult [16]. Therefore, the Level Diagrams method is used to visualize a Pareto front. In this method, each point of Pareto front must be normalized between 0 and 1 based on its minimum and maximum values

$$J^M_i = \max(J_i), J^m_i = \min(J_i), i = 1, 2, 3$$

$$\bar{J}_i = \frac{J_i - J^m_i}{J^M_i - J^m_i}$$

Provided that the origin of the n-dimensional space is considered as ideal point, the distance of the each Pareto front point is used to choose optimum points. In this work, Euclidean norm $\|J\| = \sqrt{\sum J_i^2}$ is used for this purpose. Hence the point whose distance to the origin is the minimum, that is, the lowest value of $\|J\|$ can be obtained as the most important trade-off point. The results of the 3-objective optimization process are shown in "Figure 4".

![Figure 4: Euclidean norm Level Diagrams of Pareto front](image)

As it is shown, the point with the lowest value of $\|J\|$ has the low value of each objective function. To illustrate the result of the optimization process, 4 points are chosen of, which three of them have the minimum value of each objective function and the forth one has the minimum value of $\|J\|$ [17]. The values of the pertinent objective functions are given in "Table 1".

<table>
<thead>
<tr>
<th>Number</th>
<th>$J_1$</th>
<th>$J_2$</th>
<th>$J_3$</th>
<th>$|J|$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min $J_1$ (A)</td>
<td>0.00096</td>
<td>1.3398</td>
<td>12020</td>
<td>1.0007</td>
</tr>
<tr>
<td>Min $J_2$ (B)</td>
<td>0.36866</td>
<td>0.7664</td>
<td>11947</td>
<td>1</td>
</tr>
<tr>
<td>Min $J_3$ (C)</td>
<td>0.31706</td>
<td>0.79679</td>
<td>11945</td>
<td>0.86131</td>
</tr>
<tr>
<td>Min $|J|$ (D)</td>
<td>0.18094</td>
<td>0.99637</td>
<td>11964</td>
<td>0.63283</td>
</tr>
</tbody>
</table>
According to Pareto chart, point D is applied for simulation. "Figure 5" illustrates clearly how the MOGA-SMC can effectively absorb the vehicle vibration in comparison to the passive system. The body acceleration in the MOGA-SMC design system is reduced significantly, which guarantee better ride comfort.

"Figure 6" depicts the suspension travel of an active suspension system and a passive suspension system for comparison purposes. The result shows that the suspension travel of an active suspension system better as compared to the passive suspension system. Moreover, the force actuator is also smaller using the proposed controller, which guarantees less energy consumption. Therefore it is concluded that the active suspension system with the MOGA-SMC improves the ride comfort while retaining the road handling characteristics, as compared to the passive suspension system. Furthermore, energy consumption of system is reduced acceptably.

6. Conclusions

In this work, a multi-objective optimal sliding mode control algorithm has been proposed and successfully applied to optimally design a sliding mode controller for an active suspension system. The multi-objective optimization of sliding mode controller led to the discovering some important trade-offs among some conflicting objective functions. In particular, the optimum design point D shows a very reasonable trade-off considering the Euclidean norm of J together with the amounts of other cost functions given in "Table 1". Therefore, this approach of the Pareto optimization of sliding mode controller is very efficient for problem in which objective functions are more than 2-dimensnal.

REFERENCES