Modeling a Supply Chain Network with Consideration of Stochastic Disaster

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ABSTRACT

In this paper, we present a model for reliable facility location in a supply chain that is vulnerable to disruptions. Since facility location decisions are costly to implement and difficult to reverse, these strategic decisions permit very little recourse once a disruption occurs. The goal of this research is to develop a comprehensive model that describes the integrated logistics operations in response to disruption. In order to respond to disruption in a supply chain we must choose facility locations proactively so that the system performs well even if disruptions occur. In this paper we proposed a mixed integer linear model that proposed the optimum allocation before and after a disruption. A set of numerical experiments is designed to test the proposed model and evaluate the properties of the optimization problem. Results indicate that the proposed method can provide better results than the previous solution procedures effectively.

KEY WORDS: Supply chain, Optimization, Stochastic disruptions, Facility allocation

INTRODUCTION

Decisions to support preparedness activities for distribution management are challenging due to the uncertainties of these events, the need to balance preparedness and risk, complications due to partial information and data. Many researchers studied on modeling the supply chain which are exposed by disruptions.

Reliable facility location models for supply chain networks that are exposed by disruptions are related to network reliability theory [1], which attempts to calculate or maximize the probability that a network remains connected after random link failures. Based on this theory Snyder and Daskin [2] introduced several models in which facilities may fail with a given probability. Jia et al.[3] reviewed deterministic and probabilistic facility location models used to model smaller scale distribution and they introduced three deterministic models: a covering model providing a coverage of demands points within a distance limit, a deterministic P-median model minimizing the total distance between demands and facilities, and a P-center model optimizing the worst performance of the system by minimizing the maximum distance between any demand point and its nearest service center.

Alsalloum and Rand [4] and Rajagopalan et al. [5] used goal programming for locating vehicles to maximize the coverage of expected demand while minimizing the number of vehicles in disruption time.

Church and Scaparra [6] considered that supply chain network exists and the firm has resources to prevent disruptions at some of them, thus partially fortifying the network. Their model finds the best facilities to fortify assuming that an interdictor will attempt to cause worst-case losses for the firm by disrupting a fixed number of the un-fortified facilities.

Barbarosoglu and Arda [7] utilized scenario-based two-stage stochastic model for transportation planning in earthquake response, where they seek optimal transportation plans. They define both stages in the response phase: their first stage covers the early response phase depending on the earthquake scenarios, and their second stage covers later response given impact scenarios that are more detailed branches of the earthquake scenarios. Beraldi et al. [8], they used a mixed integer formulation with probabilistic constraints to solve for the location and assignment of emergency vehicles. Beraldi and Bruni [9] formulated two-stage stochastic program. The problem of locating and distributing rescue resources for flood emergency is studied by Chang et al. [10] under possible flood scenarios with a two-stage stochastic programming model.

In this paper, we present a mathematical model to determine the main and reserved allocation planning of distribution center to the central warehouses and manufacturers. This model is a 3-tier supply chain network and the warehouses and manufacturers are exposed by disruption and may fall into the job, according to this matter we want to determine the reserved plan, that is to say we want to determine that the distribution centers should be supplied from which warehouses if their primary warehouses fail. In addition for transportation, we consider direct and indirect transportation. Each distribution can order its demand from manufacturers and warehouses. We have a two-stage stochastic model and we discretize the feasible space by making scenarios, the number of scenarios is exponentially large because there are N facilities and each can fail independently, there are 2^N failure scenarios.
The rest of the article is organized as follows. In Section 2, we present a two-stage stochastic model to determine the optimal allocation. Section 3 explains the solution approach, scenario generation and reduction, and Section 4 presents a discussion of the results and finally Section 5 concludes the paper.

2. Modeling framework

In this paper we consider a 3-tier supply chain that consists of manufacturers, warehouses and distribution centers. In this supply chain the manufacturers and warehouses are susceptible to failure. For this matter we have to determine a reserved allocation plan to reallocate the distribution centers which their warehouse has failed. In this model we assumed that the products can be transferred directly from manufacturers to distribution centers. The schema of this supply chain is shown in Fig. 1.

We use a two stage stochastic model and utilize a huge number of scenarios to discrete the feasible space of this problem. Using scenarios increases the computation efforts and complexity in solving state. Other assumption of the model are as follows:

1. The disruptions probability have been obtained by experts.
2. The occurrence of each scenario is independent of other scenarios.
3. The transportation unit cost is identical for all distance between supply chain’s facilities.
4. Disruptions occur only in warehouse.
5. The demand of each distribution center does not change after disruption occurrence
6. The model does not consider the capacity constraint for facilities.

Fig. 1 The schema of the supply chain

Before proposing mathematical model, we provide a verbal description of the model. All notation used in model are as follows:

\[ i=1,2,..., D \] Distribution centers
\[ j=1,2,..., W \] Warehouses
\[ k=1,2,..., M \] Manufacturers
\[ s=1,2,..., S \] Scenarios

The input parameters are:
\[ d_i \] Demand of distribution center \( i \)
\[ d_{cf} \] The distance between facility \( f \in \{M, W\} \) and facility \( f' \in \{W, D\} \)
\[ a_{fs} \] =1 if facility \( f \in \{M, W\} \) fails in scenario \( s \); =0 otherwise
The variables are as follows:

- \( X_j \) = 1 if warehouse \( j \) is opened; = 0 otherwise
- \( W_k \) = 1 if manufacturer \( k \) is opened; = 0 otherwise
- \( Y_{ijs} \) Fraction of demand of distribution \( i \) that supplied by warehouse \( j \) in scenario \( s \)
- \( Z_{jks} \) Fraction of demand of warehouse \( j \) that supplied by manufacturer \( k \) in scenario \( s \)
- \( U_{iks} \) Fraction of demand of distribution \( i \) that directly supplied by manufacturer \( k \) in scenario \( s \)

The entire model is mentioned as follow:

\[
\begin{align*}
\text{min} & \sum_{j \in J} f_{c_j} X_j + \sum_{k \in K} f_{c_k} W_k + \sum_{i \in I} \sum_{s \in S} q_{s} d_{i} \sum_{j \in J} Y_{ijs} + \sum_{s \in S} \sum_{j \in J} \sum_{k \in K} q_{s} d_{i} c_{jk} Z_{jks} + \sum_{s \in S} \sum_{i \in I} \sum_{k \in K} q_{s} d_{i} c_{ik} U_{iks} \\
\sum_{j \in J} Y_{ijs} + \sum_{k \in K} U_{iks} &= 1 \quad \forall i \in I, s \in S \\
\sum_{j \in J} Y_{ijs} - \sum_{k \in K} Z_{jks} &= 0 \quad \forall j \in J, s \in S \\
Y_{ijs} &\leq (1-a_{js})X_j \quad \forall i \in I, j \in J, s \in S \\
U_{iks} &\leq (1-a_{ks})W_k \quad \forall i \in I, k \in K, s \in S \\
Z_{jks} &\leq (1-a_{ks})W_k (\sum_{i} d_i) \quad \forall k \in K, j \in J, s \in S \\
Y_{ijs} &\geq 0 \quad \forall i \in I, j \in J, s \in S \\
Z_{jks} &\geq 0 \quad \forall j \in J, k \in K, s \in S \\
U_{iks} &\geq 0 \quad \forall i \in I, k \in K, s \in S
\end{align*}
\]

Objective function (1) minimizes the fixed cost and transportation cost of entire supply chain. Relation (2) assures that the total demand of each customer should be satisfied in each scenario. Relation (3) assures that the all demand which a warehouse can satisfy is equal to its received products from manufacturers. Relation (4-5) assures that the failed facilities cannot fulfill any demand. Relation (6) assures that the model does not assign a failed warehouse to a manufacturer. Relation (7-9) assure the fraction of demand that are supplied by facilities are positive and these constraints are very obvious.

3. Scenario generation and reduction

In this section we proposed our solution methodology. The main aspect of proposed model is to estimate a set of efficient scenarios. each facility in this problem has a failure probability.

Based on these probability we generate scenarios and utilize the backward scenario reduction to reduce the number of scenarios. Detailed descriptions are proposed in next section.

Scenario reduction

In this paper we generate scenarios based on failure probability of each facility. As we mentioned before the total number of scenarios exceeds a large number and our problem cannot be solved by common methods. For this matter we should reduce the number of scenarios to solve this problem in effective time and cost. for this purpose we utilize the backward scenario reduction, proposed by Heitsch et al., the steps of this method are as follows:

**Step 0:** Compute the distances of scenario pairs:

\[ c_{ij} = c_i (R^i, R^j), \quad k; j = 1 \ldots S \]

**Step 1:** for each scenario compute the distances of scenario pairs:
\begin{align*}
    c_{ij}^{[m]} &= m_{ij} \cdot c_{ij}, \quad i = 1, ..., S \quad \text{and} \\
    x_{ij}^{[m]} &= y_i \cdot c_{ij}, \quad i = 1, ..., S \\
    \text{find} \quad l_k &= \arg \min_{l} z_{ik}^{[m]} \\
    \text{set} \quad j^{[0]} = \{l_k\}
\end{align*}

**Step 0:** for remaining scenarios compute:

\begin{align*}
    c_{ij}^{[m]} &= \min_{j \in j^{[m]} \cup \Omega} c_{ij} \\
    \text{for} \quad i \in j^{[m]} \cup \Omega, \quad k \in j \in j^{[m]} \cup \Omega \quad \text{and} \\
    z_{ij}^{[m]} &= \sum_{j \in j^{[m]} \cup \Omega} \pi_k c_{ij}^{[m]} \\
    \text{choose} \quad l_k &= \arg \min_{l} z_{ik}^{[m]} \\
    \text{set} \quad j^{[0]} = j^{[0]} \cup \{l_k\}
\end{align*}

**Step S-s+1:**

\begin{align*}
    \text{set} \quad j = j^{[s]} \quad \text{is the index set of deleted scenarios. Compute optimal probabilities for the preserved scenarios.}
\end{align*}

In this method we first calculate the distance between scenarios as shown in step 0 and we find the minimum distance as shown in step 1, these minimum distances are multiplied by probability of scenarios and finally the scenario with minimum value of \( z_i \) are eliminated. This sequence is done repeatedly until we can reduce the number of scenario to a desirable number.

**4. RESULT AND DISCUSSION**

We present the results obtained by the utilization of the proposed model to obtain the optimal allocation of a large supply chain in Iran. In this supply chain there are many warehouses and distribution centers.

In this paper the supply chain should deliver products to capital of each province. All required information and data about this supply chain are provided in appendix section.

The demand of the distribution are supplied by warehouses and manufacturers and we can determine the best allocation without considering any disruption. First of all we solve the problem without considering disruption and solve the problem to gain the optimal solution of this supply chain. Fig. 2 shows the optimal solution.

From Fig. 2 we can know the best allocation of our problem without considering any failure for facilities. For example the manufacturer number 7 supply the warehouses number 17 and 26 and directly provides demand of
distribution number 18 and 14. In other hand the warehouse number 17 provides the demand of distribution center number 13 and 24.

According to the failure probability of each facility we can calculate the imposed cost to supply chain as the result of manufacturers and warehouses failure. Table 1 shows the imposed cost because of each warehouse and manufacturer failure.

**Table 1** the imposed cost by failure in each facility

<table>
<thead>
<tr>
<th>Failed facility number</th>
<th>Type</th>
<th>Demand percent that supplied by this facility</th>
<th>Total cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original model without failure</td>
<td>--</td>
<td>--</td>
<td>2828609</td>
</tr>
<tr>
<td>1</td>
<td>manufacturer</td>
<td>11.3</td>
<td>3106872</td>
</tr>
<tr>
<td>7</td>
<td>manufacturer</td>
<td>51.7</td>
<td>4062888</td>
</tr>
<tr>
<td>10</td>
<td>manufacturer</td>
<td>10.3</td>
<td>3298050</td>
</tr>
<tr>
<td>16</td>
<td>manufacturer</td>
<td>16</td>
<td>3235631</td>
</tr>
<tr>
<td>20</td>
<td>manufacturer</td>
<td>10.8</td>
<td>3129888</td>
</tr>
<tr>
<td>17</td>
<td>warehouse</td>
<td>12.6</td>
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<td>22</td>
<td>warehouse</td>
<td>8.4</td>
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<tr>
<td>26</td>
<td>warehouse</td>
<td>6</td>
<td>2862867</td>
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</table>

As it is obvious the total cost of supply chain without any disruption is 2828609 and if one warehouse or one manufacturer fails this cost increases. For example if the manufacturer 1 that supplies %11.3 of whole demand of manufacturer fails the total cost increases from 2828609 to 3106872 unit.

Fig.3 show the best allocation if manufacturer 1 fails.

If the manufacturer 1 fails its distribution center should gain their demand from warehouse 17 to satisfy the demand of their customers.

Fig.4 also shows the optimal solution if the manufacturer number 7 fails. In this case the distribution number 14 should provide its demand from manufacturer 10. also warehouse 26 that supplied distribution 23 should gain its
demand from manufacturer 10. Fig (8-10) also show the optimal allocation is facilities 10, 16 and 20 and break in supply chain respectively.

5. Conclusion

This paper studied a problem of supply chain in tactical level that all manufacturers and warehouses are susceptible to failure, since the continuous product flow of this supply chain we should consider the scenarios that in each scenarios some facilities fails and their distribution centers should provide their demand from another facility with considering the cost and amount of demand. We proposed a mathematical model to find the optimal allocation of facilities and proposed the best allocation with consideration of each facility failure. The results of extensive computational tests indicated that the proposed method is both effective and efficient for a wide variety of problem sizes and structures.

6. REFERENCES


Appendix
The demand and failure probability of each facility are shown in Table 2

<table>
<thead>
<tr>
<th>Facility number</th>
<th>Fixed cost of facility</th>
<th>Demand of each facility</th>
<th>Probability of failure</th>
<th>Facility number</th>
<th>Fixed cost of facility</th>
<th>Demand of each facility</th>
<th>Probability of failure</th>
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