A New Image Denoising Algorithm Using Morphological Diffusion

Hamid Reza Moetamedzadeh1*, Nazanin Afrasiabi1, Mohammad Reza Hairi Yazdi2

1Graduate Student, University of Tehran, Tehran, Iran;  
2Associate Professor, School of Mechanical Engineering, University of Tehran, College of Engineering,  
Karegar St. North, Tehran, Iran

ABSTRACT

In this paper, a new algorithm for multivariate noisy environment is proposed. Anisotropic diffusion has a good performance for gaussian noise, but the response of filter is not satisfactory for nonlinear noise. In fact, the diffusion coefficient is a function of gradient magnitude and for nonlinear noise, ordinary gradient estimation fails. The morphological filters can be used for nonlinear noise reduction. The proposed method uses the morphological operators to preprocess the noisy image and uses the denoised image for gradient estimation. The technique proposed in this method enables the linear and nonlinear noisy environment to be well handled. Based on the experimental results, the proposed method achieves a good visual quality and PSNR in presence of multivariate noise.

KEYWORDS: denoising, diffusion, morphological filters, multivariate noise.

1. INTRODUCTION

Image denoising has a crucial role in the vision tasks. If the noise of image is not removed properly, the feature selection sections in computer vision systems may work improperly. In fact, the noise is detected as a feature and consequently the intelligent systems decision is down by wrong information and it could be so damaging. Image denoising is an attractive topic in image processing society [1]-[4]. There are many research works in this area since several decades ago. Noise reduction can be done either in frequency domain by filtering the noise frequencies or in the spatial domain by directly removing the noise components [5]. One of the best algorithms in spatial domain is based on the diffusion or heat equation. This algorithm diffuses the pixel intensity in the image domain based on the heat equation or linear diffusion equation [6]:

\[ I_t = c \nabla I, \quad I(0) = \text{noisy image} \quad (1) \]

where \( I \) and \( c \) is the denoised image and diffusion coefficient respectively and the noisy image is considered as an initial condition. The linear diffusion can be considered as Gaussian smoothing where the standard deviation of the Gaussian kernel is increased by the time. Hence, some features are blurred besides the noise. Perona-Malik(PM) [7] suggested to use non constant diffusion coefficient. They proposed a diffusion coefficient as a decreasing function of the image gradient:

\[ I_t = \text{div}(c(|\nabla I|)\nabla I) = \text{div}(c(x, y, t)\nabla I) \quad (2) \]

Here, \( c \) is a function of spatial and time coordinates and the diffusion is nonlinear. By this equation, the features with high gradient such as edges are preserved through the diffusion for linear noisy environment such as additive gaussian noise. It removes linear noise because of using ordinary gradient estimation. For nonlinear noises such as salt&pepper noise and speckle one, the ordinary gradient estimation does not work properly. In fact, the nonlinear noise can not be well distinguished by ordinary gradient estimation. One of the issues that should be considered in the denoising is the type of the noise that contaminates the image. There is the possibility that, one algorithm that well denoises the linear noises, fails in removing nonlinear noises and vice versa.

In general point of view, a good denoising algorithm should handle both linear and nonlinear noises and any combination of them. Here, an operator for gradient estimation in multivariate noisy environment with enough accuracy is needed. Morphological operators such as open-closing operator with flat structuring element removes the nonlinear noises like salt&pepper noise [8]. The performance of the Morphological filters in presence of linear noise for estimating the gradient is good enough. It can be used as a preprocessing step for applying the gradient operator to the image for calculating the diffusion coefficient. It sufficiently reduces the noise power specially the nonlinear noise. We utilize this characteristic of morphological filters to improve the performance of diffusion based denoising technique. In other words, we use two denoising algorithms. The morphological operators filter the image to better estimate the gradient magnitude in multivariate noisy environment for calculating the diffusion coefficient and simultaneously applying the diffusion equation to the image. The rest of the paper is organized as follows: Section2 discusses the diffusion based denosing method, the proposed algorithm is introduced in section3, experimental results are reported in section4 and the paper is concluded in section5.

*Corresponding author: Hamid Reza Moetamedzadeh, Graduate Student, University of Tehran, Tehran, Iran.  
E-Mail: Moetamedzadeh@ut.ac.ir; Mobile: 0098-9126080188
2. REVIEW

In this section, some traditional and state-of-the-art denoising algorithms based on diffusion idea are briefly reviewed.

2.1 PM Diffusion

As mentioned above, [7] suggested to use a non-constant diffusion coefficient which is the monotonically decreasing function of gradient magnitude:

\[
c(\|\nabla I\|) = \frac{1}{1 + \left(\frac{\|\nabla I\|}{k}\right)^2}
\] (3)

Where k is a smoothing threshold. The area of the image which has gradient magnitude lower than k is blurred. In the edge regions, image gradient has high value and therefore the diffusion coefficient in those regions based on (3) has lower value. Hence, the diffusion coefficient and consequently diffusion speed in that area is low and as a result, the edges are preserved during the diffusion process. While, in smooth regions the speed has high value because the gradient magnitude in that region is small.

2.2 ROF Method

Rudin, Osher, and Fatemi (ROF) [9] proposed to minimize the total variation norm of noisy image under a constraint that related to standard deviation of noise.

\[
\min_I \int_{\Omega} \|\nabla I\|dx dy \frac{1}{\|\nabla I\|} \int_{\Omega} (I - I_0)^2 dx dy = \sigma^2
\] (4)

Where \(I_0\) and \(\sigma^2\) are the noisy input image and the power of noise, respectively. In the original work, the noise is supposed to be additive gaussian with known noise power. Equation (4) can be solved using Lagrange multiplier and Euler equation [10]:

\[
I_t = \text{div}\left(\frac{1}{\|\nabla I\|}\nabla I\right) + \lambda (I_0 - I)
\] (5)

Where \(\lambda\) is Lagrange multiplier called fidelity term:

\[
\lambda = \frac{1}{\sigma^2} \int_{\Omega} \text{div}\left(\frac{1}{\|\nabla I\|}\nabla I\right)(I - I_0) dx dy
\] (6)

ROF similar to the PM method is gradient based method and hence preserves the edges if the ordinary gradient estimation works well.

2.3 Weickert Diffusion Tensor


\[
I_t = \text{div}(D(x, y, t)\nabla I)
\] (7)

Where D is diffusion tensor:

\[
D = \lambda_0(x, y, t) V_0 V_0^T + \lambda_1(x, y, t) V_1 V_1^T
\] (8)

\(V_0\) is the orientation of gradient and \(V_1\) is the orientation of isophot. By applying (7) to the image, the diffusion is occurred along the image and not across them. Hence, flow-like structures are preserved.

2.4 General Variational Model [12]

[12] proposed a general variational restoration model based on minimizing of a convex function of the gradient in normalized region \(\Omega = (0,1) \times (0,1) \subset \mathbb{R}^2\) under minimal growth conditions:

\[
-\text{div}(B(\nabla I)) + I = I_0
\] (9)

where

\[
B(u) = \frac{2 + \|u\|}{(1 + \|u\|)^2}
\] (10)

2.5 Directed Diffusion Equation [13]

[13] Discussed the use of a primary approximation of the original image in the diffusion equation:

\[
I_t = \eta_1 \text{div}(c(\|\nabla \ast G_{\sigma}\|, \nabla I)) + \eta_2 \text{div}(c(\|\nabla b \ast G_{\sigma}\|, \nabla b))
\] (11)
Where b is the primary approximation. They suggest to use wavelet denoising with soft thresholding to estimate b. The coefficient $\eta_1$ and $\eta_2$ avoid the edge smoothing. By using (11) the signal information that is lost in b will come back to the image.

3. Proposed Algorithm

All diffusion based methods mentioned in the previous section are gradient based methods. In fact, in these methods the gradient magnitude controls the diffusion coefficient and hence, has an important role in the denoising. They use ordinary gradient estimation which is appropriate choice for linear noises such as additive gaussian. For nonlinear noise, the ordinary gradient does not work properly. Fig.1 shows the results of denoising for two types of nonlinear (salt & pepper) and linear (additive gaussian) noises. As can be seen in this figure, the performance of denoising in case of linear noise is good while for nonlinear noise it is significantly decreased and the resulting image is not satisfactory. To overcome this issue we propose to use Morphological denoising (MD) as the preprocessing step for calculating the diffusion coefficient.

Fig2 shows the gradient estimation by ordinary method and the MD. As this figure confirms, in ordinary gradient estimation, noise components are wrongly detected and these points are vanished in MD gradient estimation. MD has a good denoising performance for nonlinear noise reduction and its performance for linear noise is good enough as a preprocessing step.

We use the combination of open-closing and close-opening as MD operators:

$$I_t = div(c(MD)VI)$$

(12)

Where

$$MD = \alpha (I \bullet se \circ se) + (1 - \alpha) (I \circ se \bullet se)$$

(13)

In this equation $\bullet$ and $\circ$ refer to closing and opening operators, respectively. $se$ is $3 \times 3$ flat structuring element. $\alpha$ is weighting coefficient that should be chosen in appropriate range $0 \leq \alpha \leq 1$.

![Fig1](image1.png)

(a) (b) (c) (d)

Fig1. The effect of type of noise on the performance of diffusion base denoising. a) image corrupted by nonlinear salt & pepper noise, b) denoised image, c) image corrupted by linear additive Gaussian noise, and d) denoised image.

![Fig2](image2.png)

(a) (b) (c) (d)

Fig2. Gradient estimation. (a) original image, (b) image contaminated by salt&pepper, (c) ordinary gradient estimation, and (d) MD gradient.

![Fig3](image3.png)

Fig3. PSNR curve as a function of $\alpha$. Input image is Lena corrupted by salt & pepper noise.
The proposed algorithm can be discretized as follows:

\[ I^{n+1} = I^n + dt (c_x I_x^n + c_y I_y^n + c I_{xy}^n) \]  

(14)

Where \( I^n \) indicate to image at \( n^{th} \) iteration. \( dt \) is the time step that must be chosen in the range \( 0 \leq dt \leq 0.25 \). \( I_x^n \), \( I_y^n \) and \( I_{xy}^n \) are the first and second derivative of image with respect to \( x \) and \( y \) coordinates at \( n^{th} \) iteration, respectively. \( c_x \) and \( c_y \) are first derivative of \( c \) with respect to \( x \) and \( y \) coordinates. \( c \) is calculated as follows:

\[ c (MD) = \frac{1}{1 + (MD / k)^2} \]  

(15)

Where MD is calculated by (13) at each iteration. The threshold \( k \) in (15) can be calculated as follows [7]:

\[ k = 0.9 \frac{1}{\Omega} \int_{\Omega} |\nabla I_0| \, dx \, dy \]  

(16)

In the next section the performance of the proposed algorithm is discussed.

4. Experiments

In this section some experiments are done to evaluate the performance of the proposed method. We use the Peak-to-peak Signal to Noise Ratio as a quantitative assessment:

\[ PSNR(dB) = 10 \log \left( \frac{1}{NxNy} \sum_{i=1}^{Nx} \sum_{j=1}^{Ny} (I(i, j) - I_{original}(i, j))^2 \right)^{1/2} \]  

(17)

The PSNR is reported in decibel. \( Nx \) and \( Ny \) are the number of pixels vertically and horizontally. To do the experiment we use Boat, Lena, and Peppers images. The images are artificially corrupted by the noise. Additive White Gaussian Noise (AWGN), speckle noise, and salt&pepper noise are used as linear and nonlinear noises. We set \( dt = 0.2 \) in (14).

<table>
<thead>
<tr>
<th>Method</th>
<th>IMAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Boat</td>
</tr>
<tr>
<td>weickert</td>
<td>24.02</td>
</tr>
<tr>
<td>PM</td>
<td>23.82</td>
</tr>
<tr>
<td>ROF</td>
<td>18.65</td>
</tr>
<tr>
<td>[12]</td>
<td>24.36</td>
</tr>
<tr>
<td>Proposed</td>
<td>24.79</td>
</tr>
</tbody>
</table>

Table 1 shows the quantitative comparison between methods mentioned in this paper. In this table, the normal variance for additive gaussian and multiplicative speckle noises is 0.01, the normal noise density for salt&pepper is 0.05. As can be seen, the ROF method has poor noise reduction in multivariate noisy environment. The PM method for Lena image has higher PSNR in comparison with weickert, while for boat and peppers it is vice versa. As the results in this table confirm, the proposed method has the highest PSNR for all three test images. Fig.4 illustrates the subjective comparison of best 4 methods in table1 where the input image is Peppers. Many isolated points remain in the image that is denoised by PM method. Some noise components appear in method [12]. The noise removing in the method [13] is better than [12] but the image is blurred. The image denoised by the proposed method removes the noise more than other methods while the image is not smooth.
Fig. 5 makes a visual comparison for Lena image. As can be seen in this figure, in method [13] the image is blur, in PM method the edges are strong but some isolated point still remains in the image, method [12] and the proposed method have the best visual quality.

5. Conclusion

A morphological diffusion for image denoising in multivariate noisy environment is proposed. It uses the morphological denoising operators to estimate more accurately the image gradient. To evaluate the proposed algorithm several experiments are performed. It outperforms other mentioned algorithms in PSNR sense and visual qualities.

REFERENCES


