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Integrated Lot Size Single Manufacturer Single Distributor for Product Sold with Warranty

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ABSTRACT

The determination of lot size production and economic order quantity is an important issue in inventory management. This study will develop models of the determination of joint economic lot size (JELS) for products sold with warranty, while the performance of manufacturer's production has decreased. In this study lot size production model and decreasing distribution demand model of Free-Replacement (FRW) were developed. The measure of performance in the developed model is aiming at minimizing the total cost of pre-sale and post-sale for each unit with the decision on variables of each cycle of the lot size production (Q). Numerical and sensitivity experiments were applied in this study.

KEYWORDS: chain supply, joint economic lot size, warranty product, inspection policy, sensitivity analysis

INTRODUCTION

The economic order quantity (EOQ) model was first introduced several decades ago to assist corporations in minimizing total inventory costs. It uses mathematical techniques to balance the setup and stock holding costs and derives an optimal order size that minimizes the longrun average cost [1]. In the manufacturing sector, when products are in-house made instead of being purchased from outside suppliers, the economic production quantity (EPQ) model is often used to deal with non-instantaneous inventory replenishment rate in order to obtain minimum production–inventory cost per unit time [2]. Due to the simplicity of EOQ and EPQ models, they are still broadly applied today. Many production–inventory models with more complicated and practical factors have been extensively studied (see for example, Jaber [3]; Wee and Shum [4]). The classic EPQ model implicitly assumes that all items made are in a perfect quality. However, in real world manufacturing systems, due to process deterioration and/or other factors, a generation of defective items is inevitable. Practically, these nonconforming items sometimes can be reworked. Hence, overall production costs can be reduced [1].

In the traditional inventory management among manufacturers and distributors, determining the optimal lot size is done only from the manufacturer or distributor side independently. This will lead to a distortion of information in supply chain networks that result in any liabilities on a party in the supply chain which requires an inventory management model that integrates multiple parties in the processes. Cooperation between producers and suppliers should be designed in accordance with the principles of chain management supply in order to benefit both parties. It determines the size of the production for the manufacturer or distributor for ordering measures must consider the common good and be dependent by minimizing the total cost of the combination of manufacture and distributor.

In traditional EMQ models, there are basic assumptions about production system: perfect and stationary. From this assumption, there is no means of production systems and that of continuous deterioration in producing the conforming items. However, this assumption may be invalid in practice [5]. Traditionally, production process starts in a controlled state to produce quality items. However, they would likely move toward out-of-control state that will result in defected items. An alternatively imperfect (deterioration) production process was developed to generalize the traditional EMQ models. Rosenblatt and Lee [6] developed a model of EMQ to producers with respect to product quality. They consider the influence of production process which experiences deterioration in the optimum length of the production cycle. Without considering the cost of restoration, it is shown that the optimum production cycle length is shorter than traditional EMQ models. However, the results will not be true if the high cost of restoration is considered. Jaber [3] investigated the lot sizing problem for reduction in setups, with reworks, and interruptions to restore process quality. He assumed the rate of generating defects to benefit from any changes to eliminate the defects, and thus they are reduced with each quality restoration action. A developed mathematical model and numerical examples provided with results are discussed later. Chakraborty et al.[7] investigated production lot size problem with deterioration process and machine breakdown under inspection schedule. For the reason that little attention was paid to the investigation of the joint effects of random defective rate and stochastic breakdown (under the NR inventory

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control policy) on economic replenishment run time decisions, this paper intends to fill in the gap within the EMQ formulation.

This study was developed based on Yeh et al. [5] model about products being sold with a warranty. However, Yeh et al. [5] did not consider the integration between producers and distributors. The focus on the development of this research is the integration between production decisions on the manufacturer to the distributor's decision to order the products sold under warranty. The purpose of this study is: 1. to produce models of the manufacturer-distributor inventory in the process of deterioration experience toward the products sold with warranty. 2. To conduct a sensitivity analysis of the model development to determine the effect of changes in the parameters of the model behaviour.

MATERIALS AND METHODS

Yeh and Chen [8] developed a mathematical model to incorporate the determination of optimal lot size and inspection policy about the production system whether it is experiencing deterioration when the product is sold by FRW. Because the system is experiencing deterioration, inspection schemes are proposed for the product as much as *K*, where *K* is the last product in the production of lot nonconform products inspected and found and corrected before the product is sold. This is done to reduce the cost of post-sale warranty. Then, Yeh et al. [9] developed a model of Yeh and Chen [8] by considering the influence of the free repair warranty on a periodic replacement policy for a product that could be improved. Cost model is formed from the product warranty but are not guaranteed, then it is connected to the optimal periodic replacement policy derived from minimizing the cost of long-cycle expectations. For products with an increased rate of damage functions, they are obtained from the structural properties of the optimal policy.

Sana [10] develops a model to determine the optimal product reliability and production rate that achieves the biggest total integrated profit for an imperfect manufacturing process. The basic assumption of the classical Economic Manufacturing Quantity (EMQ) model is that all manufacturing items are in a perfect quality, where in practices they often do not meet the perfect quality. Most production systems produce both perfect and imperfect quality items. In some cases the imperfect quality (non conforming) items are reworked at a cost to restore its original quality. Rework cost may be reduced by improvements in product reliability (i.e., decreasing in product reliability parameter). Lower value of product reliability parameter does not only result in increasing development cost of production but it also decreases quantity of nonconforming products. The unit production cost is a function of product reliability parameter and production rate resulting higher development cost and unit production cost. The problem of optimal planning work and reworking processes belongs to the broad field of production–inventory model which deals with all kinds of reused processes in supply chains. These processes aim to recover defective product items in such a way that they meet the quality level of 'good item'. The benefits from imperfect quality items are: regaining the material and value added on defective items and improving the environment protection.

Lu [11] developed a model of single vendor and buyer for a few different types of items. This model assumes a deterministic demand, and should be avoided. Shortage, booking lead time is constant and all buyers make a reservation at once. Delivery from suppliers to buyers can be done when the supplier has to have enough supplies, so one need not wait for the whole batch to finish (lot streaming). In this model the researcher determines the optimal buyer's order interval that can be combined to minimize the total cost. Pujawan and Kingsman [12] developed a supplier-buyer inventory model for an infinite time horizon. In this model the delivery to the buyer who wants to order is as much as *n* times. If the delivery is done in a number *q*, then the buyer's ordering lot is defined as *nq* and production lot as *mq*. They conducted a comparison between the model without lot streaming and with lot streaming for two different cases, namely: (i) if the decision is made for each party, and (ii) if the decision is made jointly. The obtained solution shows that a good synchronization between suppliers and purchasers in determining the frequency of delivery and production time will significantly result in savings to the total inventory costs.

For products sold with warranty, post-sale cost is closely related to the quality of products produced in the production process that has deterioration. Therefore, it is important to pay attention to the cost of the warranty in EMQ models to reflect the practical situation. Djamaludin et al. [13] consider lot size issues by including warranty costs in the calculation. In the model he assumes that the production process is modelled by a two-state discrete-time: (1) Markov chain and (2) product quality which are characterized into two distributions of damage. Regardless to the inspection, maintenance's and inventory's holding cost during the production cycle proposed cost model to get the optimum lot size to control warranty costs per item for products sold under free repair warranty (FRW). Yeh et al. [5] consider EMQ models for products sold with a warranty in the imperfect process, where the cost to serve a warranty claim (called a guarantee fee) can affect the optimal lot size large.

Inventory models such as the Joint Economic Lot Size (JELS), which integrates the management of inventory in the supply chain, have been done by Goyal [14]. His model assumes solutions generated from this model and it can provide significant savings on the combined total inventory costs. Banerjee [15] also discovered the supplier-buyer inventory model with lot for lot policy where suppliers produce each shipment to the buyer in a separate batch production.

MODEL DEVELOPMENT

Mathematical Model

In this model Q_d distributor's order number is deterministic and the amount to be produced by the manufacturer is n times the demand for distributor $(Q_P = n.Q_d)$. Production at the producer level is assumed fixed at P, where the production rate is greater than demand rate $(P> D)$. Most products are ordered by the distributor in each period.

Notation used in this model include:

D	: total demand (units / year)
S	: setup cost for the manufacturer on each setup ($/ setup)
A	: ordering cost of product for every order ($/ order)
r	: level of inventory handling costs are expressed as fractions.
P	: average production level of the manufacturer (units / year)
Cp	: unit production costs incurred by the manufacturer ($/ unit)
Co	: purchase price per unit paid by the distributor ($/ unit)
Cr	: cost of restoration on the production system by the manufacturer ($/ unit)
C_{mr}	: minimal repair cost of repair per unit by the manufacturer ($/ unit)
θ_1	: percentage defective in controlled conditions (in-control)
θ_2	: percentage of defective under conditions of control (out-of-control)
$q\ (Q)$	: proportion of products that do not qualify prior to inspection
$h_1(\tau)$	: hazard rate of qualified products (conforming item) with parameters $\lambda 1$ and $\beta 1$
$h_2(\tau)$	: hazard rate products that do not qualify (nonconforming item) with the parameter $\lambda 2$ and $\beta 2$
ω	: warranty period

These models assumed the process of production system performance decreased. The production process can have the status of the transfer "in-control state" to "state out-of-control." It is assumed that the in-control state, elapsed time, X, follows an exponential distribution with finite mean $1 / \lambda$. When the system moves to state out-of control, the production process continues until the end of the completed production process. After the production process is completed, the system will be set at a cost of $S> 0$.

In the state out-of control, the probability that the system generates an item is not eligible (non-conforming item). It is greater than the current system in a state in-control. To restore the state from out-of control to in-control required an additional fee of $Cr> 0$ for the next production process. Assumed that for all items produced are operational and can be classified into two types of eligible items (conforming items) and unqualified items (nonconforming items) depending on the performance of the item whether they are in accordance with the specifications or not. $h_1(t)$ and $h_2(t)$ are the hazard rates for conforming and nonconforming items by assuming h1 (t) <h2 (t) for $t \geq 0$. In a production system that will produce non-conforming item to the probability of θ_1, the system is in state in-control. While in the state out-of-control, production systems will result in the probability of non-conforming items θ_2 where $\theta_1<\theta_2$, products are sold with a minimum warranty repair during the warranty period ω where all costs are borne by the manufacturers with warranty claims. Manufacturers bear the cost of minimal repair of C_{mr}.

To get the expected cost of post-sale warranty, first calculate the expected number of unqualified items, N, while production for the time t is:

$$N = \theta_1 pt, \qquad \text{for } X \geq t,$$
$$= \theta_1 pX + \theta_2 p(t-X) \text{ for } X < t,$$

The expected value for N is

$$E(N) = \int_t^{\infty} \theta_1 pt\lambda e^{-\lambda x} dx + \int_0^t \left[\theta_1 px + \theta_2 p(t-x)\right]\lambda e^{-\lambda x} dx$$

$$E(N) = \theta_2 pt + p(\theta_1 - \theta 2)\frac{1-e^{-\lambda t}}{\lambda} \tag{1}$$

Thus, the expected number of conforming items in the production cycle length t, is $pt\text{-}E(N)$. The fraction of non conforming items, which are denoted q (t) in the production cycle, is

$$q(t) = \frac{E(N)}{pt} = \theta_2 + (\theta_1 - \theta_2)\frac{1 - e^{-\lambda t}}{\lambda} \tag{2}$$

The used type of warranty is free minimal repair warranty. A failed process of conforming items (or nonconforming item) is known as nonhomogeneous process with intensity $h_1(t)$ or $h_2(t)$. Expected number of minimal repair for conforming items (or nonconforming item) in the warranty period ω is $\int_0^{\omega} h_1(\tau)d\tau$ or $\int_0^{\omega} h_2(\tau)d\tau$. Thus, expectations of post-sale warranty cost per item are obtained as follows

$$W(t) = C_{mr}\left[(1 - q(t)\int_0^{\omega} h_1(\tau)d\tau + q(t)\int_0^{\omega} h_2(\tau)d\tau\right] \tag{3}$$

Thus, the total expected cost of the manufacturer each year can be modelled as follows:
$TC_{Manufacturer}$ = production cost + setup cost + holding cost + restoration costs + warranty costs.

$$TC_{Pd}(Q) = D.C_p + \frac{D}{n.Q_D}.S + D.Q_D(\frac{2-n}{2.P} + \frac{n}{D} - \frac{n+1}{2.D}) + \frac{C_r.D.(1 - e^{-\frac{\lambda.n.Q}{P}})}{n.Q} + C_{mr}.D.[(1 - q(Q))\int_0^{\omega} h_1(\tau)d\tau + q(Q).\int_0^{\omega} h_2(\tau)d\tau] \tag{4}$$

Where :

$$h(\tau) := \lambda^{\beta}.\beta.\tau^{\beta-1} \tag{5}$$

While the expectations of distributor cost per year is modelled as follows:
TC $_{Distributor}$ = purchase cost + ordering cost + holding cost + cost of product warranty administration.

$$TCD_t(Q) = C_o.D + \frac{D}{Q}A + \frac{Q}{2}\cdot r \cdot C_o \tag{6}$$

So that the total combined cost can be formulated as

$$JTC(Q) = D.C_p + \frac{D}{n.Q_D}.S + D.Q_D(\frac{2-n}{2.P} + \frac{n}{D} - \frac{n+1}{2.D}) + \frac{C_r.D.(1 - e^{-\frac{\lambda.n.Q}{P}})}{n.Q} + C_{mr}.D.[(1 - q(Q))\int_0^{\omega} h_1(\tau)d\tau + q(Q).\int_0^{\omega} h_2(\tau)d\tau] + C_o.D + \frac{D}{Q}A + \frac{Q}{2}\cdot r \cdot C_o \tag{7}$$

To determine the number of orders in the production for distributors and manufacturers for the optimal number, then the total expected cost is minimized by locating the first derivative of the function of *JTC (Q)* of *Q*.

$$\frac{d}{dQ}JTC(Q) = \frac{-D}{n\cdot Q^2}\cdot S + D\cdot\left[\frac{1}{2}\cdot\frac{(2-n)}{P} + \frac{n}{D} - \frac{1}{2}\cdot\frac{(n+1)}{D}\right] - \frac{D}{n\cdot Q^2}\cdot C_r\cdot\left(1 - \exp\left(-\lambda\cdot n\cdot\frac{Q}{P}\right)\right) + \frac{D}{Q}\cdot C_r\cdot\frac{\lambda}{P}\cdot\exp\left(-\lambda\cdot n\cdot\frac{Q}{P}\right) + D\cdot C_{mr}\cdot\left[\left(\theta_1 - \theta_2\right)\cdot\frac{\exp\left(-\lambda\cdot n\cdot\frac{Q}{P}\right)}{Q} - P\cdot\left(\theta_1 - \theta_2\right)\cdot\frac{\left(1 - \exp\left(-\lambda\cdot n\cdot\frac{Q}{P}\right)\right)}{\lambda\cdot n\cdot Q^2}\right]\cdot\left(\int_0^{\omega} h_2(\tau)\,d\tau - \int_0^{\omega} h_1(\tau)\,d\tau\right) - \frac{D}{Q^2}\cdot A + \frac{1}{2}\cdot r \tag{8}$$

Then, Q becomes optimal with $JTC'(Q) = 0$.

Theorem:

If the *JTC (Q)* is a function of the combined total cost as shown in equation 7, then there is a unique Q with $Q^* > 0$ and it generates minimum *JTC(Q)*.

Proof:

Define $g(Q) = Q^2\,[dJTC_1(Q)/dQ]$, then the function $g(Q)$ will have the same properties as $dJTC(Q)/dQ$ for $Q > 0$. Since $g(Q)$ is a continuous function, then the Q value substituting the close to 0, will be obtained:

$$\lim_{Q \to 0} g(Q) = -\frac{D}{n}\cdot S - (D.A) \quad .$$

Because the right hand side of equation is negative then it could be argued that $g(Q)$ has a negative value, whereas the Q value substituting the approaches infinity will be obtained: $\lim_{Q \to \infty} g(Q) = \infty$ or we can say that $g(Q)$ has a positive value to infinity.

Since $g(Q)$ is a continuous function, then the above conditions indicate that at least one function $g(Q)$ moves the sign from positive to negative and there is a value of Q so that $g(Q) = 0$. The same condition applies to the $dJTC(Q)/dQ$. This shows that Q satisfies the equation of unique $dJTC(Q)/dQ$ that exists.

To show that Q is an extreme minimum, then a sufficient condition to be fulfilled is the second derivative of the $JTC(Q)$ of Q namely greater zero.

Numerical Examples and Analysis

Parameters used in this analysis refer to numerical examples that exist in the model of Yeh et.al [5].

Notation	Value	Notation	Value
P	600 unit/year	θ_2	65 %
D	400 unit/year	λ	0,1
C_p	\$ 10 /unit	$h_1(\tau)$:	
S	100/setup	λ_1	1/36
C_o	\$ 20 /unit	β_1	2
A	\$ 15 /order	$h_2(\tau)$	
R	0.1	λ_2	1/12
C_r	\$ 200/restoration	β_2	2
Cp_{ins}	\$ 0.1/unit	C_{mr}	\$ 0.1 /unit
C_c	\$ 0.15/unit	Ω	1 tahun
θ_1	15 %		

The optimal value of combined lot size, and the total cost of combined $JTC(Q)$, is obtained by inserting the numerical example in Mathcad 2001i Professional software. Order size (Q) and production (nQ) in the JELS model or in an independent policy can be seen in Table 1 below:

Table 1 Numerical results of Model I

N	Condition	QP (unit)	QD (unit)	TCP (\$)	TCD (\$)	JTC (\$)
1	Joint	185.817	185.817	4290	8218	12508
	Independent	77.46	77.46	4556	8155	12711
2	Joint	263.5	131.75	4231	8177	12408
	Independent	154.919	77.46	4310	8155	12465
3	Joint	323.419	107.806	4209	8163	12372
	Independent	232.379	77.46	4237	8155	12392
4	Joint	374.111	93.528	4198	8158	12356
	Independent	309.839	77.46	4207	8155	12362
5	Joint	418.881	83.776	4192	8155	12347
	Independent	387.298	77.46	4194	8155	12349
6	Joint	459.427	76.571	4189	8155	12344
	Independent	464.758	77.46	4189	8155	12344
7	Joint	496.761	70.966	4188	8156	12344
	Independent	542.218	77.46	4190	8155	12345
8	Joint	531.543	66.443	4188	8157	12345
	Independent	619.677	77.46	4193	8155	12348
9	Joint	564.235	62.693	4188	8158	12346
	Independent	697.137	77.46	4199	8155	12354
10	Joint	595.172	59.517	4189	8160	12349
	Independent	774.597	77.46	4206	8155	12361
11	Joint	624.609	56.783	4190	8162	12352
	Independent	852.056	77.46	4214	8155	12369
12	Joint	652.744	54.395	4192	8165	12357
	Independent	929.516	77.46	4223	8155	12378
13	Joint	679.735	52.287	4194	8167	12361
	Independent	1007	77.46	4233	8155	12388

From Table 1 above, it shows that the sixth *n* reaches a combined total minimum cost. This happens in both joint and independent states. For more details on the curve, they can be seen in fig.1 below:

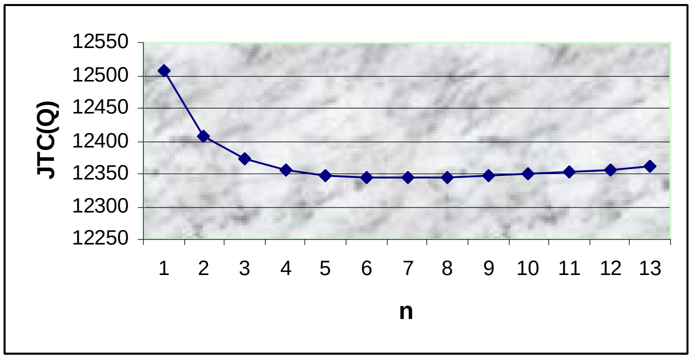

Figure 1. JTC curve (Q) for a model with a warranty JELS

SENSITIVITY ANALYSIS

Changes are made on the model parameters of JELS to analyse the behaviour of the model by making changes in the cost of restoration, repair and minimal cost of the warranty period with respect to Q and the total combined cost of JTC (Q).

Restoration Cost Changes

Table 2. Comparison of Q values and JTC (Q) for Different Restoration Costs

Cr	Initial value	5%	10%	20%
Q_S	459.426	459.462	459.492	459.564
Q_D	76.571	76.577	76.582	76.594
JTC(Q)	$ 12340	$ 12340	$ 12350	$ 12350

From Table 2, it shows that the greater the cost of restoration is, the greater the total cost incurred by producers will be. An increase in the cost of restoration will be responded by the manufacturer to reduce the frequency of restoration. A decrease in the frequency of restoration would result in increasing number of defects in production. With the increasing number of defects, then the proportion of nonconforming products, *q (Q)*, will tend to increase. This leads to warranty costs to be incurred and increased as well, because the products are sent to the distributor who includes a lot of nonconforming products. So it can be concluded that the restoration cost increases will result in an increase of the number of nonconforming products and warranty costs for producers, where in turn it will contribute an increase in total costs incurred by manufacturer.

For distributors, they should bear the total cost of increasing supplies. The increasing cost for the distributor is caused by the increase in the number of nonconforming products. As a whole, it is seen that an increase of restoration cost will result in the increase of total cost combined.

Changes in Minimal Repair Costs

Table 3 Changes in the value of C_{mr} with respect to Q * and *JTC (Q)*

Cmr	Initial value	$0.2	$0.5	$2
Q_S	459.426	459.42	459.42	459.306
Q_D	76.571	76.57	76.57	76.551
$JTC_1(Q)$	$ 12340	$ 12340	$ 12340	$ 12350

From Table 3, it shows that the greater the cost of minimal repair is, the greater the total cost incurred by producers will be. A minimal increase in the cost of repair will be responded by manufacturers to produce conforming products, so the number of lots that are produced will decrease. It aims to reduce the fraction of nonconforming products. By reducing the nonconforming product, it means the products deliver conform to the distributor who has a greater fraction. Therefore, the probability of warranty claim occurrence will be reduced. With this, the minimal repair cost incurred the decrease.

Changes in Warranty Period

Table 4. Changes in the value of ω to Q and *JTC (Q)*.

ω	Initial value	4	6	12
Q_S	459.426	459.336	459.21	458.538
Q_D	76.571	76.556	76.535	76.423
$JTC_1(Q)$	$ 12340	$ 12350	$ 12350	$ 12350

From Table 4, it shows that the longer the warranty period is, the total costs incurred by the producer will be even greater. Increasing duration of the warranty period will be responded by manufacturers to produce conforming products, so the number of lots that are produced will decrease. It aims to reduce the fraction of nonconforming products. By reducing the nonconforming product, it means the products deliver conform to the distributor who has a greater fraction. Thus, the probability of warranty claim occurrence will be reduced.

CONCLUSION

After the research has been done, some important conclusions can be drawn:

1. Retrieving order lot and production lot is unique and it minimizes the total expected costs combined with an optimum at 6 n. When parameter changes in the cost of restoration, minimal cost and a long period of warranty repair are done, it shows that three elements are important ones in determining the ordering and production lot.
2. Changes in the cost of restoration can be summed up lots so lots of production orders will be greater than the EMQ models when the cost of restoration is greater than the cost of the nonconforming product.

RECOMMENDATION

Recommendations can be given for further research to improve the research that has been done as follows:

1. Acceptance of sampling inspection policy to the manufacturer or the distributor.
2. Warranty policy of Pro Rate Warranty (PRW).

REFERENCES

1. Chiu,Yuan-Shyi P., Lin,Hong-Dar.,Chang,Huei-Hsin.(2011). *Mathematical modeling for solving manufacturing run time problem with defective rate and random machine breakdown*. Computers and Industrial Engineering.
2. Silver, E. A., Pyke, D. F., & Peterson, R. (1998). *Inventory management and production planning and scheduling*. New York: John Wiley & Sons, Inc..
3. Jaber, M. Y. (2007). *Lot sizing with permissible delay in payments and entropy cost.* Computers and Industrial Engineering, 52, 78–88
4. Wee, H-M., & Shum, Y-S. (1999*). Model development for deteriorating inventory in material requirement planning systems*. Computers and Industrial Engineering, 36, 219–225.
5. Yeh, R.H., Ho, W.S., Tseng, S.T.(2000) “Optimal production run length for products sold with warranty”, *European Journal of Operational Research* 120, 575-582
6. Rosenblatt, M.J., dan Lee, H.L., (1986). “Economic production cycles with imperfect production processes”, *IIE Transactions*., 18: 48-55
7. Chakraborty, T., Giri, B. C., & Chaudhuri, S. (2009). *Production lot sizing with process deterioration and machine breakdown under inspection schedule*. Omega, 37,257–271.
8. Yeh, R.H., Chen,T.H., (2006) “Optimal lot size and inspection policy for products sold with warranty”, *European Journal of Operational Research* 174, 766-776.
9. Yeh, R.H., Chen,M.Y., Lin,C.Y., (2007) “Optimal periodic replacement policy for repairable products under free-repair warranty”, *European Journal of Operational Research* 176, 1678-1686
10. Sana, Shib Sankar.(2010). A production–inventory model in an imperfect production process.European *Journal of Operational Research* 451-464.
11. Lu, L. (1995) “A one-vendor multi-buyer integrated inventory model” *European Journal of Operational Research* 81:312-323
12. Pujawan, I N., dan Kingsman, Brian G. (2002), “Joint optimisation and timing synchronisation in a buyer supplier inventory sistem”, *International Journal of Operations and Quantitative Management* 8:93-110
13. Djamaludin,I., Murthy D.N.P., Wilson R.J.(1994) “Quality control through lot sizing for items sold with warranty”, *International Journal of Production Economics* 33 : 97–107
14. Goyal, S.K. (1976) “An integrated inventory model for a single supplier – single customer problem” *International Journal of Production Research* 15:107-111
15. Banerjee, A., A joint economic-lot-size model for purchaser and vendor, *Decision Sciences,* 17(1986), 292-311